Effective Pedagogy in Mathematics/Pāngarau

Best Evidence Synthesis Iteration [BES]

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This report is one of a series of best evidence synthesis iterations (BESs) commissioned by the Ministry of Education. The Iterative Best Evidence Synthesis Programme is seeking to support collaborative knowledge building and use across policy, research and practice in education. BES draws together bodies of research evidence to explain what works and why to improve education outcomes, and to make a bigger difference for the education of all our children and young people.

Each BES is part of an iterative process that anticipates future research and development informing educational practice. This BES follows on from other BESs focused on quality teaching for diverse learners in early childhood education and schools. Its use will be informed by other BESs, focused on teacher professional learning and development and educational leadership. These documents will progressively become available at: [http://educationcounts.edcentre.govt.nz/goto/BES](http://educationcounts.edcentre.govt.nz/goto/BES)

Feedback is welcome at best.evidence@minedu.govt.nz

Note: the references printed in purple refer to a list of URLs in Appendix 2. These are a selection of potentially useful sources for teachers to engage more deeply with the range of issues raised in this best evidence synthesis iteration.
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About the writers

Glenda Anthony and Margaret Walshaw, both from the School of Curriculum and Pedagogy at Massey University, bring to this Best Evidence Synthesis (BES) decades of mathematics classroom teaching and educational research experience. They are acutely aware of the challenge that educators face in constructing a democratic mathematical community with which all students can identify. For them, making a positive difference to diverse learners’ outcomes is a central educational issue. At the heart of their work is a concerted effort to illuminate how this issue is best addressed. In this synthesis, they report on the outcome of their deliberations over, and search for, what makes a difference for diverse learners in mathematics/pāngarau.

Advisory Group

A core Advisory Group membership was selected to provide expertise and critique in relation to the various focuses of the BES, including Māori and Pasifika learners, early childhood, primary and secondary sectors, and teacher education. The authors wish to thank the members of this group:

- Dr Ian Christensen (Massey University and He Kupenga Hao i te Reo)
- Dr Joanna Higgins (Victoria University of Wellington)
- Roberta Hunter (Massey University)
- Garry Nathan (Auckland University)
- Dr Sally Peters (Waikato University)
- Assoc. Prof. Jenny Young-Loveridge (Waikato University)

We also wish to acknowledge the supportive formative feedback received from Faith Martin (Director, Massey Child Care Centre), Brian Paewai (Runanga Kura Kaupapa Māori), Professor Anne Smith (University of Otago) and Johanna Wood (Principal, Queen Elizabeth College, Palmerston North).

Ministry of Education advisory team

The Ministry of Education, led by Dr Adrienne Alton-Lee, has guided the development of the synthesis. The team at the Ministry also gave us access to additional literature and demographic and trend data. We thank all of the team.

External quality assurance

Professor Paul Cobb from Vanderbilt University, US, has provided invaluable assistance. We would like to acknowledge his scholarly critique and thank him for his knowledgeable contribution to the synthesis.

Formative quality assurance was also provided by: Maggie Haynes (Unitec), Professor Derek Holton (University of Otago), Tamsin Meaney (EARU, University of Otago), Lynne Peterson, Tony Trinick (Auckland University), initial and ongoing Teacher Education (Victoria University of Wellington), the New Zealand Educational Institute and representation from the Post Primary Teachers’ Association (Jill Gray). We wish to thank them all for their contributions.
Acknowledgments

The Ministry of Education extends special thanks to those who have contributed to different stages of this BES development through their participation in the BES Management Group. The advice and guidance from principal Diane Leggett, NZEI, and Judie Alison, Advisory Officer (Professional Issues), PPTA, have greatly strengthened this BES development. Particular thanks also to Robina Broughton and Linda Gendall, New Zealand Teachers Council, and to Ministry of Education colleagues.

The Chief Education Adviser acknowledges in particular the support and guidance provided by Malcolm Hyland and Ro Parsons through the partnership between BES and the Numeracy Development Project. The model of collaboration across research, practice and policy exemplified in that project has been an inspiration for the Iterative Best Evidence Synthesis Programme. Thanks to all those in the wider NDP community who have informed the BES development.

The Ministry of Education thanks Dr Fred Biddulph for his ongoing role in providing advice from the earliest formulation of the request for proposals through to a consideration of the final draft.

The Ministry of Education is indebted to Professor Bill Barton, Mathematics Education Unit, University of Auckland, for taking a proactive leadership role in bringing together teacher, teacher educator and research colleagues from across New Zealand to assist in scoping this BES at the outset.

Thanks for the deeply valued contribution made to the formative quality assurance and other advice offered to this BES development by Professor Paul Cobb, Vanderbilt University; Irene Cooper, Sandie Aikin, Cheryl Baillie and colleagues, NZEI; Jill Gray and Patrick McIntee, PPTA; Dr Mere Skerrett-White, Dr Maggie Haynes, Unitech, Dr Jo Higgins and colleagues at the Victoria University of Wellington College of Education; Dr Tamsin Meany, Professor Derek Holton, Lynne Petersen, Peter Hughes, Lynn Tozer and Michael Drake.

Thanks also for the valued participation of colleagues from initial teacher education institutions across New Zealand and Derek Glover, Secretary, New Zealand Association of Mathematics Teachers, in the formative quality assurance forum held for this BES development.

Thanks also for the significant contribution made to this and other BES developments through the advice given in the development of the Guidelines for Generating a Best Evidence Synthesis Iteration by the BES Standards Reference Group; The BES Māori Educational Research Advisory Group, the BES Pasifika Educational Research Advisory Group and Associate Professor Brian Haig, University of Canterbury.
Forewords

International

Even the casual visitor is struck by the dramatic changes that have occurred in New Zealand in the last 15 years. I have tuned in to local media on each of my four visits to get an initial sense of people’s current concerns and issues. Based on this narrow sampling, the New Zealand of 1991 was an immensely likeable country that had seen better days and was struggling to find its place in a rapidly changing world. Although innovation and experimentation appeared to be the watchwords of the day, there seemed to be an undercurrent of apprehension and anxiety as people attempted to cope with economic disruption. Today, New Zealand continues to be an immensely likeable place, but the visitor immediately notices a quiet, understated self-assurance. It has become a largely prosperous country that, in a very real sense, has reinvented itself as a leading information economy in an increasingly globalised world. Refreshingly for the visitor from the United States, there appears to be widespread belief that government will approach problems pragmatically and is capable of solving them. If the Iterative Best Evidence Synthesis Programme is representative of New Zealand government in action, this belief would appear to be well founded.

Put quite simply, the Iterative BES Programme is the most ambitious effort I have encountered that uses rigorous scientific evidence to guide the ongoing improvement of an education system at a national level. The programme has a strong pragmatic bent and is clearly grounded in the hard-won experience of synthesising research findings to inform both policy and teachers’ instructional practices. Four aspects of the programme are particularly noteworthy. The first is the overriding commitment to make the development of the best evidence syntheses transparent. This commitment takes concrete form in the exacting evaluation and feedback process that all BES reports undergo at each phase of their development, from the initial identification of relevant bodies of research literature through to the final critique and revision of the report. This is in the best traditions of science, where claims are justified in terms of the means by which they have been produced.

The second notable characteristic is a mature view of evidence and an emphasis on methodological and theoretical pluralism. This is important, given that attempts have been made in a number of countries, including the United States, to legislate what counts as scientific research in education on the basis of ideological adherence to a particular methodology. In taking an inclusive approach, the Iterative BES Programme acknowledges that different types of knowledge are of greatest use to teachers and to policymakers. Teachers make pedagogical decisions on the basis of a detailed understanding of specific students in particular classrooms at particular points in time. Policymakers, in contrast, typically need knowledge of trends and patterns that hold up across classrooms to make decisions that affect large numbers of students and teachers in multiple schools. Different methodologies are appropriate for developing these equally important types of knowledge.

The third noteworthy characteristic of the programme is its focus on the explanatory power and coherence of theories. Priority is given to theories that give insight into learning processes and the specific means of supporting their realisation in classrooms. This pragmatic criterion is important in a field where theoretical perspectives continue to proliferate.

The final notable characteristic of the programme is its explicit attention to the issues of language and culture. This emphasis is clearly critical if New Zealand teachers and policymakers are to address the inequities inherent in the disturbingly large gaps in school achievement between children of different ethnic and racial groups. In keeping with the tenet of methodological and theoretical pluralism, the Iterative BES Programme uses group categories such as socioeconomic status, ethnicity, and culture as key variables in assessing efforts to achieve
effective pedagogy must avoid stereotyping children of particular racial, ethnic, or language groups by acknowledging the complexity of individual identity when explaining inequities in children’s learning opportunities. Furthermore, the programme emphasises ecological models of learning that link what is happening in classrooms both to the institutional contexts in which classrooms are located and to issues of race, culture, and language. It is here that the full ambition of the programme becomes apparent: few viable models of this type currently exist in education. The BES writers are therefore charged with the task of synthesising in the true sense of the term, that is, to combine disparate and sometimes fragmented bodies of research into a single, unified whole. At the risk of understatement, this is a formidable challenge.

The writers of this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau, Drs. Glenda Anthony and Margaret Walshaw, have risen to the challenge. They were charged with the daunting task of reviewing, organising, and synthesising all mathematics education research from the early childhood years through secondary school that relates classroom processes to student learning. On my reading, the resulting synthesis of over 600 research studies is directly relevant to teachers and will be educative for policymakers. The educative value of the report stems from Anthony and Walshaw’s focus on what goes on in mathematics classrooms, thereby providing a window on the complexity of effective pedagogy. The forms of pedagogical practice that they identify as effective are ambitious because they involve high expectations for all children’s mathematical learning. The goals at which these forms of pedagogy aim are best illustrated in chapter 7, A Fraction of the Answer, in which Anthony and Walshaw pull together the key insights of the proceeding chapters as they present an integrated series of cases that focus on the learning and teaching of fractions. As this chapter makes clear, the instructional goals for fractions are not limited to ensuring that children can add, subtract, multiply, and divide fractions successfully. Instead, the instructional objectives also focus on children’s development of a deep understanding of fractions as amounts or quantities. At an elementary level, children who are coming to understand fractions as quantities know that 1⁄6 is smaller than 1⁄5 because there will be more pieces when something is divided into 6 pieces than into 5 pieces, so the pieces must be smaller. At a more advanced level, students will be able to describe real world situations that involve multiplying and dividing fractional quantities. More generally, ambitious pedagogy focuses on central mathematical ideas and principles that give meaning to computational methods and strategies.

Anthony and Walshaw’s review of the relevant research indicates that central mathematical ideas and principles cannot be directly transmitted to children. However, the research also shows that discovery approaches that place children in rich environments and simply encourage them to inquire are also ineffective. Effective pedagogy is complex because it requires teachers to achieve a significant mathematical agenda by taking children’s current knowledge and interests as the starting point. As Anthony and Walshaw clarify, these forms of pedagogy involve a distinctive orientation towards teaching. First and foremost, the emphasis is on building on students’ existing proficiencies rather than filling gaps in students’ knowledge and remediating weaknesses. As a consequence, the teacher’s focus when planning for instruction is not on students’ limitations but on their current mathematical competencies and interests, as these constitute resources on which the teacher can build. More generally, effective mathematical pedagogy places students’ reasoning at the center of instructional decision making. As a consequence, the ongoing assessment of students’ reasoning is an integral aspect of instruction, not a separate activity conducted after the fact to check whether goals for students’ learning have been achieved. A key characteristic of accomplished teachers is that they continually adjust instruction, as informed by these ongoing assessments.

One of the strengths of Anthony and Walshaw’s synthesis is that it provides the reader with a concrete image of what effective mathematical pedagogy looks like. Anthony and Walshaw emphasise that a respectful, non-threatening classroom atmosphere in which all students feel comfortable in making contributions is necessary but not, by itself, sufficient. As they document, the research findings indicate unequivocally that it is also essential that classroom activity
and discourse focus explicitly on central mathematical ideas and processes. The selection of instructional tasks is therefore critical. On the one hand, it is important that task contexts or scenarios are accessible to all students, regardless of cultural background. On the other, the teacher should be able to capitalise on students’ solutions to support their development of increasingly sophisticated forms of mathematical reasoning. Thus, when designing and selecting tasks, the teacher has to take account both of students’ current competencies and interests and their long-term learning goals. As Anthony and Walshaw discuss in chapter 5, an important way in which the teacher can build students’ solutions is by introducing judiciously chosen tools and representations. A second, equally important way in which the teacher can capitalise on the potential of worthwhile mathematical tasks is to engage students in justification, abstraction, and generalisation (see chapter 4), by doing which they learn to speak the language of mathematics.

The image of effective mathematical pedagogy that emerges from Anthony and Walshaw’s synthesis is of teaching as a coherent system rather than a set of discrete, interchangeable strategies. This pedagogical system encompasses:

- a non-threatening classroom atmosphere;
- instructional tasks;
- tools and representations;
- classroom discourse.

To see that these four aspects of effective pedagogy constitute a system, note that the way in which instructional tasks are realised in the classroom and experienced by students depends on the classroom atmosphere, the tools and representations available for them to use, and the nature and focus of classroom discourse. And because effective pedagogy is a system, it makes little sense to think of student learning as being caused by isolated teacher actions or strategies. It is for this reason that Anthony and Walshaw speak of mathematical learning being occasioned by teaching. In using this term, Anthony and Walshaw emphasise the teacher’s proactive role in supporting students’ development of increasingly sophisticated forms of mathematical reasoning.

In addition to highlighting the systemic character of effective mathematical pedagogy, Anthony and Walshaw make good on the charge to develop an ecological model of learning that links what is happening in the classroom to issues of race, culture, and language, and to the school contexts in which teachers develop and revise their instructional practices. A concern for issues of equity permeate the entire report but come to the fore in the discussion of school–home partnerships that take the diverse cultures of students and their families seriously and treat them as instructional resources.

Anthony and Walshaw make it clear that it is essential to view school contexts as settings for teachers’ ongoing learning. In a very real sense, these settings mediate the extent to which high quality teacher professional development will result in significant changes in teachers’ classroom practices. Anthony and Walshaw’s synthesis documents that mathematics instruction that places students’ reasoning at the center of instructional decision making is demanding, uncertain, and not reducible to predictable routines. The available evidence indicates that a strong network of colleagues constitutes a crucial means of support for teachers as they attempt to cope with these uncertainties and the loss of established routines. Consequently, there is every reason to expect that improvement in teachers’ instructional practices and student learning will be greater in schools where mathematics teachers participate in learning communities whose activities focus on central mathematical ideas and how to relate them to student reasoning. The value of teacher learning communities in turn foregrounds the critical role of the principal as an instructional leader.

Historically, teaching and school leadership have been loosely coupled, with the classroom being treated as the preserve of the teacher while school leaders managed around instruction. Recent research findings demonstrate the limitations of this type of school organisation
in supporting the improvement of teaching on any scale. These findings also indicate that principals can play a key role in supporting the emergence of a shared vision of what effective mathematical pedagogy looks like and in supporting teacher collaboration that focuses on challenges central to the development of effective pedagogy. This alternative type of school organisation is characterised by reciprocal accountability. Teachers are accountable to principals for developing increasingly effective pedagogical practices and principals are accountable to teachers to create opportunities for their ongoing learning. Changes of this type in the relations between teachers and school administrators are far reaching and might be viewed as too radical. It is, however, sobering to note that previous large-scale efforts to improve the quality of classroom instruction have rarely produced lasting changes in teachers’ practices. Research into educational leadership and policy indicates that this history is due in large part to the failure to take into account the institutional settings in which teachers develop and refine their instructional practices.

The broader policy and leadership literature strongly indicates that the improvement of mathematics instruction on the scale being attempted in New Zealand is not simply a matter of providing high quality teacher professional development. It also has to be framed as a problem for schools as educational organisations that structure the institutional settings in which teachers develop and revise their instructional practices. My reading of this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau is that Anthony and Walshaw have distilled valuable lessons from the available research, thereby positioning New Zealand educators to succeed where others have failed.

Paul Cobb
Professor of Mathematics Education
Vanderbilt University, Tennessee

Note: The second Hans Freudenthal Medal of the International Commission on Mathematical Instruction (ICMI) was awarded to Professor Paul Cobb in 2005, “whose work is a rare combination of theoretical developments, empirical research and practical applications. His work has had a major influence on the mathematics education community and beyond.”

**Early Childhood Education**

This Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau is a ‘must read’ for those in the early childhood sector who want an insight into what effective mathematical pedagogy looks like in an early childhood service. The synthesis acknowledges the vital role that quality early childhood education plays in the mathematical development of infants and young children. It also provokes early childhood teachers to reflect on practice: their mathematical awareness of the environment, the depth of their mathematical knowledge, and the importance of effective teaching and learning strategies that will support children’s optimal engagement in mathematical experiences. The extensive, wide-ranging research information is effectively balanced by vignettes which involve the reader in meaningful mathematical experiences that illustrate the possibilities for supporting mathematical learning. Effective distribution of the synthesis would enhance teaching and learning outcomes in early childhood services.

Faith Martin
Director, Massey Child Care Centre
NZEI Te Riu Roa

NZEI Te Riu Roa welcomes this Best Evidence Synthesis of Effective Pedagogy in Mathematics/ Pāngarau, particularly as it takes for its starting point the assertion that “all children can learn mathematics”. This key message is at the heart of every teacher’s commitment to the mathematical learning of his or her students.

The synthesis recognises the complexity of teaching, particularly given the diverse learning needs of the students in our classrooms and centres and the necessity for specialised knowledge of mathematics. But the writers consistently underline the power that teachers have to make a difference: “It is what teachers do, think and believe (that) significantly influences student outcomes.”

A teacher’s role, whether in a school or a centre, includes the design of activities that help students to construct meaning and think for themselves. To achieve such outcomes, teachers need to appreciate the part that mathematics plays in the world around them, what the big mathematical ideas are, and how the concepts that they teach fit in with those ideas. They need to know how to teach knowledge and skills, how to match new learning with students’ prior knowledge, and which activities effectively encourage understanding and learning. Teachers also need to be conscious of developing attitudes and values. They need to create opportunities for their students to develop a critical eye and, in the context of this synthesis, a critical mathematical eye.

The primary purpose of the synthesis is to identify evidence that links pedagogical practice with effective mathematics outcomes for students. To achieve this, the writers have drawn on national and international research that contributes to our understanding of what works in mathematics education.

When reviewing the synthesis in its draft form, NZEI teachers were particularly pleased to read the chapter, Mathematics Practices Outside the Classroom, which they saw as contributing to a constructive environment and encouraging of good practice. The synthesis explores ways in which parents can contribute to their children’s mathematical development and ways in which schools can strengthen links with the home. If teachers are to successfully fulfil expectations, such links are likely to be vital. Teachers were also pleased to see the importance of school leadership recognised.

NZEI sees the Effective Pedagogy in Mathematics/Pāngarau BES as being of great benefit to teachers, teacher educators, and policymakers. The research identified in the synthesis, together with the case studies and vignettes, has the potential to stimulate much constructive professional discussion. To maximise its potential for teachers, it will need to be accompanied by professional learning opportunities and time for reflection and discussion in the school or centre setting.

Irene Cooper
National President
Te Manukura
NZEI Te Riu Roa
Post Primary Teachers’ Association

Tēnā koutou, tēnā koutou, tēnā tatou katoa.

PPTA welcomes this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau. It is the result of a very thorough process, inclusive of the expertise of practitioners. The final report reflects and caters to their realities, and provides some very interesting and thought-provoking reading for teachers themselves, and for those involved in the pre-service and in-service education of mathematics teachers. At the same time, the research highlights the shortage of outcomes-linked research evidence specific to secondary school mathematics teaching and we hope that as a result of this BES, New Zealand researchers will step up to fill this gap.

Debbie Te Whaiti
President
New Zealand Post Primary Teachers’ Association

Teacher Educators

The Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau succeeds in providing a systematic treatment of relevant outcomes-based evidence for what works for diverse learners in the New Zealand education system. One of the strengths of the document is the central positioning given by its authors to a broad notion of diversity.

Teacher educators, both initial and ongoing, will find that the BES is an invitation to engage—as teachers and as researchers—with a wide range of national and international studies. The document succeeds in preserving the complexity of pedagogical approaches through careful structuring and presentation. Well chosen classroom vignettes capture the essence of pedagogical issues for use in initial and ongoing teacher education. The CASEs are likely to prove particularly valuable for teachers by demonstrating how research can inform classroom practice.

The BES also presents a challenge to New Zealand researchers by identifying areas in which there is a paucity of outcomes-based evidence. Such evidence is scarce for Māori-medium mathematics classrooms. The senior secondary area is generally not well represented and a wider range of early childhood contexts needs to be investigated. The CASEs highlight for teacher educators the possibilities of writing up research projects undertaken as part of ongoing teacher education initiatives, and encourage them to gather further evidence to support practice.

The importance to mathematics education of the outcomes-based research evidence represented in this synthesis cannot be overstated. It is to be hoped that the value of the Iterative BES programme is widely recognised, and that it has the impact on policy and practice that it ought.

Joanna Higgins
Director, Mathematics Education Unit and Associate Director,
Jessie Hetherington Centre for Educational Research
Victoria University of Wellington
For the last 20 years, the teaching of pāngarau (mathematics) has played a significant role in the revitalisation of te reo Māori. The Effective Pedagogy in Mathematics/Pāngarau BES recognises the close relationship that exists between language and the learning and teaching of mathematics.

The BES identifies a range of major considerations and challenges for teachers and all those involved in Māori-medium education. The research makes it clear that mathematical outcomes for students are affected by a complex network of interrelated factors and environments, not just individual preferences or the language of instruction. By identifying the key elements in this network and discussing the relevant research, the writers have created what should prove a very useful resource.

The BES highlights the paucity of research into Māori-medium mathematics education, particularly in the area of teacher practice.

Tony Trinick  
Māori-medium mathematics educator  
Faculty of Education  
The University of Auckland

The Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau has drawn together a comprehensive synthesis of evidence that relates to quality mathematics pedagogical practices. Its particular strength is that it provides stimulating and thought-provoking reading for a range of stakeholders and at the same time affirms that there is no one, specific, ‘quality’ pedagogical approach. Rather, it directs attention to many effective approaches which make a difference for all mathematics learners. The vignettes are an added strength; they make the theoretical structures they illustrate accessible to a wider audience.

The synthesis highlights the shortage of outcomes-linked research evidence concerning quality teaching and learning for Pasifika students at all levels of schooling. It also highlights the importance of a culture of care. How this translates into quality outcomes for Pasifika students requires the attention of New Zealand researchers.

Roberta Hunter  
Senior Lecturer  
School of Education Studies  
Massey University, Albany Campus
The Effective Pedagogy in Mathematics/Pāngarau BES sets out to uncover and explain the links between what we do in mathematics education and what the outcomes are for learners. The result is a valuable resource that can be used to enhance a wide range of outcomes for diverse learners. These include the ability to think creatively, critically, strategically and logically; mathematical knowledge; enjoyment of intellectual challenge; self-regulatory, collaborative and problem-solving skills; and the disposition to use, enjoy and build upon that knowledge throughout life.

The BES reflects the outstanding scholarly work and professional leadership of co-authors Drs Glenda Anthony and Margaret Walshaw of Massey University. They are the first to use the new Guidelines for Generating a Best Evidence Synthesis and follow the collaborative development process that is central to the Iterative BES Programme. They have consulted tirelessly and responsively with a wide range of early childhood teachers, primary and secondary teachers, principals, advisers, researchers, policy workers and teacher educator colleagues from across New Zealand, and with international colleagues. The Ministry of Education acknowledges and values all these contributions—and those of the formative quality assurers, whose affirmations and challenges have been so helpful in optimising the quality and potential usefulness of this BES.

The BES celebrates and returns to early childhood educators, teachers, teacher educators and researchers a record of their professional work, highlighting the complexity of that work, and suggesting how research evidence can be a valuable resource to inform their ongoing work and that of their colleagues. From the first vignette explaining how mathematical learning can be embedded in waiata (Māori song) and dance, the vignettes bring children’s learning in mathematics to life. The underlying explanations and theoretical findings have the power to inform practice in ways that are relevant and responsive to the learners in any particular centre or classroom.

The challenge now is for us all is to use this resource in ways that will support further systemic development in mathematics education, with strengthened outcomes for diverse learners. In many cases, the BES will affirm what is already happening, but it will be the points of challenge that take us forward. Individual teachers have already engaged with the BES in its draft form, and some report remarkable insights and developments in their practice. But it is only through the wider and systemic development of the conditions that support effective practice for diverse learners that improvements will proliferate and become self-sustaining. The findings emerging from the outcomes-linked professional learning and development BESs should be an invaluable resource in determining how to generate changed practice on such a scale.

Many teachers and early childhood educators have indicated that they want to read this BES for themselves, and to do this they need time. They need time to read, discuss and consider how they can use relevant BES findings in response to diagnostic information about the mathematical understandings of the children and young people they teach. They also need time to participate in professional learning communities. The Teacher Professional Learning and Development BES finds that such participation doesn’t guarantee better outcomes for students, but it is a consistent feature of teacher professional learning that does have a strong positive impact.

The same BES highlights the important role that external expertise with strong pedagogical content knowledge can play in facilitating and supporting changes in practice that impacts positively on student outcomes. Such expertise can be vital in engaging teachers’ theories and challenging problematic discourses. The findings do, however, caution that ‘experts’ need more than good intentions—in the worst-case scenario, teacher professional development can actually impact negatively on student achievement. This finding calls for careful and iterative evaluation of the effectiveness of all professional development.
The teacher education community in New Zealand has already made a foundational contribution to this BES with its engagement in the research and development reported in this BES, and its advice to the BES writers. As the Teacher Professional Learning and Development BES will show, some of our most effective professional development has been taking place as part of the Numeracy Development Projects (NDP)—with effect sizes twice those attained in England. The primary and early childhood teachers’ union, NZEI, confirms what the evaluation reports have been saying: that teachers who have been involved in the NDP value the transformational experiences this professional learning has afforded them. Two teachers from a Hawkes Bay school explained to me recently that, as a result of professional learning undertaken through the NDP, they have changed the way they work across the curriculum—they now listen more, are more diagnostic, and they place much more emphasis on children articulating and sharing their learning strategies. The dynamic, reflective, nation-wide learning community of researchers, teacher educators, teachers, and other educators created by the NDP and its Māori-medium counterpart, Te Poutama Tau, has been inspirational for BES.

If the mathematics BES is to serve New Zealand education well, the teacher education and research communities must make it a ‘living’ BES by building on the powerful insights and exemplars it makes available, addressing the gaps, and ensuring a cumulative and increasingly dynamic shared knowledge base about what works for learners in New Zealand education. To assist in this collaborative work, the New Zealand Council for Educational Research is creating a database of relevant New Zealand education theses. It has already built a database to support this document, with live links to the electronic version so that readers can quickly access either the full text or bibliographic details for some of the most helpful articles that have informed the synthesis. These links are also listed in the print version.

It is our hope that this BES will stimulate readers to let the Iterative Best Evidence Synthesis Programme know of other/new research and development that should feature in future iterations of the synthesis. Such research needs to clearly document demonstrated or triangulated links to student outcomes (see the Guidelines for Generating a Best Evidence Synthesis Iteration, found on the BES website), and preferably show larger positive impacts on desired outcomes for diverse learners. We are especially seeking studies of research and development in New Zealand contexts, but we are also interested in information on overseas studies that show particularly large impacts on diverse learners. Please send details to best.evidence@minedu.govt.nz.

In the New Zealand context, where schools and centres are self-managing, principals and centre leaders have a critical role to play in supporting their staff to realise the potential of this BES. The Teacher Professional Learning and Development BES indicates that, in the case of the most effective school-based interventions, principals and others in leadership roles have actively supported the development of a learning culture amongst their teachers.

For centuries, societies have required their education systems to sort children into successes and failures. Knowledge societies, such as our own, require much more. Our challenge is to ensure that all our children flourish as learners, strong in their own identities, and confident global citizens.

To achieve such goals, we need to value, build upon, and go beyond the craft practice traditions that require each teacher to ‘rediscover the wheel’. The Effective Pedagogy in Mathematics/Pāngarau BES has been designed to serve as a resource and catalyst for strengthened practice, innovation, and systemic learning. By using it, and by making learner outcomes our touchstone, we can work together to give our children a mathematics education that prepares them well for the opportunities and challenges that will be their future.

Adrienne Alton-Lee
Chief Education Adviser
Iterative Best Evidence Synthesis Programme
New Zealand Ministry of Education


3 Ibid.

4 Timperley et al., to be published 2007.


6 Timperley et al., to be published 2007.


8 http://educationcounts.edcentre.govt.nz/goto/BES
Authors’ Preface

What is a Best Evidence Synthesis in Mathematics?

A best evidence synthesis draws together available evidence about what pedagogical approaches work to improve student outcomes in Mathematics/Pāngarau. This synthesis is part of the Iterative Best Evidence Synthesis (BES) Programme established late in 2003 by the Ministry of Education to deepen understanding of what works in education. The programme involves policy, research, and practice in collaborative knowledge building, aimed at maximising desirable outcomes for the diverse learners in the New Zealand education system.

This best evidence synthesis in Mathematics/Pāngarau plays a key role in knowledge building for New Zealand education. As a capability tool, it identifies, evaluates, analyses, and syntheseses what the New Zealand evidence and international research tell us about quality mathematics teaching. It shows us how different contexts, systems, policies, resources, approaches, practices, and influences all impact on learners in different ways. Importantly, it illuminates what the evidence suggests can optimise outcomes for diverse mathematics learners.

The importance of dialogue

The development of this BES has been shaped by the Guidelines for Generating a Best Evidence Synthesis Iteration (Alton-Lee, 2004) and informed by dialogue amongst policy makers, educators, researchers and practitioners. Right from the very early stages of its development, the health-of-the-system perspective taken in this synthesis has ensured that we have listened to and responded to the viewpoints of a wide range of constituencies. Our interactions with these multiple communities have revealed to us the key roles that infrastructure, context, settings, and accountabilities play in a system that is functioning effectively for all its learners. Our various stakeholders have challenged us not only to produce better and more relevant educational research but to consider how this knowledge base might best be used. It is our hope that this discussion across sectors will be ongoing.

We have received a strong and positive response to the best evidence synthesis work from New Zealand’s primary and post-primary teacher associations. Both have reported on how helpful the synthesis is to their core professional work. For example, the New Zealand Educational Institute (NZEI) writes: “In our view, the writers have drawn on national and international research which contributes to an understanding of what works in mathematics education; they have identified the significance of the context and ways in which to strengthen practice … We liked the … underpinning view that all children can learn mathematics” (p. 2). The representative for the Post Primary Teachers’ Association at the Quality Assurance Day is reported as saying: “There are numerous wonderful ideas in the synthesis, and I found myself repeatedly jolted into possibilities for my own classroom resources.” In addition, a group of initial and ongoing mathematics teacher educators have welcomed the “sophisticated treatment of diversity” and the way in which “the complexity of pedagogical approaches is preserved” (Victoria University of Wellington College of Education, 2006, p. 1).

Writing for multiple audiences

Our task was to make the findings of the synthesis accessible to and of benefit to a range of educational stakeholders. At one level of application, it is intended to provide a strengthened basis of knowledge about mathematics pedagogical practices in New Zealand today. The evidence it produces is expected to inform teacher educators within the discipline of mathematics education about effective pedagogical practice. At another level, the synthesis attempts to make transparent to policy makers and social planners an evidential basis for quality pedagogical approaches in mathematics. At a third level, the synthesis is expected to benefit practitioners and assist them in doing the best possible job for diverse learners in their classrooms.
Our approach to the “almost overwhelming task” (Cobb, 2006) of writing with several levels of application in mind has been to draw on both formal and informal approaches. We have offset the ‘academic’ language of the BES by including a series of vignettes that expand upon broad findings. We have received feedback from a range of sources that these vignettes bring the reality of classroom life to the fore and, in particular, do not minimise the complexities of actual practice. We hope that researchers, policy makers and practitioners alike will see in the vignettes theoretical tools that have been adapted and used by actual teachers.

**The BES as a catalyst for change**

This best evidence synthesis in mathematics does more than synthesise and explain evidence about what works for diverse learners. By bringing together rigorous and useful bodies of evidence about what works in mathematics, the project plays an important function as a catalyst for change. It is designed to help strengthen education policy and educational development in ways that effectively address both the needs of diverse learners and patterns of systemic underachievement in New Zealand education. It is written with the intent of stimulating activity across practitioners, policy makers, and researchers and so to strengthen system responsiveness to educational outcomes for all students.

The writers anticipate that reflection on the findings will lead to sustainable educational development that has a positive impact on learners. It will create new insights into what makes a difference for our children and young people. Reflection on the findings will also spark new questions and renewed, fruitful engagement with mathematics education. These new questions, in turn, will render the BES a snapshot in time—provisional and subject to future change.

**Key features**

Key features of the BES are:

- Its teacher orientation. Its view is towards a strengthened basis of knowledge about instructional practices that make a difference for diverse groups of learners.
- Its cross-sectoral approach. Its scope takes in the teaching of children in early childhood centres through to the teaching of learners in senior secondary school classrooms.
- Its inclusiveness. It documents research that reveals significant educational benefits for a wide range of diverse learners. It pays particular attention to the mathematical development of Māori and Pasifika students and documents research that captures the multiple identities held by New Zealand learners.
- Its breadth of search coverage. It reports on the characteristics of effective pedagogy, following searches through multiple national databases and inventories as well as masters’ projects and theses. It provides comprehensive information about effective teaching as evidenced from small cases, large-scale explorations, and short-term and longitudinal investigations.
- Its local character. It makes explicit links between claims and bodies of evidence that have successfully translated the intentions and spirit of the Treaty of Waitangi. It identifies research relevant to the particular conditions and contexts in New Zealand, both in mathematics education in particular and in education in general, in relation to the principles and goals of *Te Whāriki* for early childhood settings and of *The Curriculum Framework*, for teachers in English or Māori-medium settings.
- Its global linkages. It connects local sources with the international literature. It identifies important Australian and international work in the area and evaluates that wide-ranging resource in relation to similarities and differences in cultures,
populations and demographics between the country of origin and New Zealand.

- Its responsiveness to concerns about democratic participation. It heeds the concern about the development of competencies that equip students for lifelong learning. This orientation coincides with the national mathematics curriculum objective of developing those knowledges, skills, and identities that will enable students to meet and respond creatively to real-life challenges.

- Its quality assurance measures. It is guided by principles of transparency, accessibility, relevance, trustworthiness, rigour, and comprehensiveness. These principles form the backdrop to the selection and systematic integration of evidence.

- Its strategic focus on policy and social planning. It uses a health-of-the-system approach to address one of the most pressing problems in education, provide a direction for future growth, and push effective teaching beyond current understandings.

- Its provisional nature. The project is an important knowledge-building tool, creating new insights from what has gone before, and will be updated in the light of findings from new studies. The findings are, above all, 'of the moment' and open to future change.

References


Executive Summary

The Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration (BES) was funded by a Ministry of Education contract awarded to Associate Professor Glenda Anthony and Dr Margaret Walshaw at Massey University. The synthesis is part of the Iterative Best Evidence Synthesis (BES) Programme, established by the Ministry of Education in New Zealand, to deepen understanding from the research literature of what is effective in education for diverse learners. The synthesis represents a systematic and credible evidence base about quality teaching in mathematics and explains the sort of pedagogical approaches that lead to improved engagement and desirable outcomes for learners from diverse social groups. It marks out the complexity of teaching and provides insight into the ways in which learners’ mathematical identities and accomplishments are occasioned by effective pedagogical practices.

The search of the literature focused attention on different contexts, different communities, and multiple ways of thinking and working. Priority was given to New Zealand research into mathematics in early childhood centres and schools, both English- and Māori-medium. Personal networks enhanced the library search and facilitated access to academic journals, theses and reports, as well as other local scholarly work. The New Zealand literature was complemented by reputable work undertaken in other countries with similar population and demographic characteristics. Indices, both print and electronic, were sourced, and the search covered relevant publications within the general education literature as well as specialist educational areas. In the end, 660 pieces of research, ranging from very small, single-site studies to large scale, longitudinal, experimental studies, found their way into the report.

Key findings highlight practices that relate specifically to effective mathematics teaching and to positive learning and social outcomes in centres/kōhanga and schools/kura. The findings stress the importance of interrelationships and the development of community in the classroom. They also reveal that both the cognitive and material decisions made by teachers concerning the mathematics tasks and activities they use, significantly influence learning. The findings demonstrate the importance of children’s early mathematical experiences and stress that constituting and developing children’s mathematical identities is a joint enterprise of teacher, centre/school, and family/whānau.

Key findings

In this section, key findings are organised and presented according to five themes: the key principles underpinning effective mathematics teaching, the early years, the classroom community, the pedagogical task and activity, and educational leadership and centre–home and school–home links.

Key principles underpinning effective mathematics teaching

Teachers who enhance positive social and academic outcomes for their diverse students are committed to teaching that takes students’ mathematical thinking seriously. Their commitment to students’ thinking is underpinned by the following principles:

- an acknowledgement that all students, irrespective of age, have the capacity to become powerful mathematical learners;
- a commitment to maximise access to mathematics;
- empowerment of all to develop mathematical identities and knowledge;
- holistic development for productive citizenship through mathematics;
- relationships and the connectedness of both people and ideas;
- interpersonal respect and sensitivity;
- fairness and consistency.
The early years

Young children are powerful mathematics learners. Quality teaching guarantees the development of appropriate relationships and support as well as an awareness of children’s mathematical understanding. Research has consistently demonstrated how a wide range of children’s everyday activities, play and interests can be used to engage, challenge and extend children’s mathematical knowledge and skills. Researchers have found that effective teachers provide opportunities for children to explore mathematics through a range of imaginative and real-world learning contexts. Contexts that are rich in perceptual and social experiences support the development of problem-solving and creative-thinking skills.

There is now strong evidence that the most effective settings for young learners provide a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive, play activities. Opportunities for learning mathematics typically arise out of children’s everyday activities: counting, playing with mathematical shapes, telling time, estimating distance, sharing, cooking, and playing games. Teachers in early childhood settings need a sound understanding of mathematics to effectively capture the learning opportunities within the child’s environment and make available a range of appropriate resources and purposeful and challenging activities. Using this knowledge, effective teachers provide scaffolding that extends the child’s mathematical thinking while simultaneously valuing the child’s contribution.

The classroom community

Research has shown that opportunities to learn depend significantly on the community that is developed within centres and classrooms. Thus, people, relationships, and classroom environments are critically important. Whilst all teachers care about student engagement, research quite clearly demonstrates that pedagogy that is focused solely on the development of a trusting climate does not get to the heart of what mathematics teaching truly entails. Teachers who truly care about their students have high yet realistic expectations about enhancing students’ capacity to think, reason, communicate and reflect upon their own and others’ understandings. Research has provided conclusive evidence that effective teachers work at developing inclusive partnerships, ensure that the ideas put forward by learners are received with respect and, in time, become commensurate with mathematical convention and curricular goals.

Studies have provided conclusive evidence that teaching that is effective is able to bridge students’ intuitive understandings and the mathematical understandings sanctioned by the world at large. Language plays a central role. Mathematical language involves more than vocabulary and technical usage; it encompasses the ways that expert and novice mathematicians use language to explain and to justify concepts. The teacher who has the interests of learners at heart ensures that the home language of students in multilingual classroom environments connects with the underlying meaning of mathematical concepts and technical terms. Teachers who make a difference are focused on shaping the development of novice mathematicians who speak the precise and generalisable language of mathematics.

Mathematics teaching for diverse learners creates a space for the individual and the collective. Whilst many researchers have shown that small-group work can provide the context for social and cognitive engagement, others have cautioned that students need opportunities and time to think and work quietly away from the demands of a group. There is evidence that some students, more than others, appear to thrive in class discussion groups. Many students, including limited-English-speaking students, are reluctant to share their thinking in class discussions. Research has also shown that an over-reliance on grouping according to attainment is not necessarily productive for all students. Teachers who teach lower streamed classes tend to follow a protracted curriculum and offer less varied teaching strategies. This pedagogical
practice may have a detrimental effect on the development of a mathematical disposition and on students’ sense of their own mathematical identity.

**Pedagogical tasks and activities**

From the research, it is evident that the opportunity to learn is influenced by what is made available to learners. For all students, the ‘what’ that they do is integral to their learning. The ‘what’ is the result of sustained integration of planned and spontaneous learning opportunities made available by the teacher. The activities that teachers plan, and the sorts of mathematical inquiries that take place around those activities, are crucially important to learning. Effective teachers plan their activities with many factors in mind, including the individual student’s knowledge and experiences, and the participation norms established within the classroom. Extensive research in this area has found that effective teachers develop their planning to allow students to develop habits of mind whereby they can engage with mathematics productively and make use of appropriate tools to support their understanding.

Choice of task, tools, and activity significantly influences the development of mathematical thinking. Quality teaching at all levels ensures that mathematical tasks are not simply ‘time fillers’ and is focused instead on the solution of genuine mathematical problems. The most productive tasks and activities are those that allow students to access important mathematical concepts and relationships, to investigate mathematical structure, and to use techniques and notations appropriately. Research provides sound evidence that when teachers employ tasks for these purposes over sustained periods of time, they provide students with opportunities for success, they present an appropriate level of challenge, they increase students’ sense of control, and they enhance students’ mathematical dispositions.

Effective teaching for diverse students demands teacher knowledge. Studies exploring the impact of content and pedagogical knowledge have shown that what teachers do in classrooms is very much dependent on what they know and believe about mathematics and on what they understand about the teaching and learning of mathematics. Successful teaching of mathematics requires a teacher to have both the intention and the effect to assist pupils to make sense of mathematical topics. A teacher with the intention of developing student understanding will not necessarily produce the desired effect. Unless teachers make good sense of the mathematical ideas, they will not have the confidence to press for student understanding nor will they have the flexibility they need for spotting the entry points that will move students towards more sophisticated and mathematically grounded understandings. There is now a wealth of evidence available that shows how teachers’ knowledge can be developed with the support and encouragement of a professional community of learners.

**Educational leadership and links between centre and home/school**

Facilitating harmonious interactions between school, family, and community contributes to the enhancing of students’ aspirations, attitudes, and achievement. Research that explores practices beyond the classroom provides insight into the part that school-wide, institutional and home processes play in developing mathematical identities and capabilities. There is conclusive evidence that quality teaching is a joint enterprise involving mutual relationships and system-level processes that are shared by school personnel. Research has provided clear evidence that effective pedagogy is founded on the material, systems, human and emotional support, and resourcing provided by school leaders as well as the collaborative efforts of teachers to make a difference for all learners.

Teachers who build whānau relationships and home–community and school–centre partnerships go out of their way to facilitate harmonious interactions between the sectors. There is convincing evidence to suggest that these relationships influence students’ mathematical development. The home and community environments offer a rich source of mathematical experiences on which to build centre/school learning. Teachers who collaborate with parents, families/whānau and
community members come to understand their students better. Parents benefit too: through their purposeful involvement in school/centre activities, by assisting with homework, and in providing suitable games, music and books, they develop a greater understanding of the centre’s or school’s programme. Their involvement also provides an opportunity to scaffold the learning that takes place within the centre or school.

**Overall key findings**

This Best Evidence Synthesis examines the links between pedagogical practice and student outcomes. Consistent with recent theories of teaching and learning, it finds that quality teaching is not simply a matter of ‘knowing your subject’ or ‘being born a teacher’.

Sound subject matter knowledge and pedagogical content knowledge are prerequisites for accessing students’ conceptual understandings and for deciding where those understandings might be heading. They are also critical for accessing and adapting task, activities and resources to bring the mathematics to the fore.

The importance of building home–community and school–centre partnerships has been highlighted in a number of studies of effective teaching.

Early childhood centre researchers have provided evidence that the most effective settings offer a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive, play activities.

Within centres and classrooms, effective teachers care about their students and work at developing interrelationships that create spaces for learners to develop their mathematical and cultural identities.

Extensive research on task and activity has found that effective teachers make decisions on lesson content that provide learners with opportunities to develop their mathematical identities and their mathematical understandings.

Studies have provided conclusive evidence that teaching that is effective is able to bridge learners’ intuitive understandings and the mathematical understandings sanctioned by the world at large.

**Gaps in the literature and directions for future research**

The synthesis provides research information about effective mathematics teaching. Although the scope of researchers’ studies is broad and far-reaching, a number of gaps in the literature are apparent. Research has so far provided only limited information about effective teaching in New Zealand at the secondary school level. Additionally, there is little reported research that focuses on quality teaching for Pasifika students. Few researchers in New Zealand are exploring mathematics in early childhood centres. The New Zealand literature lacks longitudinal, large-scale studies of teaching and learning. Also missing are studies undertaken in collaboration with overseas researchers. Such research is crucially important for understanding teachers’ work and the impact of curricular change. The scholarly exchange of ideas made possible through joint projects with the international research community contributes in numerous ways to the capability of our local researchers.

It is important to keep in mind that, as a knowledge building tool, the synthesis provides insights based on what has gone before. A snapshot in time, it is subject to change as new kinds of evidence about quality teaching become available. Important mathematics initiatives are underway in New Zealand schools and centres. The Numeracy Development Projects, new assessment methods, projects involving information technology, and a greater focus on statistics in the curriculum are just three examples of changes that are currently taking place. All new initiatives require research that monitors and evaluates their introduction and ‘take up’ by centres/schools and the changes in teaching and learning that take place as a result. Such research is necessary to guide future directions in schools, educational policy, and curriculum design.
This chapter is all about the way teachers arrange for learning within their classroom communities. It provides evidence of the sort of pedagogical arrangements that contribute to positive outcomes for diverse students. In it, we explore how teachers work at establishing a web of relationships within the classroom community with a view to developing their students’ mathematical competencies and identities. It is in the classroom community that students develop the sense of belonging that is essential if they are to engage with mathematics. It is within this community that the teacher creates a space for individual thinking and for collaborative mathematical explorations.

Of course, creating a caring learning environment for individuals and groups is not the only thing that effective teachers do in their classrooms. What they do is very much dependent, in the first place, on sound content knowledge. This chapter provides convincing evidence that teacher knowledge is a prerequisite for accessing and assessing students’ thinking. It is also critical if teachers are to provide responsive support directed towards structuring and advancing their students’ thinking. We look at the ways in which effective teachers scaffold learning through the development of classroom norms of participation, and we see how these are used as a starting point for developing productive mathematical experiences.

Enhancing academic and social outcomes is the objective of all quality teaching. We acknowledge the importance of the teacher’s explicit instruction in achieving this objective. Language plays a key part in shaping students’ mathematical experiences. Teachers who use mathematical language effectively use it for cognitive structuring. They also use student explanation and justification as a basis for their on-the-spot decision making about the best ways to advance student understanding.

**Mathematics teaching for diverse learners demands an ethic of care**

Teachers who create effective classroom communities care about student engagement. They demonstrate their caring in their relations with their students (Noddings, 1995), establishing a classroom space that is hospitable as well as ‘charged’ (Palmer, 1998). They do more than comply with “the politeness norm that dominates most current teacher discourse” (Ball & Cohen, 1999, p. 27). Teaching practice founded on an ethic of care takes pains to ensure that students do not develop a permanent dependency on their teachers. Instead, it works at developing interrelationships that open up spaces for students to develop their own mathematical identities, providing them with opportunities to ask why the class is doing certain things and with what effect (Noddings, 1995). Teachers who care work hard to find out what helps and what hinders students’ learning (Cobb & Hodge, 2002). Caring relationships are oriented towards enhancing students’ capacity to think, reason, communicate, reflect upon and critique what they do and say in class. Such relationships always involve reciprocity and a pedagogical attention that moves students towards independence (Hackenberg, 2005).

There is a substantial body of literature that reveals that teachers who care are those who identify, recognise, respect, and value the mathematics of diverse cultural groups (e.g., Goos, 2004). Caring about students from diverse cultural backgrounds requires teachers to move closer to their students, which carries with it the implication that teachers also have something to learn from their students (Perso, 2003). Mathematics teaching in kura kaupapa embraces the concepts, practices, and beliefs of te ao Māori (the Māori world) and is deeply committed to Māori culture and language (Macfarlane, 2004). Teachers in kura kaupapa Māori hold that “affectionate nurturing breeds happy hearts and lissome spirits and, thereby, warm and caring people” (Ministry of Education, 2000, p. 23). They maintain that such nurturing in a caring learning environment will contribute to positive lifelong futures. Mutual responsibilities...
are created in a caring, supportive environment as older children care for younger ones and assist in their learning activities.

Research by Stipek, Salmon, Givvin, Kazemi, Saxe, and MacGyvers (1998) found that “a positive affective climate that promoted risk-taking was positively associated with students’ mastery orientation, help-seeking and positive emotions associated with learning fractions” (p. 483). In their comprehensive study of 24 teachers’ practice in grades 4–6, the researchers compared the practices of non-traditional and traditional teachers. The teachers were videotaped for two or more episodes as they taught the same content. Three teacher practice categories were created from an analysis of the videotaped lessons: (a) learning orientation, (b) positive affect, and (c) differential student treatment. The following vignette illustrates what teachers in the research did to contribute to students’ cognitive and emotional development.

Promoting Student Development

In the study undertaken by Stipek and colleagues (1998) teachers who scored high on the Learning Orientation subscale conveyed to students that effort and persistence would pay off. In whole-class settings this orientation was demonstrated by the teacher staying with one student for a substantial length of time in an effort to get a clearer explanation, to provide alternative suggestions, or, in general, to make sure that the student understood the concept or problem. During student-work time this orientation was observed when the teacher encouraged students to keep working or thinking about a problem, gave them instrumental help that facilitated their progress, allowed plenty of time for students to complete their work, required students to go back and try again when they had reached inadequate solutions, or encouraged them to come up with multiple strategies.

Teachers who scored high on this subscale neither embarrassed students nor ignored wrong answers in whole-class instruction. Rather, they used students’ inadequate solutions and mistakes to enhance the instruction. They commented on the problem-solving process or the strategies students were employing, often making reference to the particular mathematical concepts that students were learning, and they held students to high standards, asking them to explain their thinking in writing as well as verbally.

Teachers with high scores on the Learning Orientation scale also fostered student autonomy by pointing out resources in the classroom, encouraging students to engage in self-evaluation, encouraging and accepting students’ own strategies for solving problems, and giving students choices in how they solved their problems or showed their work. They encouraged students to monitor their own work, set goals, plan their approaches, and move on to the next task without having to check with the teacher. Teachers scoring high on the Learning Orientation scale did not emphasise performance (e.g., receiving good grades), and they did not encourage students to avoid difficult tasks (e.g., by saying, “You’re not ready for those yet”).

Teachers who scored high on the Positive Affect subscale were sensitive and kind (without being artificially sweet). They showed an interest in what students had to say, listened to their ideas, avoided sarcasm or put-downs, and did not allow students to put each other down. The teachers appeared genuinely to like and respect their students. They also appeared to enjoy mathematics, and they made an effort to make mathematics problems interesting. The teachers conveyed that they valued all students’ contributions by, for example, calling on students having difficulties and pointing out what could be learned from mistakes. They never threatened students with “being called on” (and potentially embarrassed) as a means of ensuring their attention.

Teachers who scored high on both Learning Orientation and Positive Affect created an environment that promoted risk-taking which, in turn, contributed to students’ cognitive and emotional development.

From Stipek et al. (1998)
Relationship building

The development of self and others, principled upon an ethic of care, is not always evident in mathematics classrooms. Many Pākehā classrooms tend to undervalue the whakawhanau-ngatanga, or relationship building, that supports a caring and knowledge-producing community (Ballard, 2003; Bishop & Glynn, 1999). Bishop and Glynn speak of “a pattern of dominance and subordination and its constituent classroom interaction patterns (pedagogy) that perpetuates the non-participation of many young Māori people in the benefits that the education system has to offer” (p. 131). This pattern often emerges alongside a teacher’s low expectations for some students, relative to the expectations held for others. Jones (1986) quotes a teacher at an all-girls secondary school:

Some of these [Pacific Island] girls have expectations way too high. We get employers ringing up for shop assistants and they [students] don’t want the job! It’s very hard to get them to lower their sights. They shouldn’t have been so high on the first place ...” (p. 489)

Higgins (2005) looked specifically at the practices, and the beliefs underpinning those practices, of an effective mathematics teacher in an English-medium primary school classroom of mainly Māori students. In this classroom, the teacher used the metaphor of the waka to describe the class; the members of the class were all heading in the same direction, but different students had different talents and were able to do different things. The teacher also used the koru as a metaphor—to describe how the students were growing mathematically, were emerging in different ways, and were helping others to emerge more successfully than would be the case if they were learning alone.

Angier and Povey (1999) investigated the culture within one year 10 mathematics classroom. They provide evidence that students’ academic and social outcomes were greatly enhanced by the inclusive pedagogy of mathematics that this teacher had established. This was a culture that did not minimise individuals’ experiences and contributions within the classroom; nor were collective experiences downplayed. Participation in this classroom went hand in hand with students’ responsibility for themselves and for their own learning. The teacher provided students with opportunities to exercise that responsibility judiciously with respect to one another and to her. Students in the class commented:

She treats you as though you are like ... not just a kid. If you say look this is wrong she’ll listen to you. If you challenge her she will try and see it your way. (p. 157)

She doesn’t regard herself as higher. (p. 157)

She’s not bothered about being proven wrong. Most teachers hate being wrong ... being proven wrong by students. (p. 157)

It’s more like a discussion ... you can give answers and say what you think. (p. 157)

We all felt like a family in maths. Does that make sense? Even if we weren’t always sending out brotherly/sisterly vibes. Well we got used to each other ... so we all knew how to work ... We all knew how to work ... it was a big group ... more like a neighbourhood with loads of different houses. (p. 153)

Valuing students’ contributions

The value that is given to their thinking and their contributions influences the way in which students view their relationship with mathematics. Whitenack, Knipping, and Kim (2001) document how a teacher communicated the value of student effort and knowledge generated in individual, paired or whole-class activity. By validating contributions and asking further questions with the intent of allowing other students to access knowledge, the teacher used students’ ideas to shape instruction and to occasion particular mathematical understanding in the classroom. Bartholomew (2003), however, found that teachers do not always value students’
contributions equally. She found that mathematics teachers at a London school valued the experiences and contributions of top-stream students more highly than the experiences of other students. This evaluation was communicated to students in a range of subtle ways. With his low-stream class, a teacher was authoritarian in manner, “insisting that students queue outside the room in absolute silence and eventually counting them in and seating them alphabetically. They had to remain in their seats in silence, were given no opportunities to ask questions, with the result that many students were extremely confused as to what they were meant to be doing” (p. 131). By contrast, the teacher’s interactions with his top-stream class at the same year level were friendly and jocular. Within this culture, many boys appeared to thrive but most girls did not. Bartholomew found that whilst the girls were performing academically just as well as the boys, they perceived themselves as struggling with the work. Others in the class, too, came to believe in the girls’ poorer ability.

In their New Zealand Progress at School study, Nash and Harker (2002) illustrate how profoundly inequitable instructional attention can affect students. They found that teachers who distribute their attention differentially tend to offer less encouragement to students that they have stereotyped as ‘not mathematical’. One student in their study said: “Like when you ask the teachers you think, you feel like you don’t know, you’re dumb. So it stops you from asking the teachers, yeah, so you just try to hide back, don’t worry about it. Everyday you don’t understand, you just don’t want to tell the teacher” (p. 180). The same inclination to hold back from asking the teacher was expressed by secondary school students in a study by Anthony (1996):

Karen:  I honestly thought it was called a pictograph. I don’t want to say anything in case it is so far wrong I embarrass myself.

Lucy:  You feel a bit dumb asking questions. I sometimes ask, but if I got one wrong and the rest right I wouldn’t really worry.

Jane:  Some of the time I don’t understand the stuff enough in mathematics to answer questions ’cause I’ll probably get it wrong. I only answer questions if I know the answers.

Brooks and Brooks (1993) have observed that students’ unwillingness to answer a teacher’s questions (unless they are confident that they already know the sought-after response) is a direct consequence of the teacher’s use of questions. “When asking students questions, most teachers seek not to enable students to think through intricate issues, but to discover whether students know the ‘right’ answers” (p. 7). The caring teacher, on the other hand, uses power and influence legitimately (Bishop, 1988) to construct more equitable relationships through the discursive practices of the classroom. Hoyles (1982, p. 353) provides a classroom example from her research in which power relations are less than benevolent:

P:  … the teacher was always picking on me.

I:  Picking on you?

P:  Yes, and in one lesson she jumped on me; I wasn’t doing anything but she said come to the board and do this sum fractions it was. My mind went blank. Couldn’t do nothing, couldn’t even begin.

I:  What did you feel then?

P:  Awful, shown up. All my mates was laughing at me and calling out. I was stuck there. They thought it was great fun. I felt so stupid I wanted the floor to open up and swallow me. It was easy you know. The teacher kept me there and kept on asking me questions in front of the rest. I just got worse. I can remember sweating all over.

Boaler, William, and Brown (2000) traced the effects on students’ perceptions of mathematics as they moved from a year 8 to a year 9 class. Using questionnaires given to 943 students, interviews with 72 students, and about 120 hours of observations in classrooms, they found
that students in lower streamed classes had fewer instructional opportunities to learn. Teachers ignored their backgrounds and needs and addressed them in ways that exacerbated their difference from more able students. Instructional strategies in these low-stream classes were narrowly defined, resulting in profound and largely negative learning experiences. One secondary school student reflected on the previous year’s experience: “… in my primary school we weren’t in groups for like how good you are in certain subjects. We were just in one massive group, we did everything together. You’ve got some smart people and you’ve got some dumb people in the class, so you just blend in, sort of so you don’t have to be that good and you don’t have to be that bad” (p. 645).

Social nurturing and confidence building

“Caring and support is integrated into pedagogy and evident in the practice of teachers and students” (Alton-Lee, 2003, p. 89). Caring within the community of learners is fuelled by the values that underwrite classroom events. An increasing body of literature within mathematics education explains that the values espoused and taught in the classroom play a central role in establishing a sense of social and mathematical identity for mathematics students and significantly shape students’ level of engagement with mathematical activity. FitzSimons, Seah, Bishop, and Clarkson (2001), in their Values and Mathematics Project (VAMP), investigated the kinds of values that teachers in primary and secondary classrooms espoused and taught. The teachers maintained that they encouraged clarity, flexibility, consistency, open-mindedness, persistence, accuracy, efficient working, systematic working, enjoyment, effective organisation, creativity, and conjecturing. However, the researchers found that, on the whole, the values the teachers claimed they were implementing were actually not the ones witnessed in their classrooms. This finding, and other similar findings (e.g., Bana & Walshaw, 2003; Lin & Cooney, 2001), caution us that the way in which teachers describe their work does not always correspond to the way in which observers characterise that same practice. These findings point to the need to either substantiate teacher assertions or look beyond the ‘espoused’ forms of practice for evidence.

Social nurturing and confidence building are among the many important aspects of teaching. In a study by Waxman and Zelman (cited in Bishop, Hart, Lerman, & Nunes, 1993), one preservice teacher talked about the way in which her mathematics confidence was destroyed:

“... I hate maths … I recall heading a page of my maths book ‘funny sums’. For this I was sent to stand outside the class, told I was a ‘bloody imbecile’ and that I was the worst student he had ever had the misfortune to teach ...” (p. 27)

By way of contrast, we provide the following vignette of a teacher focused on social nurturing and confidence building. Students in a classroom study undertaken by Cobb, Perlwitz, and Underwood-Gregg (1998) had been investigating the problem: “How many runners altogether? There are six runners on each team. There are two teams in the race.”

**How Many Runners?**

<table>
<thead>
<tr>
<th>Teacher:</th>
<th>Jack, what answer – solution did you come up with?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jack:</td>
<td>Fourteen. How did you get that answer?</td>
</tr>
<tr>
<td>Teacher:</td>
<td>Fourteen. Because 6 plus 6 is 12. Two runners on two teams ... (Jack stops talking, puts his hands to the sides of his face and looks down at the floor. Then he looks at the teacher and then at his partner, Ann. He turns and faces the front of the room with his back to the teacher and mumbles inaudibly.)</td>
</tr>
<tr>
<td>Teacher:</td>
<td>Would you say that again. I didn’t quite get the whole thing. You had — say it again please.</td>
</tr>
<tr>
<td>Jack:</td>
<td>(Softly, still facing the front of the room.) It’s six runners on each team.</td>
</tr>
</tbody>
</table>
Once he realised that his answer was incorrect, Jack interpreted the situation as one that warranted acute embarrassment.

Teacher: (Softly.) Oh, okay. Is it okay to make a mistake? Andrew?
Andrew: Yes.
Teacher: Is it okay to make a mistake, Jack?
Jack: Yes.
Teacher: You bet it is. As long as you’re in my class it is okay to make a mistake. Because I make them all the time, and we learn from our mistakes, a lot. Jack already figured out, “Oops, I didn’t have the right answer the first time,” (Jack turns and looks at the teacher and smiles) but he kept working at it and he got it.

From Cobb, Perlwitz, and Underwood-Gregg (1998)

Social nurturing and confidence building was also investigated by Morrone, Harkness, D’Ambrosio, and Caulfield (2004). In their study of middle school students, they revealed that a caring culture within the classroom community contributed to the development of mathematical as well as rounded social identities. Morrone and colleagues found that care within the community of practice was related to an overall goal structure that included consistent affective support. This support conveyed the message that student ideas were valued. In turn, the positive support from teachers encouraged further student effort.

In the New Zealand context, Hyett (2005) explored how teachers created a caring community for diverse students in year 11 mathematics classrooms. One of the three teachers in the study noted: “To me it’s all about building relationships with my students ... with them having a positive approach to the subject then they develop the will and the desire and maybe the need at times to do what is required, not to please me but because they realise how far it gets them” (p. 23). Friendliness also means maintaining professional distance:

You can have a friendly relationship with them but at all times I am the teacher. And I’m quite strict about that too, that for my own self, I am a role model; I am a teacher. I need to earn their respect. It’s a two-way process. You have to model the behaviour that you want the kids to do ... I don’t have many rules in my classroom. The basic one of course is respect for me and other students and that if I’m talking then they’re not. (p. 24)

Christensen (2004) also provides evidence of teachers who have simultaneously developed students’ social and cognitive development. Te Poutama Tau teachers, working in an intensive professional development programme for Māori-medium classroom teachers, have improved students’ achievement in numeracy as they have been responsive to the goal of Māori language revitalisation. Like the English-medium Numeracy Development Project (NDP), the project is part of the Ministry of Education’s Numeracy Strategy, where the primary objective is to improve student achievement in numeracy through lifting teachers’ professional capability. Student engagement through active participation in mathematical discussion is advocated in Te Poutama Tau (Trinick, 2005). Teachers involved in the project are encouraged to assess the ways in which the social norms of classroom interaction influence student engagement levels.

In 2005 Trinick looked closely at the effects that the project had had on students. He reported a significant change over the duration of the project in terms of student attitude to Mathematics/Pāngarau. In particular, the students expressed a greater comfort level, an increased knowledge base, and higher confidence as a result of the way in which numeracy was taught. There are a number of possible explanations for this improvement. It can be explained by teachers’ enhanced mathematical understanding and resulting confidence, following their involvement
in the project. It might also be attributed to the structure that the project provides: the model for assessing students’ strategy and knowledge levels, and the suggested learning experiences and equipment that correspond to the different levels of development. Irwin (2005) reported similar findings from the Effective Numeracy Teaching in Years 1 to 6 (Pasifika Focus) project. This project involved 35 teachers from three schools and consisted of extra workshops for teachers and a redefined role for the numeracy leader or facilitator. This redefinition included allowance for additional teacher observations to be made at teachers’ request. The teachers involved reported a new empowerment to make changes in their practices.

Barton, Paterson, Kensington-Miller, and Bartholomew (2005) offer preliminary support for enhanced participation and achievement in mathematics from predominantly Māori, Pasifika and new immigrant senior secondary school students. The students involved in the five-year project are from eight schools in a low socio-economic area of Auckland. The Mathematics Enhancement Project (MEP) is driven by a care and concern about what mathematics can do for students who are economically and materially disadvantaged. Over 80% of the students involved are bilingual, and more than half of these understand at least three languages. The project works on four levels: teacher development, learning support for students, image enhancement of mathematics, and classroom-based research. “Coming to these sessions in groups, it’s made me look at what my students feel when they’re in the classroom. And I think initially … when somebody comes up with something, you’re taken aback a bit … Because anything that’s unknown is scary” (p. 83). The episode signals the crucial importance of establishing a caring community of practice, both for students and their teachers.

Caring about the development of mathematical proficiency

In her landmark study with low-attaining students, Watson (2002) found that teachers who care promote mathematical thinking and reasoning. Specifically, teachers in the study believed that students want to learn in a ‘togetherness’ environment, that students’ questions should propel teaching and learning, and that teaching should foster an awareness of learning. They believed that teaching should not offer students simplified tasks but should challenge them and provide support for them to task risks. The contrasting deficit model of student ability is, however, evident in some schools. For example, the eight teachers in Bergqvist’s (2005) research tended to underestimate their students’ reasoning ability: they believed that only a few students in a class were able to use higher level reasoning in mathematics. This underestimation, it was argued, may have strong consequences for students’ learning in classrooms. Hiebert et al. (1997) discuss a teacher who did not underestimate the mathematical reasoning powers of a student with Down’s syndrome. Just like the others in the group, this student took her turn to take on the role of teacher by sharing her word problem and asking questions. Although the problem was relatively easy, the group responded to the student with the same respect and intellectual support they had shown to other members of the group. One student, new to the group, who had called out “that’s easy!” was summarily put in her place by another who pointed out that it was not easy for this particular student. The teacher had surrounded the group with the resources of a caring community, intent on promoting the mathematical advancement of all members within the group.

Horowitz, Darling-Hammond, and Bransford (2005) offer the following vignette that describes what a first-year teacher did to organise a supportive, enriching environment for young mathematicians.
The Reams of Paper Problem

On a spring morning just before the last week of school, when many students are just biding time, Jean Jahr’s classroom of 28 second- and third-grade students is intently engaged in a mathematical investigation. A first-year teacher, Jean teaches at an elementary school. The multiracial, multilingual class of students is working in small groups on a single problem. Some children use calculators; others do not. Some have drawn clusters of numbers; others have developed a graphic display for their problem. As they finish, everyone takes their solutions with them as they sit on the carpeted meeting area facing the board. Jean begins by reading the problem with the group: In September, each person in classroom 113 brought one ream of paper. There are 500 sheets of paper in one ream. There are twenty-eight children in class 113. How many pieces of paper were there altogether?

She opens the discussion with an invitation, “Let’s talk about how different people solved the problem, and why you decided to solve it that way.” Over the next twenty minutes, students show, draw, and discuss seven different strategies they have used to solve the problem. Jane questions them to draw out details about their solution strategies and frequently recaps what students say. With patience and careful choice of words, she helps each member of the group understand the thought processes of the others. As the session nears its end, she asks if everyone understands the different solutions. Three children from one group seem in doubt and raise their hands. Jean asks one of the girls to come up and show “her way.” The teacher and the other children observe patiently, obviously pondering the girls’ thought process. Suddenly, Jean’s face lights up as she sees what they have done. Her response clarifies their work: “That’s how you did it! I was wondering if you had used tens groupings, but you had a totally different pattern. You started as if there were 30 children and then you subtracted the 1,000 sheets that would have been brought by the additional two children from the total number. You rounded to a higher number and then you subtracted. Wow. I get it. Let me see if I can show it to the others.”

The young girl is pleased when the teacher shows the group “her” system. When everyone seems clear, Jean asks, “Does anyone remember where this problem came from?” A girl raises her hand and says: “That was my problem a long time ago.” “You’re right,” her teacher responds. “You asked that problem during the first week of school when all of you were asked to each bring a ream of paper for the year. You saw all those reams of paper stacked up in front of the room, and you wanted to know how many sheets of paper we had. I told you that we would find out some day but that at that point in the year it was hard to figure it out because you had to learn a lot about grouping, and adding large numbers. But now you all can do it and in many different ways.” Another child recapitulates noting, “That means that we used 14,000 sheets of paper this year,” Jean says, “You got it!” The problem stays on the board for the day, along with the students’ multiple solutions.

This first-year teacher’s practice demonstrates that she understands how to organise a developmentally supportive classroom so that young children are productively engaged in meaningful work.

From Horowitz, Darling-Hammond, and Bransford (2005)

Mathematics teaching for diverse learners creates a space for the individual and the collective

Establishing norms of participation

Quality teaching facilitates students’ growing awareness of themselves as legitimate creators of mathematical knowledge. Yackel and Cobb (1996) make the important observation from their research that the daily practices and rituals of the classroom play an important part in how students perceive and learn mathematics. Students create ‘insider’ knowledge of mathematical behaviour and discourse from the norms associated with those daily practices. This knowledge evolves as students take part in the “socially developed and patterned ways” (Scribner & Cole, 1981, p. 236) of the classroom. By scaffolding the development of those patterned ways, the teacher regulates the mathematical opportunities available in the classroom. Cobb, Wood, and Yackel (1993) have found in their research that cognitive development begins with a taken-as-shared sense of the expectations and obligations of mathematical participation.
How and when does the teacher set up practices that will contribute to mathematical thinking? [Wood (2002)] researched six classes over a two-year period, investigating the patterns of interaction within the classrooms. From data collected on a daily basis during the first four weeks of school, Wood examined the ways in which the six teachers set up the social norms for classroom interaction. Further data were gathered at a later date to compare and contrast discursive interactions when the same instructional activity took place in different classrooms. Wood found variation in students’ ways of seeing and reasoning and these were attributed in the first place to the particular differences established in classrooms early in the year concerning when and how to contribute to mathematical discussions and what to do as a listener. The discourse principles propping up classroom participation regulated the selection, organisation, sequencing, pacing, and criteria of communication. Varying classroom expectations and obligations served to create marked differences in the cognitive levels demanded of the students and in the opportunities the students were given to engage in justification, abstraction, and generalisation. In the following vignette, Wood reveals how one teacher’s early expectations of how seven-year-olds would participate in class discussion became a reality within her everyday teaching.

### The 52 – 33 Problem

Teacher: What did you get for an answer for this problem?

Fred: 25.

Sara: 19.

Adam: 21.

Teacher: Any other answers? Okay. Fred tell us how you got 25.

Fred: We used the unifix cubes. 52 and then we took away 33. First I took away the tens. 42, 32, 22. Then I counted back the ones, 21, 20, 19.

Karen: But you said it was 25.

Fred: I know, but now I think it is 19, because I counted it again with the cubes.

Teacher: John, what do you want to say? (He has his hand raised).

John: I went back to 52 take away 30 is 22 (points to second problem on the paper). And I took away 3 more and that was 19. So I think it is 19.

Teacher: Okay. But why did you take away 3?

John: Because 52 take away 30 is 22, and 33 is 3 more than 30 so it was 19.

Teacher: How did you know that it was 19?

John: Because if 52 take 30 is 22, 52 take away 33 is 3 more than 30, so then I had to take away 3 more from 22, and that would be 19.

Teacher: That makes sense. Sarah, what would you like to say?

Sara: Well if you take 30 from 50, then you would have 20. Then you would have 2 and that 3, so you could take 1 from 20, and that would be 19.

Mark: This is too confusing for me. Sarah, I don’t understand why you took the 1 from 20.

Sara: Because you have 2 minus 3 and so you need 1 off the tens.

Mary: But if you took 1 from the 20, what happened to the 2 and the 3?

Sara: I took 1 from the tens and added it to the 2 to make 3. Then 3 minus 3 is 0. So then I had to take 1 from the tens–20, and that makes the answer 19.

Ryan: Well if you check it by adding 19 and 33, you get 52, so 19 is the answer.

Karen: I think the answer must be 19, because we did it so many different ways to figure it out. And we got 19.

Class: Agree. It is 19.

*From Wood (2002)*
Establishing participation processes and responsibilities for class discussion is an important pedagogical strategy. Classroom expectations and obligations concerning who might speak, when and in what form, and what listeners might do was the focus of a study by Nathan and Knuth (2003). One teacher’s classroom was studied over a two-year period. During the second year, the teacher worked at creating social norms surrounding behaviour and participation in mathematical discussion. More student-centred than in the first year of the research, the established discourse principles facilitated students’ participation in classroom interactions and ensured that students shared their thinking and listened attentively to each other. For example, the teacher told the class: “If you don’t have an opinion, will you try and get one so we can keep this [discussion] going a little longer” (p. 195). As a result of the changed norms of participation, there was a marked increase in student contribution. However, the researchers point out that the teacher’s focus was merely on changing the social norms of participation. Although the teacher sustained the interactions, it was with a view to keeping the conversation going rather than to nudge the conversation in mathematically enriching ways.

Students involved in the New Zealand Numeracy Development Project (NDP) have revealed that they like to be actively involved and they like to share their mathematical thinking with others (Young-Loveridge, Taylor, & Hawera, 2005). Students who were not involved in the NDP were less inclined to articulate the merits of listening to other students’ strategies. NDP students, on the whole, saw an advantage in solving challenging problems, explaining personal solutions to their peers, as well as listening to and trying to make sense of someone else’s explanations. Honouring students’ contributions is an important inclusive strategy. Yackel and Cobb (1996) found that classroom teachers who facilitate student participation and elicit student contributions, and who invite students to listen to one another, respect one another and themselves, accept different viewpoints, and engage in an exchange of thinking and perspectives, are teachers who exemplify the hallmarks of sound pedagogical practice. In their exploration into the teaching practice of one teacher, McClain and Cobb (2001) reported that although the teacher accepted all students’ ideas, it was an acceptance that did not differentiate between the mathematical integrity of those ideas. A pedagogical practice that does not attempt to synthesise students’ individual contributions (Mercer, 1995) does not advance mathematical thinking. These findings support the earlier theoretical work of Doyle (e.g., Doyle & Carter, 1984) on classroom participation, which found that teachers use the strategy of accepting all answers as a way of achieving student cooperation in an activity.

Some students, more than others, appear to thrive in whole-class discussions. In their respective research, Baxter, Woodward, and Olson (2001) and Ball (1993) have found that highly articulate students tend to dominate classroom discussions and tend to offer valuable insights to the mathematical conversation. Typically, low academic achievers remain passive; when they do participate visibly, their contributions are comparatively weaker and their ideas sometimes muddled. Quality teaching ensures that participation in classroom discussion is safe for all students that the norms of student participation and contribution are equitable.

From their research, Planas and Gorgorió (2004) illustrate the ways in which teachers sometimes unwittingly create inconsistent social norms of participation. The study was undertaken in Spain, investigating social interactions at the beginning of the school year in a secondary mathematics classroom with a high percentage of immigrant students. Unlike local students, immigrant students were not permitted to participate in mathematical argumentation and hence did not have personal experience of how participation could help clarify and modify thinking. The researchers observed the teacher’s ‘subtle, systematic refusal’ of immigrants’ attempts to explain and justify their strategies for solving problems. Instead of participating actively, the immigrant students were required to remain passive observers while local students explained their thinking. As Planas and Gorgorió report, the reduced social obligations and lesser cognitive demands placed on these students had the effect of excluding them from full engagement in mathematics and hence constrained their development of a mathematical disposition.
Differential access to knowledge and the production of a mathematical identity was also reported by Clark (1997) and by Boaler, Wiliam, and Brown (2000). From their longitudinal study involving six UK schools, Boaler and colleagues document that lower streamed classes followed a protracted curriculum and experienced less varied teaching strategies. This curriculum polarisation had a marked effect on the students’ sense of their own mathematical identity. The findings of these researchers are supported by other international research into the detrimental effects of such policies on the teaching and learning of students in lower streams (e.g., Gamoran, 1992; Slavin, 1990).

Working in groups

Many researchers have shown that small-group work can provide the context for social and cognitive engagement. Slavin (1995), in his meta-analysis of research on group learning, found a median effect size of .32 across studies using small groups of four heterogeneous members. These positive effects were characteristic not only of mathematics but also of other curriculum subjects. In another meta-analysis of the effects of small group learning in mathematics, Springer, Stanne, and Donovan (1997) found that such processes have significant and positive effects on undergraduates. The effect sizes recorded (.51), in fact, exceeded the average effect of .40 for classroom-based interventions on student achievement, as noted by Hattie, Marsh, Neill, and Richards (1997).

Thornton, Langrall, and Jones (1997) illustrate from a small study how classrooms organised for group work can provide a rich forum for diverse students to develop their mathematical thinking. They cite a study by Borasi, Kort, Leonard, and Stone (1993) in which a student who had a severe motor disability in writing, in addition to a ‘numerical’ disability, learned from his peers about how to share ideas and articulate his thinking. The student offered his explanation about a tessellating problem to the researcher and completed the recording task with support and questioning from the researcher over two days. As the researchers note:

This one-on-one work seemed really important and productive ... It was our hope that this experience would show [the student] what he could really do, and provide a model for the future; we do not expect him now to be able to do similar writing on his own yet, but perhaps he might be able to do it a second time around with less help, and gradually learn to do the same without the adult support. (p. 152)

Research has shown that gifted students, as well as low attainers, benefit from collaboration with peers. From a study involving six mathematically gifted students, Diezmann and Watters (2001) provide evidence that small homogeneous group collaboration significantly enhanced knowledge construction. Group participation also developed students’ sense of self-efficacy. In particular, collaborative work that was focused on solving challenging tasks produced a higher level of cognitive engagement than that produced by independent work. The supportive group provided a forum for the giving and taking of critical feedback and building upon others’ strategies and solutions. From their investigation, Diezmann and Watters claim that the positive effects of homogenous groupings for gifted students outweigh those offered through heterogeneous arrangements.

Doyle (1983) provides a theoretical grounding and empirical evidence for the ways in which effective groups operate. Successful group process depends on (a) the spatial configuration and interdependencies among participants, (b) how familiar the students are with the activity, (c) the rules established and the teacher’s managing skills, and (d) students’ inclinations to participate and their competencies. It is the teacher’s responsibility to ensure that roles for participants, such as listening, writing, answering, questioning, and critically assessing, are understood and implemented. Cohen (1986) maintains that “if the group is held accountable for its work, there will be strong group forces that will prevent members from drifting off task” (p. 17). Cohen’s extensive research into small-group effectiveness reveals that groups should be small in size. In particular, groups of four or five tend to be most effective. They should be
mixed in relation to academic achievement and any status characteristic. They need space for easy interactions and freedom from distractions.

The New Zealand NDP provides a teaching model that encourages both whole-class and small-group teaching. Through interaction with others within these teaching groups, the intention is that students will gradually develop the skills of and dispositions towards mathematically accepted ways of thinking and reasoning. Higgins (2005) has documented the way in which a teacher involved in the NDP used group organisation effectively as an instructional tool to enhance student engagement. The teacher set up the learning environment as a collective of groups working at varying levels of mathematical sophistication, each contributing to the overall class discussion and debate.

Groups provide opportunities to work with and learn from peers

Advocates of grouping claim that the organisational practice gives every student the opportunity to articulate thinking and understanding without every classroom eye and mind on what is being said. Wood and Yackel (1990) provide examples of how, in the course of working through problems with others, students extended their own framework for thinking. Benefits accrued as they listened to what their peers were saying and tried to make sense of it and coordinate it with their own thoughts on the situation. Whitenack, Knipping, and Kim (2001) also recorded the advantages for students when they explored the teacher’s role in sustaining and enabling classroom mathematical practices in a second grade classroom. They noted how the teacher recognised important aspects of the students’ interpretations as they worked individually, or with partners or in whole-class discussions. A collective conceptual shift was made by the class as they began to talk in ways that were similar to the quality explanations contributed by peers.

White (2003) found that students with limited English were more inclined to share their thinking with a friend rather than with the whole class. The teacher noted: “A lot of time they won’t share something with the whole group. But they will share it with somebody sitting next to them, or they can sometimes get ideas from other kids who are sitting next to them” (p. 42). Peers serve as an important resource for developing mathematical thinking and for finding out about the nature of task demands and how those demands could be met (Doyle, 1983). Quality teaching pays attention to the important fact that students’ willingness to contribute in the public arena of the classroom is influenced not only by the nature of the community established; it is also “affected by a student’s ability to function in social situations and interpret the flow of events in a discussion. For some students the social skills needed for classroom lessons are not necessarily fostered at home or other nonschool settings” (Doyle, p. 180).

Artzt and Yaloz-Femia (1999) have shown that collaborative activity within a small supportive environment allows students not only to exchange ideas but also to test those ideas critically. Through groups, they learn to make conjectures as well as learn to engage in argumentation and validation. Helme and Clarke (2001) found in their secondary school classroom study that peer interactions, rather than teacher–student interactions, provide opportunity for students to engage in high-level cognitive activity. These researchers stress the important role the teacher plays in establishing social rules governing participation. They point out the way in which those rules serve to regulate the way cognitive engagement is taken up and expressed.

Baxter, Woodward, Voorhies, and Wong (2002) explored student group processes in one classroom over a seven-month intervention period. Of the 28 students in the class, two were categorised as at-risk and one other received the assistance of a teacher aide. The intervention was focused on the academic development of these low achievers. The target students participated in different mixed-ability groupings during small-group discussions. The teacher aide’s key responsibility during these discussions was to provide support for the target students to actively participate in group discussions. Specifically, she ensured that they understood the problem and, where necessary, adapted the level of difficulty for them. She made sure that
they listened to the contributions of others, that they offered their own contributions, and that they could articulate the group’s strategy for solving the problem. Baxter et al. report that the students were exposed to a wide range of ideas, strategies and solution pathways from their more academically able peers. Their peers’ more advanced cognitive levels provided richer social-emotional as well as cognitive outcomes for the target students than would have been possible in a remedial classroom setting.

This same finding is reported in Alton-Lee (2003). In the *Quality Teaching for Diverse Students in Schooling: Best Evidence Synthesis*, Alton-Lee reports that the teachers in a study by Webb (1991) who set aside time to instruct students about the intricacies of effective group processes invariably enhanced students’ outcomes. Students who learned to help each other learned that to make the group work effective, communication and feedback within the group needed to be centred on mathematical explanations and justifications rather than on single answers to problems. Alton-Lee concludes that “Webb’s finding is particularly important because it shows the potential benefit to high achievers as well as low achievers” (p. 28).

Holton and Thomas (2001) have called two-way peer tutoring “reciprocal scaffolding” (p. 99). They note that students interacting need to have sufficient competence and experience to allow them to ask appropriate questions of themselves and each other. Rawlins (in progress) finds that although students feel comfortable asking the teacher for help, their first preference is to discuss mathematical problems with their peers. Year 12 students studying for the National Certificate of Educational Achievement (NCEA) reported in a group interview:

S1: We discuss how we arrived at our conclusion. For example, in algebra we talk about how we interpreted the application question and how we turned that information into a number problem.

S2: Normally if we have a question wrong we compare and look at the others working to find out where we went wrong. Then we discuss how we view the questions.

S3: Just what their answer was and how they got it. Normally we try to help each other out as much as we can.

Similarly, in a study of year 12 students learning calculus, Walshaw (2005) reports the same value placed by students in peer tutoring. As one student says:

Kate and I study a lot together and we help each other because she understands. She seems to understand a lot more of it than I do. I don’t get much of it at all this year. It’s just going straight past me. But she’s understanding it so she helps me out with that. We help each other and if she doesn’t understand it then I’d ring one of the guys who are friends. Martin would help. He got like 94% in School C. So I’d ring him. My older brother would help but he’s not hugely good with maths. I mean, he just scraped through last year, so he sort of helps me with what he can but he gets to a point where it’s beyond him and he can’t cope with it so I have to turn to friends who I know will be able to help me with it. And it’s fresh in their minds. (p. 26)

Goos (2004) documents the way in which one student views his interactions with peers in his senior mathematics class as an enriching learning experience:

Adam helps me … see things in different ways. Because, like, if you have two people who think differently and you both work on the same problem you both see different areas of it, and so it helps a lot more. More than having twice the brain, it’s like having ten times the brain, having two people working on a problem (p. 278).

The following vignette shows Adam guided the learning of Luke and helped him see his error.
Adam and Luke Supporting Each Other’s Learning

The students were investigating the iterative processes underlying fractals and chaos theory. The class had considered the example of the Middle Thirds Cantor Set, a fractal constructed by starting with a line of length 1, removing the middle third, then removing the middle third of the remaining segments and repeating this process infinitely many times. The point of the example was to prove that the sum of all lengths removed is equal to the length of the original line, a surprising and counterintuitive result. Students were then asked in a subsequent activity to find how much space is removed from the Middle Fifths Cantor Set. A common error made by many students was simply to substitute 1/5 for 1/3 in the proof provided in the worked example. The worked example for Middle Thirds Cantor Set was available to the students. Adam and Luke are investigating the problem.

Luke: It’s going to be a fifth instead of a third [pointing to example, no response from Adam].

Adam: It’s going to be a fifth [points to example].

Adam: Just think ... start, work through it from the beginning.

Luke: I am. [Writes.]

Adam: [Glances at Luke’s work.] Work through it! [Emphatically]

Luke: [Not looking up.] I am. [Looks up, puzzled.] What am I doing? [Checks example.] The size remaining’s right, isn’t it? [Adam looks at Luke’s work and chuckles.] That’s right!

Adam: OK, you just do it.


Adam: [Opens his own book and checks his working, grins at Luke.] Wrong!

Luke: [Expression of disbelief on his face, looks at his working.] How and where? I cannot see where it could possibly be wrong!

Adam: [Pauses, raises eyebrows, makes the decision to rescue Luke.] OK, explain this to me. Explain ... explain this to me [pointing to Luke’s working].

By way of contrast, Thomas (1994) explored the primary school classroom to find out what primary students actually say when they are engaged in classroom talk. In analysing the talk of 46 New Zealand primary school students during mathematics lessons, Thomas reported that “in the many hours of recorded and transcribed talk there were few instances of the children engaged in talk which could be directly linked with learning in the sense of a child obviously understanding something as a result of their talk with another child” (p.ii).

Peter-Koop (2002) provides evidence of students’ refusal to interact with others. In a study that explored group processes in third and fourth grade classrooms, the researcher demonstrated the difficulty that students can have in engaging with a new line of thought, given the distractions of group discussion. A New Zealand study undertaken by Higgins (1997) revealed that young students’ group work (new entrant to J2) was not as effective as their teachers believed it to be. Higgins showed that student explanations appeared to be constrained by the group process. In later research, Higgins (2000) demonstrated that teachers are often unclear about their role during student group work. However, when the mathematical intent of the group activity was articulated at the beginning and again during the feedback episode, and when student contributions were evaluated in terms of that intent, students appeared to engage more actively with the mathematics.

In her investigation into classroom group processes at the senior secondary school level, Barnes (2005) found that both poor social relationships and poor communication within groups contributed to limited student mathematical engagement in an activity. Barnes analysed video data to explore precisely who introduced ideas, the response of others, and and who controlled,
sustained, or impeded the discussion. She provides evidence that two students were frequently interrupted and their efforts ignored by others during group work. These ‘outsiders’ were assigned their position by others who did not recognise their rights to explain, question, or challenge. Barnes reports that these students learned less, and although the video data revealed that they offered the group distinctive mathematical insight, these students tended to lose confidence in their mathematical ability. Barnes suggests that pedagogical practice that regularly includes all students in group work reinforces the norms of careful and courteous listening.

**Individual thinking time**

A number of studies have provided evidence of the benefits for some students of independent learning approaches. For example, McMahon (2000) reported on successful individualised teaching in a Mathematics Recovery Programme targeting year 2 students. The two teachers involved had undertaken a year-long programme to develop purposeful instructional resources and strategies for their individualised teaching sessions. They focused the level appropriately, encouraged independent learning, and gave the students time to reflect on their thinking and methods. The success of the intervention depended crucially on the teachers’ sophisticated craftsmanship, which involved both anticipating and supporting students’ responses. The researchers report that the students increased their confidence in their ability and their mathematical understanding.

Walshaw (2004) reports on one student who advanced her learning more through independent thinking than through collaborative efforts with peers. A belief in the value of student collaboration to enhance mathematical learning was at the forefront of research undertaken by Sfard and Kieran (2001). The researchers report on two students, Gur and Ari, who were set a task by the teacher and expected to work together towards producing a solution. Classroom observations led the researchers to believe that the students were working together but further scrutiny revealed otherwise:

> While having a close look at a pair of students working together, we realised that the merits of learning-by-talking cannot be taken for granted. Our analyses compel us to conclude that if Gur did make any real progress, it was not thanks to his collaboration with Ari but rather in spite of it, and if this collaboration did, in the end, spur Gur’s development, it was probably mainly in an indirect way, by providing him with an incentive to learn. Our experiment has shown that the interaction between the two boys was unhelpful to either of them. The present study, therefore, does not lend support to the common belief that working together can always be trusted to have a synergetic quality. It is not necessarily true that two people who join forces can do more than the sum of what each one of them can do alone. (p. 70)

Sfard and Kieran showed how articulating mathematical thinking to oneself can have beneficial effects for the individual. These highly respected researchers conceptualise ‘talking to oneself’ as a form of communication and record from their research how an invisible and inaudible discourse with self creates mathematical thinking. They make the point that “interaction with others, with its numerous demands on one’s attention, can often be counterproductive. Indeed, it is very difficult to keep a well-focused conversation going when also trying to solve problems and be creative about them” (p. 70).

Students need some time alone to think and work quietly away from the demands of a group. Reliance on classroom grouping by ability may have a detrimental effect on the development of a mathematical disposition and students’ sense of mathematical identity. What effective teachers do is create a space for both the individual and the collective. They use a range of organisational processes to enhance students’ thinking and to engage them more fully in the creation of mathematical knowledge. More significantly, over and above establishing structures for participation, the effective teacher constantly monitors, reflects upon, and makes necessary
Mathematics teaching for diverse learners demands explicit instruction

Shaping students’ mathematical language

Quality teaching bridges students’ intuitive understandings and the mathematical understandings sanctioned by the world at large. Language plays a central role in building these bridges: it constructs meaning for students as they move towards modes of thinking and reasoning characterised by precision, brevity, and logical coherence (Marton & Tsui, 2004). The teacher who makes a difference for diverse learners is focused on shaping the development of novice mathematicians who speak the precise and generalisable language of mathematics. In order to enculturate their students into the mathematics community, effective teachers share with their students the conventions and meanings associated with mathematical discourse, representation, and forms of argument (Cobb & Yackel, 1996; Wood, 2002). Khisty and Chval (2002), among others, have reported that the language that students use derives from the language used by their teacher. The responsibility for distinguishing between terms and phrases and sensitising students to their particular nuances weighs heavily with the teacher, who profoundly influences the mathematical meanings made by the students.

Competency in mathematics allows the student to understand discussions about mathematics, to learn the subject, and to comprehend the mathematical way of speaking. It demonstrates control over the specialised discourse (Gee & Clinton, 2000). But the specialised language of mathematics can be problematic for learners. Particular words, grammar, and vocabulary used in school mathematics can hinder access to the meaning sought and the objective for a given lesson. Words, phrases, and terms can take on completely different meanings from those that they have in the everyday context. Walkerdine (1988), for example, has reported the difficulties young students encounter in establishing the mathematical meaning of ‘less than’ and ‘more’, given their idiosyncratic meanings within specific home practices. Similarly, Christensen (2004) has provided evidence that, in some cases, mathematical words can assume quite different meanings in everyday life. He documents the difficulties encountered by Te Poutama Tau students in using mua and muri for number sequencing, and in using whakamua and whakamuri for forward and backward directional counting.

Developing this point for Australian students, Sullivan, Mousley, and Zevenbergen (2003) found that students with a familiarity of standard English (usually children from middle-class homes) had greater access to school mathematics. As the teachers in their study said, the students were able to ‘crack the code’ of the language being spoken. One teacher of students from non-English-speaking backgrounds makes the point about meanings of words: “[Y]ou need to reinforce: ‘Tell me what I mean when I say estimating?’ or ‘Where are some things that you estimate?’ Ground it in their world because for a child for whom English is not their first language, if there are numbers they’ll be right, but if you say ‘estimating’ they won’t have a clue what that might mean” (p. 118).

Initiating and eliciting

A fruitful approach, aimed at clarifying descriptions and explanations, is for the teacher to purposefully provide information or ask questions. Lobato, Clarke, and Ellis (2005) refer to this as an aspect of teacher ‘telling’ (p. 102). The form of ‘telling’ that Lobato, Clarke, and Ellis advocate is one that facilitates learning by initiating student reflection on the concept and on the process. The approach is directed at developing students’ conceptual knowledge rather than their memory skills. This form of telling does not take away from students the agency for making sense of mathematics (Hiebert & Wearne, 1993). Lobato, Clarke, and Ellis (2005)
develop the pedagogical approach further. They expand on teacher telling by describing two strategies. The first, they call *initiating*, by which they mean a group of actions and behaviours used to introduce new ideas that function as prompts for the way in which students construct mathematics. The purpose is conceptual: it is not aimed at showing procedural steps, rather, its intent is to shape ideas and make connections between ideas in a coherent and sensible fashion. The manner in which this is done includes the use of symbols, images and graphics, the summarising of student work and adding in of new material, counter-examples, questioning, and the providing of new representations. Lobato, Clarke, and Ellis differentiate this approach from a second strategy, which they call *eliciting*. Following on from initiation, the teacher *elicits* information to determine how the students have interpreted the mathematical concept.

Initiating and eliciting pedagogical strategies are illustrated in studies by Turner and colleagues (1998, 2002). These researchers found that what distinguished high-involvement year 5 and 6 classrooms was the engagement of the teachers in forms of instruction focused on shaping students’ understandings. In particular, teachers negotiated meaning through ‘telling’ tailored to students’ current understandings. They shared and then transferred responsibility so that students could attain greater autonomy. They also fostered intrinsic motivation by sparking curiosity and by supporting students’ goals. In these classrooms, telling was followed by a pedagogical action that had the express intent of finding out students’ understandings and interpretations of the given information.

**Multilingual contexts**

A number of studies have investigated the challenges of teaching mathematics in multilingual contexts. Adler (2001), Khisty (1995) and Moschkovich (1999), for example, have all studied the tensions that arise in multilingual classrooms between mathematics and language, and have explored the teacher’s role in this relationship. Neville-Barton and Barton (2005) looked at these tensions as experienced by Chinese Mandarin-speaking students in New Zealand schools. Their investigation focused on the difficulties that could be attributable to limited proficiency with the English language. It also sought to identify language features that might create difficulties for students. Two tests were administered, seven weeks apart. In each, one half of the students sat the English version and the other half sat the Mandarin version, ensuring that each student experienced both versions. There was a noticeable difference in their performances on the two versions. On average, the students were disadvantaged in the English test by 15%. What created problems for them was the syntax of mathematical discourse. In particular, prepositions, word order, and interpretation of difficulties arising out of the contexts. Vocabulary did not appear to disadvantage the students to the same extent. Importantly, Neville-Barton and Barton found that the teachers of the students in their study had not been aware of some of the student misunderstandings.

Like the students in the study undertaken by Neville-Barton and Barton (2005), Pasifika students in Latu's (2005) research had difficulty with syntax. Word problems involving mathematical implication and logical structures such as conditionals and negation were a particular issue for students from senior mathematics classes. They also found technical vocabulary, rather than general vocabulary, to be problematic. Latu noted that English words are sometimes phonetically translated into Pasifika languages to express mathematical ideas when no suitable vocabulary is available in the home language. The same point was made by Fasi (1999) in his study with Tongan students. Concepts such as ‘absolute value’, ‘standard deviation’, and ‘simultaneous equations’ and comparative terms like ‘very likely’, ‘probable’, and ‘almost certain’ have no equivalent in Tongan culture, while some English words, such as ‘sikuea’ (square), have multiple Tongan equivalents. The suggestion is that special courses in English mathematical discourse be delivered with the express intent of connecting the underlying meaning of a concept in English with the students’ home language.

Other pedagogical strategies for multilingual classrooms have been advocated by Cohen (1984). Cohen has found that small-group work can assist in overcoming potential and real
language difficulties. Students with limited English proficiency need to hear their peers use mathematical language. The most effective way of doing this is to place students in groups with English speakers and a proficient bilingual student. To ensure the effectiveness of the group and the enhancement of mathematical knowledge of all its members, the teacher needs to clarify the important role that the bilingual student will play in bridging understandings between members. Cohen reports on research that found that in classrooms in which two languages were used by the teacher and students, bilingual students enjoyed the highest social status.

**Home language and code switching**

Latu (2005) records that both Sāmoan and Tongan students encountered problems with relational statements. Fasi investigated the discursive approaches of two teachers, one Sāmoan and the other Tongan, both of whom had been educated in their native country before moving to New Zealand to complete their higher education. He found that the teachers switched between the language of instruction and the learners’ main language in order to explain and clarify the concepts to students. Adler (1998), Setati and Adler (2001), Dawe (1983), and Clarkson (1992) all found evidence of language switching (code switching) for bilingual students, particularly when students could not understand the mathematical concept or when the task level increased. Code switching involved words and phrases as well as sentences and tended to enhance student understanding.

In Latu’s study, low general proficiency in both home language and in the language of instruction was predictive of students’ difficulty with mathematics. This finding is supported by Christensen (2004), who reported a positive correlation between pāngarau achievement and language proficiency. Clarkson and Dawe (1997) also found that, compared with monolingual students, bilingual students with low proficiency in both the home language and the language of instruction are at a cognitive disadvantage and achieve less successfully. However, De Avila (1980) offered an alternative conclusion. In a study of year 1, 3, and 5 Hispanic students, De Avila found that language proficiency was not strongly related to mathematics achievement. Linguistic proficiency is unlikely by itself to contribute to poor performance. Other factors, such as ‘linguistic distance’ (Dawe, 1983) between the home and instructional languages might help to explain the relationship.

Researching in South Africa, Adler (2001) has highlighted a number of dilemmas experienced by teachers in that country. One is whether to focus explicitly on correct (accepted-at-large) mathematical language or to focus on mathematical reasoning. Teachers have found that a focus on language form compromises the mathematical content and vice versa. Woodward and Irwin (2005) explored this issue when they observed two teachers of year 5 and 6 students, mostly Pasifika, involved in the New Zealand Numeracy Development Project. Under investigation was the teachers’ use of vocabulary in probability lessons and the linguistic structures employed in discursive interactions between teachers and students and between peers. The researchers report that the language that dominated the lessons was everyday English and, as a consequence, little effort was expended on teaching students the nuances of mathematical language. It would have been particularly useful for these students if the teachers had been able to capture the subtleties of mathematical language.

Within our theorising of a productive learning community, mathematical language involves more than vocabulary and technical usage. In this section, we have seen that mathematical language encompasses the ways that expert and novice mathematicians use language to explain and to justify a concept. Listening carefully to students’ expressions of mathematical content is a sound first-step pedagogical strategy. Quality teaching puts the spotlight on mathematics; this focus directly implicates students’ relationships, both with the concepts and with each other.
Mathematics teaching for diverse learners involves respectful exchange of ideas

Students’ articulating thinking

There is now a large body of empirical and theoretical evidence that demonstrates the beneficial effects of students articulating their mathematical thinking (e.g., [Fraivillig, Murphy, & Fuson, 1999], Lampert, 1990; O’Connor, 1998). By expressing their ideas, students provide their teachers with information about what they know and what they need to learn. Fraivillig and colleagues have found that effective teachers do more than sustain discussion; they nudge conversations in mathematically enriching ways, they clarify mathematical conventions and they arbitrate between competing conjectures. In short, they pick up on the critical moments in discursive interactions and take learning forward. Hiebert and colleagues (1997) have found that relevant and meaningful teacher talk involves drawing out the specific mathematical ideas encased within students’ methods, sharing other methods, and advancing students’ understanding of appropriate mathematical conventions. Reframing student talk in mathematically acceptable language provides teachers with the opportunity to enhance connections between language and conceptual understanding.

Articulating comprehensible explanations about mathematical concepts is a learned strategy. The effective teacher is aware that “the art of communicating has to be taught” (Sfard & Kieran, 2001, p. 70). It is, however, a major challenge to make classroom discourse an integral part of an overall strategy of teaching and learning (Hicks, 1998; Lampert & Blunk, 1998). Doing so successfully involves significantly more than developing a respectful, trusting and non-threatening climate for discussion and problem solving. Quality teaching involves socialising students into a larger mathematical world that honours standards of reasoning and rules of practice (Popkewitz, 1988). O’Connor and Michaels (1996) put it this way:

*The teacher must give each child an opportunity to work through the problem under discussion while simultaneously encouraging each of them to listen to and attend to the solution paths of others, building on each others’ thinking. Yet she must also actively take a role in making certain that the class gets to the necessary goal: perhaps a particular solution or a certain formulation that will lead to the next step … Finally, she must find a way to tie together the different approaches to a solution, taking everyone with her. At another level just as important she must get them to see themselves and each other as legitimate contributors to the problem at hand. (p. 65)*

Quality teaching familiarises students with the rules that regulate appropriate mathematical ways, including inference, analysis, and modelling. O’Connor and Michaels (1996) highlight the importance of shaping students’ higher-level thinking by fostering students’ involvement in taking and defending a particular position against the claims of other students. This instructional process depends upon the skilful orchestration of classroom discussion by the teacher. It is a skill that effective teachers have developed. It “provides a site for aligning students with each other and with the content of the academic work while simultaneously socialising them into particular ways of speaking and thinking” (p. 65).

Stigler (1988) looked at teachers who have developed the skill of scaffolding the responses of diverse students. He compared the pedagogical approaches of Japanese and American teachers and found that Japanese teachers spend more time than American teachers in encouraging their students to produce comprehensive verbal explanations of mathematical concepts and algorithms. Expanding on this aspect, Cobb, Wood, and Yackel (1993) report that effective teachers in their research initiated and guided a genuine mathematical dialogue between students. These teachers made it possible for students to share their interpretations of tasks and their solutions. In addition, the teachers influenced the course of the dialogue by picking up on students’ contributions. They did this by framing students’ interpretations and solutions as topics for discussion. Valuing and shaping students’ mathematical contributions served these important functions:
• it allowed students to see mathematics as created by communities of people;
• it supported students’ learning by involving them in the creation and validation of ideas;
• it helped students to become aware of more conceptually advanced forms of mathematical activity.

White (2003) explored how two teachers used classroom discourse to teach third graders mathematics, looking at how the discourse enhanced the educational experiences of the teachers’ diverse student populations and how it influenced the mathematical thinking of the students. The teachers were part of larger project, IMPACT (Increasing the Mathematical Power of All Children and Teachers). Classroom vignettes illustrated one of four themes that emerged from the classroom discourse: (a) valuing students’ ideas, (b) exploring students’ answers, (c) incorporating students’ background knowledge, and (d) encouraging student-to-student communication. The teachers engaged all students in discourse by first monitoring their participation in discussions and then deciding when and how to encourage each to participate. By actively listening to students’ ideas and suggestions, they demonstrated the value they placed on each student’s contribution to the thinking of the class. The teachers encouraged their students to give critical feedback on each other’s responses and asked them to reveal their assessment of each other’s ideas by giving a ‘thumbs up’ or ‘thumbs down’ signal. In one of the classrooms for limited-English-speaking students, this proved to be a particularly useful strategy for getting members to share their views with the class.

Hunter (2005) examined the discourse patterns within a low-decile year 5 classroom of predominantly Pasifika and Māori students. In this vignette, a range of pedagogical strategies is utilised by the teacher to support a productive mathematical discourse:

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**Clarifying Expectations of Classroom Discourse**

The teacher’s specific pedagogical effectiveness within classroom discussion was her use of explicit strategies to enhance the mathematical contributions of her students:

- If you don’t understand, what questions do you need to ask?
- If someone didn’t understand it though and the same thing was said to them …
- I want you to explain to the people in your group how you think you are going to go about working it out. Then I want you to ask if they understand what you are on about and let them ask you questions. Remember in the end you all need to be able to explain how your group did it so think of questions you might be asked and try them out.
- Okay so I have heard lots of talking, discussing in your groups and listening to each other and that’s good.
- Now this group is going to explain and you are going to look at what they do and how they came up with the rule for their pattern, right? Then as they go along if you are not sure please ask them questions. Tune in here, step by step, and as they go along if you can’t make sense of each step remember ask those questions.
- Pen down. Have a look and think. Now has anyone got a question they want to ask of Rewa at this point?
- Arguing is not a bad word … sometimes I know you people think to argue is … I am talking about arguing in a good way. So please feel free if you do not agree with what someone has said as long as you say it in an okay way. A suggestion could be that you might say I don’t actually agree with you, could you show that to me. Do you think you could prove it mathematically, could you, perhaps write it, or draw something to show that idea to me … and sometimes doing that the other person thinks it wasn’t quite right so they change their idea and that’s okay.

*From Hunter (2005)*
Hunter’s research reveals a pedagogical strategy that presses students for understanding. It is a strategy that aims to develop thinking and speaking in ways that are mathematical. Students are encouraged and supported in developing the skills of explanation, argumentation, and justification. Other New Zealand studies have found that students are not always able to elaborate on their mathematical reasoning. Meaney (2005) explored the responses of 35 students to questions in the 2003 Mathematics with Statistics national examination paper that required mathematical justification. The ways in which students constructed their mathematical explanations varied. Twelve students used both equations and narrative explanation to explain their thinking, while 17 used only equations. In an investigation into primary school students’ responses to tasks requiring explanations of commutativity, Anthony and Walshaw (2002) noted that many students were unable to offer explanations that reflected structural understanding. Moreover, several students experienced unease with the expectation that they justify their thinking. At the secondary level, Bicknell (1998) studied students’ written explanations and justifications for mathematical assessment tasks. She writes of the 36 year 11 mathematics students involved in her study:

[S]tudents experienced some difficulties writing explanations and had concerns about whether their explanations were satisfactory; ... most students surveyed were unable to write justifications; they lacked knowledge and confidence in justifying their solutions. (p. ii)

Lubienski (2002), as teacher-researcher, compared the learning experiences of students of diverse socio-economic status (SES) in a seventh grade classroom. She reported that higher SES students believed that the patterns of interaction and discourse established within the classroom helped them learn other ways of thinking about ideas. The discussions helped them reflect, clarify, and modify their own thinking, and construct convincing arguments. In Lubienski’s view, the lower SES students were reluctant to contribute because they lacked confidence in their ability. They claimed that the wide range of ideas contributed in the discussions confused their efforts to produce correct answers. Their difficulty in distinguishing between mathematically appropriate solutions and nonsensical solutions influenced their decisions to give up trying. Pedagogy, in Lubienski’s analysis, tended to privilege the ways of being and doing of high SES students. In a similar way, Jones’s (1991) classic study showed that the discursive skills and systems knowledge that are characteristic of high SES families align them favourably with the pedagogy that is operationalised within school settings. Set in the New Zealand context, Jones provided conclusive evidence that Pasifika girls were unwittingly penalised by the sorts of instructional approaches taken by classroom teachers.

**Opportunities for students to explain and justify solutions**

The benefits of providing regular opportunities for students to explain and justify their solutions are well documented. Many researchers have found that pedagogical practices that make provision for the development and evaluation of mathematical argument and proof contribute to the development of students’ mathematical thinking. At another level, such provision provides access to an individual’s mathematical thinking. When an individual’s thinking is accessed by a learning community, the intentions and interpretations of the individual lend themselves to modification in response to the community’s reception of their ideas (White, 2003).

In the New Zealand context, [Woodward and Irwin (2005)] report on opportunities for students to explain and justify solutions. A particular teacher involved in their study made a significant contribution to students’ mathematical development. She did this by listening attentively to her students’ queries and explanations, asking them to justify their answers, and holding back with explanations until she deemed them crucial. The researchers record one of many occasions during which she structured students’ mathematical practice: “Before you write it down I want you to justify it to your partner. So if you say there’s eight queens your partner needs to say, ‘How do you know that there’s eight queens?’” (p. 803). By using such pedagogical strategies, the teacher was able to develop mathematical ways of doing and being in all her students.
McChesney (2005) explored levels of student participation in low- and middle-band New Zealand classes at the junior secondary school level. McChesney notes that teachers who established classroom communities in which there was access to social, discursive, visual, and technological resources, were able to support students’ mathematical activity. Her research clearly demonstrates a direct relationship between the quality of teacher–student interaction and students’ negotiation of mathematical meaning. The effective teachers in this research were able to set up an environment in which conventional mathematical language migrated from the teacher to the students. Over time, students’ contributions, which were initially marked by informal understandings, began to appropriate the language and the understandings of the wider mathematical community. It was through the take-up of conventional language that mathematical ideas were seeded.

Within the intellectual space that is shared by students and their teacher, it is the teacher’s pedagogical content knowledge and expertise that makes a difference to the quality and level of mathematical discussion. What needs to be stressed, too, is that the teacher’s expertise is also related to the role he or she assumes as earnest listener and co-learner. Zack and Graves (2002) have reported that teachers who make a difference are themselves active searchers and enquirers into mathematics. O’Connor’s (2001) classroom research highlights how one teacher, through purposeful listening, facilitated a group of students towards a mathematical solution. The discussion in the following vignette illustrates how students took positions on the answer and attempted to support those positions with evidence. The teacher made her contribution by challenging the students’ claims, using counter-examples.

Gina and Bruno’s Alternative Conceptions

A class has been discussing whether fractions can be converted into decimals. One student (Bruno) directly addresses the logic of the framing question: to successfully argue that not all fractions can be turned into decimals, you need an example of a fraction that cannot be turned into a decimal. Almost immediately, a claim he made is challenged. Gina offers the counter-example of 2/5.

Gina: I disagree because five can go, uh, any fraction with five or fifths in it, can go into, can also be turned into a decimal.

Teacher: Give me an example.

Gina: Um, two fifths is, is one tenth, umm, two fifths is umm is turned into ...

Teacher: Okay, what does two fifths look like as a decimal?

Gina: Point forty.

The teacher asks Bruno to respond, and he immediately replies that fifths are covered under his original statement: fraction denominators that are factors of powers often (like fifths) all do allow transformation of fractions to decimals. (So his claim could have been stated more precisely, e.g.: ‘any odd prime number that is not also a factor of a power of ten will result in a repeating decimal.’)

At this point the class looks a bit stunned at the directness of Gina’s counter-example, and the alacrity with which her challenge is returned. There is silence. Gracefully, the teacher takes this opportunity for a meta-comment that depicts the two as collaborating in an important mathematical practice:

Teacher: Great, now I hope you’re listening because what Gina and Bruno said was very important. Bruno made a conjecture and Gina tested it for him. And based on her tests he revised his conjecture because that’s what a conjecture is. It means that you think that you’re seeing a pattern so you’re gonna come up with a statement that you think is true, but you’re not convinced yet. But based on her further evidence, Bruno revised his conjecture. Then he might go back to revise it again, back to what he originally said or to something totally new. But they’re doing something important. They’re looking for patterns and they’re trying to come up with generalisations.
The teacher has sensed that this excellent display of the mathematical practice of finding and testing counter-examples to a claim is not without its potential risks. By portraying the exchange as positive, she is reassuring all students that the practice does not connote hostility, as it might in any informal or everyday setting.

From O'Connor (2001)

Goos (2004) described how a secondary school mathematics teacher developed his students’ mathematical thinking through scaffolding the processes of enquiry. There was a tacit agreement amongst members of the learning community that instructional practices demanded students’ mathematical talk. For his part, the teacher orchestrated mathematical events by first securing student attention and participation in the classroom discussion. Specifically the “teacher call[ed] on students to clarify, elaborate, critique, and justify their assertions. The teacher structured students’ thinking by leading them through strategic steps or linking ideas to previously or concurrently developed knowledge” (p. 269). In a series of lesson episodes, Goos provides evidence of how the teacher pulled learners “forward into mature participation in communities of mathematical practice” (p. 283). He scaffolded thinking by providing a predictable structure for enquiry through which he enacted his expectations regarding sense making, ownership, self-monitoring and justification. As the year progressed, the teacher gradually withdrew his support to push students towards more independent engagement with mathematical ideas. For their part, the students responded by completing tasks with decreasing teacher assistance and by proposing and evaluating alternative solutions. Engagement for them was “a complex process that combine[d] doing, talking, thinking, feeling, and belonging” (Wenger, 1998, p. 56).

Constructive feedback

Constructive feedback, as one form of exchange of ideas, has a powerful influence on student achievement (Hattie, 2002). In keeping with our ethic of care, we have evidence that praise, not of itself but taken together with quality feedback, can be a powerful pedagogical strategy (Hill & Hawk, 2000). Of course, in an environment that does not value student contribution or knowledge, feedback has decidedly negative effects (Hoyles, 1982).

What constitutes quality feedback? Research has shown that feedback that engages learners in further purposeful knowledge construction will contribute to the development of their mathematical identities. We have evidence (e.g., Wiliam, 1999) that feedback that is constructive has the effect of occasioning certain mathematical capabilities in students and assists in the development of their perception of the mathematical world. Khisty and Chval (2002) have shown that quality feedback plays a key role in students’ learning. In their research into the way in which a teacher interacted with her fifth grade Latino students (and with English-language learners), the researchers found that her focus on mathematical talk and meaning enabled the students to develop mathematical reasoning in significant ways. She facilitated learning through questioning that was concerned less about teacher exposition and more about the perceptions held by her students. The teacher opened up the discussion with each interaction and, by making use of the responses received, she was able to lend structure to their mathematical meanings.

Teachers who provide quality feedback draw on a range of pedagogical content knowledge skills that enable them to know when to and when not to intervene. When they do intervene, their feedback makes a judgment about students’ strategies, skills, or attainment (Gipps, McCallum, & Hargreaves, 2000), pinpointing the difference “between the actual level and the reference level of a system parameter which is used to alter the gap in some way” (Ramaprasad, 1983, p. 4). In other words, the feedback makes a comparison between where a student is currently at and a standard as interpreted by the teacher. We can classify as feedback much of the dialogue that occurs in the classroom to support learning (Askew & Lodge, 2000). Feedback is a rich
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resource by which students are able to gauge and “monitor the strengths and weaknesses of their performances, so that aspects associated with success or high quality can be recognised and reinforced, and unsatisfactory aspects modified or improved” (Sadler, 1989, p. 120).

This is not to advocate grades and test scores for enriching student learning, or to promote feedback as the responsibility of the teacher alone. In her study involving senior secondary school students, Anthony (1996) found that, by itself, feedback in the shape of scores and grades was of limited value. She makes a case for pedagogical practices that shift total responsibility from the teacher and include student participation in the feedback process. She recommends that teachers be explicit about the value of student reflection and that students be given the opportunity to evaluate the reasonableness of their solutions and assess the justification for their procedures.

Feedback that is provided too early or too late can be ineffective. Freeman and Lewis (1998) have demonstrated that students require an opportunity to confront the mathematical problem themselves before they are given feedback and that “the greater the delay, the less likely it is that the students will find it useful or be inclined to act on it” (p. 49). Knight (2003) interviewed six New Zealand teachers and observed the feedback they provided in their primary school numeracy classes. The study revealed that out of the 349 examples of verbal feedback recorded in the lessons, most (83%) took the form of an expression of encouragement or praise. Knight reports that the teachers were often unaware of the high frequency of such responses and their automated nature. Only 17% of the feedback reflected on the cognitive development of the students. As Knight notes, a more effective pedagogical strategy would aim to support students’ mathematical thinking as well as their motivation.

Classroom research at both primary and secondary level (e.g., Ruthven, 2002; Wiliam, 1999) has shown that much of the teacher feedback that students receive is not particularly constructive. Ruthven reported that, in the UK, teacher feedback was not assisting students “to study mathematics and think mathematically” (p. 189). Rawlins (in progress) reports that year 12 students working towards the National Certificate of Educational Achievement (NCEA) valued teacher feedback that “pointed them in the right direction.” In the following transcript, two of the participant students discuss their views on what quality feedback involves:

S1: Just stating where I went wrong … “This is what you should have done …”

S2: Like when we went back through them and she said, “OK, next Monday I want you to hand them in done” but she gave us the answers for each of the questions. I found that hopeless. I couldn’t figure out how they got to the answer and I was just sitting there an …

S1: You sit there for about an hour on your calculator and you do all sorts of thing …

S2: … and you just cannot figure it out and I found that really useless.

S1: There were a couple of questions—the really crazy ones—where she did the first bit for us and then told us what to do next, like you need to factorise, expand, solve, and the …

S2 … that was good. That was helpful. But she didn’t do that for all of them.

Wiliam (1999) reported on a comparative study of two groups of year 4 students. One group was given minimal feedback that nevertheless allowed members to advance towards a solution to a mathematics problem. The other group was given a full solution when they reached the point where they could not make further progress without help. Wiliam found that “[m]inimal intervention promoted better learning” (p. 9). As Chamberlain (2005) found in a study involving teachers of the middle grades, minimal feedback tends to more effectively promote student learning and retention. These teachers urged: “Allow the students to work on the problem
Too much feedback is counterproductive to learning. This point is illustrated by Woodward and Irwin (2005). Their study compares the talk and feedback provided by two teachers of mostly Pasifika year 5 and 6 students involved in the New Zealand Numeracy Development Project. For one of the teachers, the researchers recorded numerous instances of too much feedback or ‘teacher lust’ (Maddern & Court, 1989). While this teacher had created a positive learning environment and shown a keen desire for talk to occur in the classroom, actual mathematical talk was minimal. Cognitive space was limited by the lack of pause times for thinking, and students were occasionally ‘talked over’. Specifically, students did not have the opportunity to learn and speak the language of mathematicians. Like the teacher in a study by Khisty and Chval (2002), this teacher did not provide students with opportunities to engage in mathematical discourse so they did not develop a medium for expressing what they were learning. Khisty and Chval found that although the teacher in their study had done many of the ‘proper’ things, by contextualising the mathematics in a story that had relevance to the students, by developing a role-playing activity to assist with conceptual understanding, and by forming small working groups, they nevertheless failed to provide students with the meanings associated with mathematical knowledge.

In reporting on their study of classroom dialogue, Cobb, Wood, and Yackel (1993) describe how one teacher shaped student participation, giving it the cultural nuances of mathematical talk:

[The teacher] reformulated their explanations and justifications in terms that were more compatible with the mathematical practices of society at large and yet were accepted by the children as descriptions of what they had actually done. Thus rather than funnelling the children’s contributions, the teacher took the lead from their contributions and encouraged them to build on each others’ explanations as she guided conversations about mathematics. As a consequence, the mathematical meanings and practices institutionalised in the classroom were not immutably decided in advance by the teacher but, instead, emerged during the course of conversations characterised by ... a genuine commitment to communicate. (p. 93)

Revoicing
‘Revoicing’ is the term used by O’Connor and Michaels (1996) to describe a subtle yet effective strategy for fine-tuning mathematical thinking. By revoicing is meant the repeating, rephrasing or expansion of student talk in order to clarify or highlight content, extend reasoning, include new ideas, or move discussion in another direction. The researchers maintain that probing into student understanding provides teachers with the opportunity to model engagement within a mathematical, multi-voiced community. According to Forman and Ansell (2001), in classrooms where revoicing is used, “[t]here is a greater tendency for students to provide the explanations ... and for the teacher to repeat, expand, recast, or translate student explanations for the speaker and the rest of the class” (p. 119). In the following vignette, revoicing is used by a teacher of year 1 and 2 students as a simple yet effective pedagogical strategy for scaffolding knowledge.

**Five Monkeys in the Tree**

Students in a year 1-2 classroom had been puzzling over the problem: “If all the monkeys in a big tree and a small tree want to play in the trees, think of all the ways that we can see all five monkeys in the two trees.”

Together the students had provided the following possibilities: 5,0; 2,3; 3,2; 0,5; 4,1; 1,4. The teacher responded:

Teacher: Is there a way that we could be sure and know that we’ve gotten all the ways?
Jordan: [Goes to the overhead screen and points to the two trees and the table as he explains.] See, if you had four in this [big] tree and one in this [small] tree in here, and one in this [big] tree and four in this [small] tree, couldn’t be that no more. If you had five in this [big] tree and none in this [small] tree, you could do one more. But you’ve already got it right here [points to 5,0]. And if you get two in this [small] tree and three in that [big] tree, but you can’t do that because three in this [small] one and two in that [big] one—there is no more ways, I guess.

Teacher: What Jordan said is that you can look at the numbers and there are only a certain ... there are only certain ways you can make five.

From Cobb, Boufi, McClain, and Whitenack (1997)

Providing cognitive structure and fine-tuning mathematical thinking

O’Connor and Michaels (1996) provide evidence that teachers who provide cognitive structure also tend to fine-tune students’ mathematical thinking. Fraivillig, Murphy, and Fuson (1999) have developed a conceptual framework for describing the ways by which teachers do this. The three key pedagogical components identified in their Advancing Children’s Thinking (ACT) framework are ‘eliciting’, ‘supporting’, and ‘extending’. Eliciting involves promoting and managing classroom interactions, supporting involves assisting individuals’ thinking, and extending captures those practices that work to advance students’ knowledge.

Shaping students’ mathematical thinking is a highly complex activity (Taylor & Cox, 1997). It is complex because teachers and students are “negotiating more than conceptual differences … they are building an understanding of what it means to think and speak mathematically” (Meyer & Turner, 2002, p. 19). Building that understanding requires the teacher to first construct sociomathematical norms (see Yackel & Cobb, 1996) for what constitutes a mathematically acceptable, different, sophisticated, efficient, or elegant explanation. Sociomathematical norms regulate mathematical argumentation and govern the learning opportunities and ownership of knowledge made available within the classroom.

Manouchehri and Enderson (1999) investigated the discursive interactions within a heterogeneously grouped seventh grade mathematics class. Cursory observations revealed an overwhelming occurrence of student talk and interaction. Further analysis unpacked the teacher’s critical role in orchestrating that mathematical activity and discourse. Through careful questioning, purposeful interventions, and her efforts to shift the students’ reliance from her towards “the guidance, support and challenge of companions who vary in skills and status” (Rogoff et al., 1993, p. 5), she provided responsive rather than directive support, all the while monitoring student engagement and understanding. Her strategy was not aimed at structuring learning by organising students’ behaviour. Rather, as Manouchehri and Enderson (p. 219) clarify, her primary objectives were to:

- facilitate the establishment of situations in which students had to share ideas and elaborate on their thinking (e.g., Would anyone else like to add anything to S13’s explanation? Could you show that to us on the board? That is an excellent question. Does anyone want to have a shot at it?);
- help students expand the boundary of their exploration (e.g., What do you think class? Do you think that this formula would work all the time for all the rows? Why don’t you extend the sequence and see if there is a pattern);
- encourage students to make connections among different discoveries and develop a deeper understanding of the interrelationships among the patterns that students identified (e.g., I wonder if we can find out how these 2 patterns are related?);
- invite multiple representations of ideas (e.g., Is there another way of representing this?).

From Cobb, Boufi, McClain, and Whitenack (1997)
The way in which one teacher orchestrated a year 1 and 2 classroom discussion is illustrated by Fraillig, Murphy, and Fuson (1999) in the following vignette. What is particularly effective is the way the teacher sustains the discussions. She has developed a sensitivity about when to ‘step in and out’ (Lampert & Blunk, 1999) of the classroom interactions and has learned how to resolve competing student claims and address misunderstanding or confusion (theirs and hers). For their part, the students listen to others’ ideas and debate to establish common meanings. In short, they participate in a ‘microcosm of mathematical practice’ (Schoenfeld, 1992), learning how to appropriate mathematical ideas, language and methods and how to become apprentice mathematicians.

High-level Thinking of Young Mathematicians

Ms Smith challenged her year 1–2 class to investigate zero under subtraction. Her students’ mathematical development was influenced by several factors—the discursive interactions, the multiple forms of mathematical representation, and particularly by the way she explicitly nudged her students towards and pressed for understanding and demonstrations of mathematical behaviour. The teacher drew on her mathematical content knowledge to make specific links between concepts. By asking them questions, listening attentively, encouraging them to look for relationships amongst concepts, checking for accuracy, and building on students’ ideas to stimulate further thought, she nurtured a mathematical disposition that incorporated risk-taking and multiple solutions. Talk in this young learners’ classroom was not merely aimed at filling a conversational void nor directed at easy solution pathways, but was strategically focused towards conjecturing and high-level thinking.

Teacher: What’s the last one? [Ms. Smith was eliciting all single-digit “doubles” from students and listing them on the board.]
S1: Zero plus zero.
Teacher: Zero plus zero.
S2: Zero plus zero is just zero.
Teacher: Zero plus zero is easy.
S3: Zero minus zero is negative one, isn’t it?
Teacher: No. Derrick [S3] You’re on zero [points to the number line] and you take zero jumps. Where are you [giving one context for deciding the answer]?
S3: Zero. [Ms. Smith motions to S3 indicating “you got it.”]
S4: Then negative five plus negative five must be negative five.
Teacher: Pardon me?
S4: Negative five plus negative five should be negative five.
Teacher: No, ’cuz you’re adding negative five and negative five, so you start at negative five and how many jumps do you take?
S4: Five.
Teacher: Well, you’re not going to end up on five [points to the negative five on the number line]; you’re not going to end up on negative five [modifying her sentence]. So, then negative five. How many jumps do you take?
S4: Five.
Teacher: So where are you going to end up?
S: Zero plus ...
Teacher: No, no, no. Negative five [pause indicating uncertainty]. You’re right, Stevie [S4]. You’re right (laughs). You see what he did? Ms. Smith was thinking the other way. Negative five (pause); ... Allan, this is hard. You might want to watch it for a minute. Negative five [she continues slowly with a questioning voice], we’re going to add negative five to it. No, it’s not right. Is it?
S5: [Students speculate as to whether or not zero is the answer.] Yeah, zero’s right.
S6: No, it’s ten. Negative goes that way [motioning toward negative ten, meaning negative five plus negative five equals negative ten].
Teacher: No, it’s ten [responding to S5]. But you’re adding negative five to it, sweetie, so you would go this way [motions toward the left of the number line]. I was right. It’s ten. You start at negative five plus five, you end up at zero. But negative five plus negative five is negative ten. I was right. [Ms. Smith whispers seemingly to reassure herself.] I was right.
S: You thought you were wrong!
Teacher: I did think I was wrong. You confused ...
S: Negative five and negative five is negative ten.
Teacher: That’s right. Negative five plus negative five is negative ten.

From Fraivillig, Murphy, and Fuson (1999)

Pedagogical practices like these, which help refine students’ mathematical thinking, are reported in research undertaken by Steinberg, Empson, and Carpenter (2004). In their study, a respectful exchange of ideas was central to a sustained change in students’ conceptual understanding. Over a period of a few months, the teacher had integrated particular pedagogical strategies into her practice, focused on probing and interpreting student understanding and on generating new knowledge. During the study, she developed a working consensus with all members of the classroom community about the form of, and social roles within, her changed instructional processes. A respectful exchange of ideas was also a feature of effective pedagogical practice in a study by Forman and Ansell (2001). The teacher in this study used repetition to highlight the particular claims and ideas of individual students, developed the understandings implicit within those ideas, negotiated meaning to establish the veridicality of a claim, and used their original ideas as a springboard for developing related new knowledge in whole-class discussions. Original ideas may be exchanged between an individual and the teacher. A New Zealand study (Walshaw, 2000) involving secondary school students documents how an exchange of ideas between teacher and student was effectively used to monitor the understanding and written work of individual students and to guide the teacher’s choice of examples and explanations at the whiteboard.

Mathematics teaching for diverse learners demands teacher content knowledge, knowledge of mathematics pedagogy, and reflecting-in-action

Teacher knowledge
Effective teaching for diverse students begins with teacher knowledge. What teachers do in classrooms is very much dependent on what they know and believe about mathematics. It is also very dependent on what they understand about the teaching and learning of mathematics (Hiebert et al., 1997). Jaworski (2004) argues that, to be successful, a teacher must have both the intention and the effect to assist students to make sense of mathematical topics: good intentions are necessary, but they are not enough. The teacher must make good sense of the mathematics involved or he or she will not be able to help students work with ideas and knowledge (Fraivillig et al., 1999; Schifter, 2001). In this section, we explore the effects of teacher knowledge on student outcomes and we develop these ideas further in chapter 5.

Expanding on the crucial importance of teachers’ subject knowledge as a resource for teaching, Ball and Bass (2000) point out that there is an intimate relationship between mathematical concepts and the way those concepts are actually conveyed by the teacher to students. The
findings of studies undertaken by Ball and Bass (2000) and many others (e.g., Carpenter & Lehrer, 1999; Doerr & Lesh, 2002; Hill, Rowan, & Ball, 2005; Kilpatrick et al., 2001; Ma, 1999; Shulman & Shulman, 2004; Warfield, 2001) signal that teachers must have sound content knowledge if they are to access the conceptual understandings that students are articulating in their methods and if they are to decide how those understandings might have come about and where they might be heading.

A core infrastructural element of effective mathematics pedagogy is knowledge in the subject area (Fraser & Spiller, 2001). In their study, Bliss, Askew, and Macrae (1996) found that when teachers demonstrated limited or confused understanding of the subject knowledge relevant to the lesson, or when their perception of the rationale or execution of the curriculum was not clear, their students struggled to make sense of the mathematical concepts. Teachers who were unclear in their own minds about the mathematical ideas struggled to teach those ideas and often used examples and metaphors that prevented, rather than helped, student development. Where teachers were insecure in their content knowledge, there was a direct, subsequent student lack of understanding. The teachers in the study acknowledged that their own limited knowledge led them to misunderstand their students’ solutions and to give feedback that was inappropriate or unhelpful.

In a study of whole-class teaching episodes at three schools, Myhill and Warren (2005) found that many strategies used by teachers worked more as devices to enable students to complete tasks rather than as learning support mechanisms that would help move them towards independence. In the year 2 and year 6 lessons observed, teachers often used ‘heavy prompts’, pointing students to the ‘right’ answers. Doyle and Carter (1984) note that this pedagogical strategy is sometimes called ‘piloting’ and is often used as an inclusive strategy to ensure that every student has the opportunity to provide a correct answer. However, using this strategy, the teachers in the Myhill and Warren study tended to miss critical opportunities for gaining insight into students’ prior knowledge or level of understanding. Teachers’ fragile subject knowledge prevented them from assessing the current level of mathematical sophistication and put boundaries around the ways in which they could develop students’ responses. One year 6 teacher told her class: “Sometimes in decimals you say point seven eight or sometimes you say point seventy-eight”, and in doing so, paved the way for student misunderstanding of place value.

**Knowing how to teach the content**

As well as documenting the importance for student outcomes of strong subject matter (content) knowledge, research has demonstrated the importance of mathematics pedagogical knowledge, that is, teachers’ knowledge of how to teach the content. It is one thing to access the level of conceptual sophistication that students are working with, and it is another to know how to utilise that content knowledge (Hattie, 2002; Hill et al., 2005). On the basis of findings from their research, Ball and Bass (2000) emphasise the important work that the teacher does in connecting mathematical ideas in real time with instructional approaches, teaching principles, and students’ contributions. Quality teaching integrates knowledge flexibly in varying contexts. As Ball and Bass and others (e.g., Hill et al., 2005; Schifter, 2001; Warfield, 2001) have made clear, teachers’ decisions about activities always fall back on the knowledge that they hold of the content to be taught.

**Teachers making connections**

Askew, Brown, Rhodes, Johnson, and Wiliam (1997) looked closely at the repertoire of mathematical knowledge that characterised effective teachers of numeracy in the UK. Their Effective Teachers of Numeracy Project revealed that quality teaching is not necessarily related to higher formal qualifications. In fact, the researchers found a slight negative relationship between the level of teachers’ formal qualifications in mathematics and the levels of attainment of their students. Evidence from case studies signalled that teachers who were highly qualified
in mathematics tended to have a more procedural view of school mathematics. In contrast, it was the teachers who were able to make connections between aspects of mathematical knowledge who recorded high academic gains for their students. The Effective Teachers of Numeracy Project revealed that “highly effective teachers of numeracy themselves had knowledge and awareness of conceptual connections between the areas which they taught in the primary mathematics curriculum” (Askew et al., 1997, p. 3). Teachers’ conceptual understanding and knowledge is critically important at any level. In New Zealand secondary schools, the 2001 Teacher Census found the proportion of teachers with a third-year university or postgraduate qualification in a given subject area to be higher for mathematics than for any other curriculum area.

Thomas, Tagg, and Ward (2002) reported on numeracy teaching in New Zealand. In a questionnaire undertaken by 148 teachers involved in the NDP, almost all (96%) maintained that their knowledge had developed through their involvement in the project. In turn, the new knowledge led to effectiveness in their teaching, as evidenced through student gains that exceeded prediction in the five aspects of number learning assessed. These achievement gains were recorded irrespective of students’ gender, age, ethnicity, or school decile rating. Specifically, teachers noted that as a result of their involvement in the NDP, changes to their practice included:

- a greater focus on both strategies and knowledge using the structure of the Number Framework;
- more effective assessment strategies and student grouping decisions;
- an increased focus on listening to students and their explanations;
- more sharing of student strategies;
- a greater understanding of student progression.

Thomas and Ward (2002) carried out case studies of 10 teachers who were identified as effective with respect to student achievement in early number. Despite their varying levels of teaching experience, all had sound knowledge of their students’ mathematical knowledge and learning capabilities, positive and enriching relationships with their students, and high expectations for their students and were thorough in their understanding and application of the Number Framework. Amongst other things, the teachers said:

He will put up his hand as he is really keen, and then he will forget what he is saying, or he won’t know, so I will usually wait longer, or I say “Keep thinking and I will come back to you.” Or I get to him one-to-one and find out what he is doing that way.

Asking students to explain their thinking and waiting for them to do so. Sometimes I get a bit impatient and want to butt in. So you are trying. Sometimes you are trying to cover too much, it’s best to just cover a little bit so that you do have time to do that.

... to go from bundling them [popsticks] up to “OK, show me this number,” that is a step in itself ... a small step, but it is still a step ... That was what I was trying to do ... go from me modelling it, to them doing it, and then take it a step further ... from a given set of equipment to a number, “Now show me this number.” (pp. 4–49)

Davies and Walker (2005) focused on the content knowledge that teachers bring to their teaching. In an experiment aimed at enhancing teacher knowledge and improving student numeracy levels, these researchers explored how teacher knowledge and pedagogical practices changed as a result of collaboration between teachers and researchers. At each meeting, the eight teachers introduced, discussed, and trialled rich tasks and problems that challenged their ‘content knowledge complexes’. As a result of their more finely tuned classroom listening and questioning skills, teachers began to notice changes in their teaching behaviours. In addition, the teachers volunteered that their planning had changed and that they had developed an increased sensitivity to the need to wait for students’ responses.
Askew and Millett (in press) investigated the relationship between teacher knowledge and student thinking. They looked particularly at critical incidents of teaching/learning—those moments that occasioned students’ thinking. What became obvious during the course of the research was the critical role that teachers’ subject knowledge plays in extending and challenging students’ conceptual ideas. Sound subject knowledge enabled teachers to mediate between the mathematical tasks, the artefacts, the talk, and the actions surrounding the teaching/learning critical incidents. Askew and Millett found that teachers with limited subject knowledge tended to focus the class on a narrow conceptual field rather than on forging wider connections between the facts, concepts, structures, and practices of mathematics.

**On-the-spot reflection**

As Askew and Millett observed, pedagogical practice that makes a difference for all learners requires professional reflecting-in-action. It requires a moment-by-moment synthesis of actions, thinking, theories, and principles (Ball & Bass, 2000). In the research undertaken by Askew and Millett, the teachers who were able to develop student mathematical understanding were those that had a sound base of subject knowledge. This knowledge informed their on-the-spot decision making in the classroom. It informed decisions about the particular content that the students would learn, the activities they carried out, how they engaged with the content, and how they conveyed to the teacher their understanding of the content.

In her research with teachers, Sherin (2002) found that they negotiate between three areas of knowledge: their understanding of subject matter, their perception of curriculum materials, and their personal theories of student learning. As they weave between these three areas of knowledge and as they deepen their own understanding of them, effective teachers are able to increase students’ levels of mathematical knowledge. The negotiation that takes place as teachers reflect in action, draws on a rich history of personally established ways of thinking and being and applying knowledge flexibly (Hattie, 2002). In particular, teachers who reflect in action are able to adapt and modify their routine practices and, in the process, contribute to the development of new pedagogical routines and new knowledge about subject matter.

Quality teaching involves a positive teacher stance towards reflecting-in-action (Schifter & Fosnot, 1993). It needs to be pointed out that reflecting-in-action can take place either within (Sherin, 2002) or beyond (Davies & Walker, 2005; Mewborn, 1999; Wood, 2001) the classroom. Jaworski (1994) offers first-hand accounts of reflecting-in-action within mathematics classrooms. Contrary to scholarly critique that claims such practice is impossible on the grounds that the metacognitive activity involved assumes more time than the classroom could possibly offer, Jaworski (2004) provides evidence of teachers noticing and then acting knowledgeably as they interact at critical moments in the classroom when students create a moment of choice or opportunity.

Teachers who reflect in action and negotiate between forms of knowledge work hard at understanding students’ viewpoints. Teachers do not, however, always accurately anticipate student thinking and behaviour. Nathan and Koedinger (2000) documented the ways in which teachers predicted students’ problem-solving activity. The researchers found that “students’ problem-solving behaviours differ in a systematic way from those predicted by teachers” (p. D184). Teachers believed that story problems and word-equation problems would be more difficult for their students than symbol-equation problems. The research students found that symbolically presented problems posed more difficulty. Bennett (2002) provides evidence that this kind of problem also creates difficulties for New Zealand secondary school students.

An effective teacher tries to delve into the minds of students by noticing and listening carefully to what students have to say (Franke & Kazemi, 2001). Jones (1986) describes how one teacher in a secondary school class encouraged a student to think through a problem:

Student: I can’t see why that 2x should be there.
Teacher: Go back to the last line. Can you explain what was done there?
Student: Yes. I can see how you had to divide by \( x^2 \) to get there.

Teacher: So what did you need to do to go from there? (p. 484)

Yackel, Cobb, and Wood (1999) report on the ways in which one year 2 teacher listened to, reflected upon, and learned from her students’ mathematical reasoning while they were involved in a discussion on relationships between numbers. Analyses of the discussion revealed that her mathematical subject knowledge and her focus on listening, observing, and questioning for understanding and clarification greatly enhanced her understanding of students’ thinking.

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**Listening and Noticing: 37 + 23 + 14**

The class is discussing the problem: 37 + 23 + 14. Three children have already given explanations as to how they arrived at 74 for a sum. In each case, the solution involved decomposing the summands into tens and ones and recombining them in various ways. The episode begins just after Latoya reported that she got 73 as an answer and the teacher asked her to explain her solution.

Latoya: I said the 10 on the 14, and I added, and then I added 30 more, that was 40.

Teacher: Where did you get the 30 from?

Latoya: Out of the 23.

The teacher assumes that Latoya added the 10 from the 14 and the 20 from the 23 to get 30 so he asks a clarifying question.

Teacher: Okay, you added, you added this 10 and this 20?

Latoya: Yeah ... and I got 40.

Teacher: You got 40.

Latoya: No, I said that I had 10 and I added 30 more.

Tonya: Where did you get the 30 from?

Latoya: Out of the 23 ... And 30 + 30 equal to 60.

Several students enter the discussion to ask Latoya about her apparent interpretation of the 3 in 23 as 30.

Carmen: How did you get the 30 out of the 23?

Latoya: Um, I said that I took away the 20 and left the 3 there, and added another 3.

Lemar: Well, how come did you take away that 2? When you said, take away that 2 and that left 3 and 3 and that was 30?

Latoya: Um.

Teacher: Maybe, let me put 23 down here, 23 (writes 23 on the chalkboard). They want to know how did you get 30 out of 23? That’s what they’re trying to figure out.

Latoya: Um.

Teacher: Listen, let me ask you a question. That 2 stands for what?

Latoya: 20.

Teacher: All right, and the 3 stands for 30?

Latoya: Yes.

Tonya: Why are you saying 20, you are saying that’s a 20 and that’s a 30? Why are you saying ... the 20 and the 30?

Teacher: Give her a chance to answer, please. She asked a nice question. She said, she said that’s a, the 2 is a 20, and 3 was a 30, and she asked you if that’s 20 - 30?

Latoya: No. The 2 stands for 20 and 3 stands for (long pause).

At this point in the episode, the teacher called on another student to “help her out.”
Conclusion

This chapter has explored how mathematics teachers create the conditions for learning in their classroom communities. We found that teaching that facilitates learning for diverse learners demands an ethic of care. Research in this area has found that effective teachers demonstrate their caring by establishing classroom spaces that are hospitable as well as academically ‘charged’. They work at developing interrelationships that create spaces for students to develop their mathematical and cultural identities. Teachers who care work hard to find out what helps and what hinders students’ learning. They have high yet realistic expectations about enhancing students’ capacity to think, reason, communicate, reflect upon, and critique their own practice, and they provide students with opportunities to ask why the class is doing certain things and with what effect.

At the same time, research quite clearly demonstrates that pedagogy that is focused solely on the development of a trusting and attentive microculture does not get to the heart of what mathematics teaching truly entails. A context that supports the growth of students’ mathematical identities and competencies creates a space for both the individual and the collective. Many researchers have shown that small-group work can provide the context for social and cognitive engagement. Quality teaching uses both individual and group processes to enhance students’ cognitive thinking and to engage them more fully in the creation of mathematical knowledge. Within the classroom, all students need time alone to think and work quietly, away from the demands of a group. This line of research has also revealed that classroom grouping by ability has a detrimental effect on the development of a mathematical disposition and on students’ sense of their own mathematical identity.

A teacher with the intention of developing student understanding will not necessarily produce the desired effect. Unless teachers make good sense of the mathematical ideas, they will not develop the flexibility they need for spotting the golden opportunities and wise points of entry that they can use for moving students towards more sophisticated and mathematically grounded understandings. Reflecting on the spot and dealing with contested and contesting mathematical thinking demands sound teacher knowledge. Studies exploring the impact of content and pedagogical knowledge have shown that what teachers do in classrooms and the way in which they manage multiple viewpoints is very much dependent on what they know and believe about mathematics and on what they understand about the teaching and learning of mathematics. A successful teacher of mathematics will have both the intention and the effect to assist pupils to make sense of mathematical topics. There is now a wealth of evidence available that shows how teachers’ knowledge can be developed with the support and encouragement of a professional community of learners.

Studies have provided conclusive evidence that teaching that is effective is able to bridge students’ intuitive understandings and the mathematical understandings sanctioned by the world at large. Consistently emphasised in research is the fact that teaching is a process involving analysis, critical thinking, and problem solving. Language, of course, also plays a central role. The teacher who makes a difference for diverse learners is focused on shaping the development of novice mathematicians who speak the precise and generalisable language of mathematics. Responsibility for distinguishing between terms and phrases and sensitising the students to their particular nuances weighs heavily with the teacher, who profoundly influences the mathematical meanings made by the students in the class. Classroom work is made more enriching when discussion involves the respectful exchange of ideas, when teachers ensure
that this exchange is inclusive of all students, and when the ideas put forward are (or become) commensurate with mathematical conventions and curricular goals. The effective teacher is able to orchestrate discussion and argumentation and facilitate dialogue, not only for the development of mathematical competencies and identities but also to ensure important social outcomes.

References


Appendix 1: Locating and Assembling BES Data

Using the ‘health-of-the-system’ approach, we sought to examine the various factors implicated in the creation of an effective learning community. We investigated a number of measures that fell naturally from the ‘what’, ‘why’, ‘how’, and ‘under what conditions’ questions concerning pedagogical approaches that facilitate learning for all students. The task was a considerable one, involving information management, the engagement of advisory and audit groups, and the seeking of contributions from the education community in general and the mathematics education community in particular. This level of engagement ensured that the Best Evidence Synthesis would be inclusive of views from across the community.

Our initial search strategy required us to pay attention to different contexts, different communities, and multiple ways of thinking and working. With this in mind, we undertook a literature search that crossed national and international boundaries. We used a range of search engines as well as personal networks to help us find academic journals, theses, projects, and other scholarly work with a focus on mathematics in New Zealand schools and centres, and by selected authors worldwide. We searched both print indices and electronic indices, endeavouring to make our search as broad as possible within the limits of manageability. This search took into account relevant publications from the general education literature and from the literature that relates to specialist areas of education. The search covered:

- key mathematics education literature including all major mathematics education journals (e.g., *Journal for Research in Mathematics Education*, *Educational Studies in Mathematics*, *Journal of Mathematics Teacher Education*, *For the Learning of Mathematics*, *The Journal of Mathematical Behaviour*), international conference proceedings (e.g., PME, ICME), Mathematics Research Group of Australasia publications, and international handbooks of mathematics education (e.g., Bishop et al., 2003);
- relevant New Zealand-based studies, reports, and thesis databases, supported by input from the professional community and the Ministry of Education;
- education journals (e.g., *American Educational Research Journal*, *British Educational Research Journal*, *Cognition and Instruction*, *The Elementary School Journal*, *Learning and Instruction*, etc.) and New Zealand work (e.g., SAMEpapers, SET, NZJES);
- specialist journals and projects, especially those located within the wider education field (e.g., *New Zealand Research in Early Childhood Education*, *Journal of Learning Disabilities*);
- landmark international studies including TIMSS, PISA, the UK Leverhulme projects.

This search strategy led us to a large body of literature that had something to say about facilitating mathematics learning: the total number of sourced references was just over 1100. Table 1 categorises these references by source:

Table 1: Initial Database Composition by Source

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<thead>
<tr>
<th>Source of data</th>
<th>Relative frequency (n ~1100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics education journals</td>
<td>24%</td>
</tr>
<tr>
<td>Mathematics education reports, books, handbooks</td>
<td>16%</td>
</tr>
<tr>
<td>Mathematics education conference proceedings</td>
<td>15%</td>
</tr>
<tr>
<td>Theses and projects</td>
<td>6%</td>
</tr>
<tr>
<td>General education reports, books, handbooks</td>
<td>10%</td>
</tr>
<tr>
<td>General education journals, reports, and proceedings</td>
<td>19%</td>
</tr>
<tr>
<td>Specialist journals</td>
<td>10%</td>
</tr>
</tbody>
</table>
All entries were stored and categorised using EndNote. To assist in the initial synthesis, we distinguished between ‘research’ and ‘discussion document’, and categorised entries according to (a) our ‘diversity’ descriptors (e.g., ethnicity, gender, socioeconomic), (b) centre/school level, and (c) country-of-origin of the data.

These categories and sub-divisions served as a useful starting point for overviewing the literature and allowed us to foreground our fundamental intent to be responsive to diversity. In addition, by classifying entries according to sector and country of origin, we gave ourselves a constant reminder of the need to be inclusive of all perspectives and interests. This inclusiveness gave us a body of literature comprising diverse frameworks and eclectic methodological and analytic approaches.

**Selecting the evidence**

Given the complexity of the teaching and learning process, it is not an easy matter to link specific outcomes with specific pedagogical approaches. In our first pass through the literature, we noted that studies could claim that student achievement was influenced by pedagogical practice much more readily than they could explain how that practice affected student achievement. Many studies offered detailed explanations of student outcomes yet failed to draw conclusive evidence about how those outcomes related to specific teaching practices. Others provided detailed explanations of pedagogical practice yet made unsubstantiated claims about, or provided only inferential evidence for, how those practices connected with student outcomes.

Granted, we were not looking for linear explanations. As Sfard (2005) points out, the complexity of the teaching–learning relationship “precludes the possibility of identifying clear-cut cause–effect relationships” (p. 407). What we were searching for were studies that were able “to offer a developing picture of what it looks like for a teacher’s practice to cultivate student [proficiency]” (Blanton & Kaput, 2005, p. 440). We were searching for studies that offered a “detailed look at how [teachers’] actions played out in the classroom and how students were involved in this” (Blanton & Kaput, 2005, p. 435) and the sorts of mathematical proficiency that resulted. Specifically, we were seeking studies that offered not just detailed descriptions of pedagogy and outcomes but rigorous explanation for close associations between pedagogical practice and particular outcomes.

Paying attention to diverse forms of research evidence required our serious consideration of the literature relating to disparate factors from different sectors and representative of different time periods. Luke and Hogan (in press) note that what is distinctive about the approach undertaken in the New Zealand Best Evidence programme “is its willingness to consider all forms of research evidence regardless of methodological paradigms and ideological rectitude, and its concern in finding contextually effective appropriate and locally powerful examples of ‘what works’... with particular populations, in particular settings, to particular educational ends” (p. 5). We have included many different kinds of evidence that take into account human volition, programme variability, cultural diversity, and multiple perspectives. Each form of evidence, characterised by its own way of looking at the world, has led to different kinds of truth claims and different ways of investigating the truth. Our pluralist stance left us free to consider the relative strengths and weaknesses of different methodological approaches.

A fundamental challenge for this BES has been to demonstrate a basis for knowledge claims. We are absolutely aware that, like data selection, assessment of evidential claims from secondary sources is a highly perspectival activity. “Even those gazing down a microscope are as capable of finding what they expect to find, or want to find, as anyone else” (Davies, 2003). In response to this challenge, studies have been reported in a way that will make the original evidence as transparent as possible. Informed by the Guidelines for Generating a Best Evidence Synthesis Iteration 2004, we included studies that:

- provided a description of the context, the sample, and the data;
• offered details about the particular pedagogy and the specific outcomes;
• connected research to relevant literature and theory;
• used methods that allow investigation of the pedagogy–outcome link;
• yielded findings that illuminated what did or did not work.

The Guidelines for Generating a Best Evidence Synthesis Iteration allowed us to deal not only with a diversity of research topics, approaches, and methods, but also to capture differences in the context, practices, and ways of thinking of researchers. The method employed in this BES for evaluating validity required us to look at the ways different pieces of data meshed together and to determine the plausibility, coherence, and trustworthiness of the interpretation offered.

Assessments about the quality of research depend to a large extent on the nature of the knowledge claims made and the degree of explanatory coherence between those claims and the evidence provided. What we were looking for was the explanatory power of the stated pedagogy–outcome link. When assessing the nature and strength of the causal relations between pedagogical approaches and learning outcomes, we were guided by Maxwell’s (2004) categorisations of two types of explanations of causality. The first type, the regularity view of causation, is based on observed regularities across a number of cases. The second type, process-oriented explanations, sees “causality as fundamentally referring to the actual causal mechanisms and processes that are involved in particular events and situations” (p. 4). Cobb argues (2006, personal communication) that regularity explanations are particularly useful for policy makers, while process-oriented explanations are relevant to teachers, who are concerned with “the mechanism through which and the conditions under which that causal relationship holds” (Shadish, Cook, & Campbell, 2002, p. 9, cited in Maxwell, 2004, p. 4). Attending to both types of explanation of causality meant including both large-scale and single-case studies. In many instances, we have found it useful to present a single case—a learner or teacher, a classroom, or a school—in the form of a vignette to exemplify the relations between learning processes and the means by which they are supported.

Research sources in this BES report

This BES report contains approximately 660 references. Included amongst these are research reports of empirical studies, ranging from very small, single-site settings (e.g., Hunter, 2002) to large-scale longitudinal studies (e.g., Balfanz, MacIver, and Byrnes, 2006). Some of the larger studies have multiple references because they include different papers/conference proceedings/book chapters or because they embrace work authored by different researchers (e.g., the New Zealand Numeracy Development Project). In addition, the references include reports containing educational statistics and policy, theoretical writings, and commentaries and reviews on multiple research findings (e.g., van Tassel-Baska, 1997).

The Guidelines for Generating a Best Evidence Synthesis Iteration point to the importance of drawing on New Zealand research in order to illuminate what works in the New Zealand context. However, despite an exhaustive search for New Zealand work, it is apparent (see chapter 8 for further discussion) that the strengths and foci of New Zealand research are not evenly distributed. In some areas—for example, early years education—there are relatively few New Zealand (or Australian) researchers working with a specific focus on mathematics education (Walshaw & Anthony, 2004). Table 2 shows the country of origin of the literature included in this BES. The numbers reflect New Zealand’s relatively new positioning within the international mathematics education research community.
Table 2: Database composition according to country

<table>
<thead>
<tr>
<th>Country</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Zealand</td>
<td>27%</td>
</tr>
<tr>
<td>Australia</td>
<td>17%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>11%</td>
</tr>
<tr>
<td>United States</td>
<td>49%</td>
</tr>
<tr>
<td>Other (e.g., Africa, Netherlands, Spain)</td>
<td>6%</td>
</tr>
</tbody>
</table>

Table 3 shows the proportion of the items included in the BES (both empirical studies and commentaries) that relates to each of the different sectors. Publications relating specifically to intermediate schools have been classified with the literature on primary schools.

Table 3: Database composition according to sector

<table>
<thead>
<tr>
<th>Sector</th>
<th>Relative Frequency (n=520)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preschool</td>
<td>18%</td>
</tr>
<tr>
<td>Primary school</td>
<td>48%</td>
</tr>
<tr>
<td>Secondary school</td>
<td>21%</td>
</tr>
<tr>
<td>Teacher education</td>
<td>13%</td>
</tr>
</tbody>
</table>

**Synthesising the data**

Our conceptual framework, outlined in chapter 2, offered a way of structuring the data. Within the community of practice frame in and beyond the classroom, we identified the following components: (a) the organisation of activities and the associated norms of participation, (b) discourse, particularly norms of mathematical argumentation, (c) the instructional tasks, and (d) the tools and resources that learners use. We began the iterative chapter-structuring process by outlining a number of key areas. These included mathematical thinking and identities, scaffolding and co-construction, tasks, activities, assessment, educational leadership, home–school/centre links, and wider school communities. Each of these served as a starting point for our exploration and was found, in the course of the investigation, to be a useful initial category for addressing questions of equity and proficiency in relation to effective mathematics teaching.

In time, we organised these categories more cohesively into groups. What we endeavoured to do was organise multiple elements, types, and levels and varying temporal conditions in line with the critical dimensions of a community of practice and the guiding principles established in chapter 2. The content of the subsequent chapters is shaped according to these dimensions and principles. Chapter 3 focuses on all three dimensions in a search for understanding of how pedagogy influences early years outcomes. Chapters 4 and 6 explore interrelationships that are centred on the joint enterprise of developing mathematical proficiency for all learners. Chapter 5 explores the role of mathematical tasks and the part that they play in enhancing students’ learning.

Reminding ourselves and readers that this BES synthesis is a product of currently accessible research, we concur with Atkinson’s (2000) view that “the purpose of educational research is surely not merely to provide ‘answers’ to the problems of the next decade or so, but to continue to inform discussion, among practitioners, researchers and policy-makers, about the nature, purpose and content of the educational enterprise” (p. 328). Rather than offering broad answers that promise much and achieve little, it is our hope that the structure we have used will foster understanding, reflection, and action concerning the characteristics of effective pedagogical approaches in mathematics.
References


Appendix 2: URLs of citations

The following 22 papers/articles/chapters/books are suggested as potentially useful sources for teachers to engage more deeply with the range of issues raised in this best evidence synthesis iteration. Readers are encouraged to source and read them. Several are available online; the others can be sourced through libraries.

The full citations are hyperlinked in the online PDF. For the convenience of those using a hard copy of the text, the URLs are listed below.

Carpenter, Thomas P ; Franke, Megan L ; Jacobs, Victoria R
A longitudinal study of invention and understanding in children’s multidigit addition and subtraction
http://nzcer.org.nz/BES.php?id=BES001

Clarke, Barbara ; Clarke, Doug
Mathematics teaching in Grades K-2: painting a picture of challenging supportive, and effective classrooms

Cobb, Paul ; Boufi, Ada ; McClain, Kay ; Whitenack, Jor
Reflective discourse and collective reflection
http://nzcer.org.nz/BES.php?id=BES020

Empson, Susan B
Low performing students and teaching fractions for understanding: An interactions analysis
http://nzcer.org.nz/BES.php?id=BES021

Fraivillig, Judith L ; Murphy, Laren A ; Fuson, Karen C
Advancing children’s mathematical thinking in everyday mathematics classrooms
http://nzcer.org.nz/BES.php?id=BES003

Gifford, Sue
A new mathematics pedagogy for the early years: in search of principles for practice
http://nzcer.org.nz/BES.php?id=BES004

Goos, Merrilyn
Learning mathematics a classroom community of inquiry
http://nzcer.org.nz/BES.php?id=BES005

Houssart, Jenny
Simplification and repetition of mathematical tasks: a recipe for success or failure?
http://nzcer.org.nz/BES.php?id=BES006

Irwin, Kathie ; Woodward, J (paper available online)
A snapshot of the discourse used in mathematics where students are mostly Pasifika (a case study in two classrooms)
http://nzcer.org.nz/BES.php?id=BES007

Kazemi, Elham ; Stipek, Deborah
Promoting conceptual thinking in four upper-elementary mathematics classrooms
http://nzcer.org.nz/BES.php?id=BES008

Latu, Viliami (paper available online)
Language factors that affect mathematics teaching and learning of Pasifika students
http://nzcer.org.nz/BES.php?id=BES009

O’Connor, Mary Catherine
“Can any fraction be turned into a decimal?” A case study of the mathematical group discussion
http://nzcer.org.nz/BES.php?id=BES010

Rietveld, Christine M.
Classroom learning experiences of mathematics by new entrant children with Down syndrome

Savell, Jan ; Anthony, Glenda Joy
Crossing the home-school boundary in mathematics
http://nzcer.org.nz/BES.php?id=BES049
Sheldon, Steven B; Epstein, Joyce L
Involvement counts: family and community partnerships and mathematics achievement
http://nzcer.org.nz/BES.php?id=BES012

Smith, Margaret Schwan Smith; Henningsen, Marjorie A
Implementing standards-based mathematics instruction: a casebook for professional development

Steinberg, Ruth M; Empson, Susan B; Carpenter, Thomas P
Inquiry into children’s mathematical thinking as a means to teacher change
http://nzcer.org.nz/BES.php?id=BES014

Watson, Anne; De Geest, Els
Principled teaching for deep progress: improving mathematical learning beyond methods and material
http://nzcer.org.nz/BES.php?id=BES015

Wood, Terry (paper available online)
What does it mean to teach mathematics differently?
http://nzcer.org.nz/BES.php?id=BES016

Yackel, Erna; Cobb, Paul
Sociomathematical norms, argumentation, and autonomy in mathematics
http://nzcer.org.nz/BES.php?id=BES017

Young-Loveridge, Jenny (paper available online)
Students views about mathematics learning: a case study of one school involved in Great Expectations Project
http://nzcer.org.nz/BES.php?id=BES018

Zevenbergen, R
The construction of a mathematical habitus: implictions of ability grouping in the middle years
http://nzcer.org.nz/BES.php?id=BES019
Appendix 3: Glossary

The page reference for the first and/or most important occurrence of the term is given in brackets.

Cognitive engagement (p. 2). The state of being engaged in thinking

Community of Practice (p. 6). The complex network of relationships within which teachers teach and students learn

Connectionist teachers (p. 97). Teachers who consistently make connections between different aspects of mathematics

Decile (p. 9). In New Zealand, a 1–10 system used by the Ministry of Education to indicate the socio-economic status of the communities from which schools draw their students; low-decile schools receive a higher level of government funding

Developmental progressions (p. 47). Sequential learning pathways categorised as a series of steps

Empirical evidence (p. 24). Data that has been collected systematically for research purposes

Equity (p. 9). The principle based on the belief that social injustices should be redressed by allocating resources according to need, not power; in education, this may mean, amongst other things, the provision of different pedagogical approaches depending upon the needs of the learners

Family Math (p. 171). A US initiative designed to develop parents’ skills so they can work with their children on their mathematics

Feed the Mind (p. 45). A media campaign funded by the New Zealand Ministry of Education and designed to support family involvement in children’s learning

High or low press for understanding (p. 121). Differing levels of cognitive engagement demanded of students by teachers for clarification of thinking

Kahoa (p. 36). A festive necklace (Tongan)

Kōhanga reo (p. 49). Màori-medium early childhood centres

Kura kaupapa Màori (p. 10). Màori-medium schools (kura = school), based on a Màori philosophy of learning (see pp. 54–5)

Manipulatives (p. 133). Any concrete materials used by students to model mathematical relationships

Mathematical argumentation (p. 123). Presenting a case to support or refute a premise developed by mathematical thinking

Mathematical identity (p. 19). How a student sees him/herself as a learner and doer of mathematics

Metacognition (p. 38). The knowledge and processes involved in thinking about and regulating one’s own thinking, which is essential for reflecting, self-monitoring, and planning

Norms of participation (p. 54). The rules, spoken or unspoken, that govern the way students behave and contribute in the classroom

Number Framework (p. 109). A model, structured in 8 stages, showing how students typically develop understanding of number and number operations (New Zealand, NDP)

Number sense (p. 98). An understanding of the relationships, patterns, and fundamental reasonableness that lie behind all mathematical operations

Numeracy (p. 28). The ability to use mathematics effectively, fluently, and with understanding in a wide variety of contexts

Numeracy Development Project (NDP) (pp. 9, 17). The central part of the New Zealand Ministry of Education’s Numeracy Strategy, which has as its primary objective the raising of student achievement in numeracy through lifting teachers’ professional capability

NumPA (p. 9). A structured, diagnostic interview used by teachers to place students on the early stages of the Number Framework (New Zealand, NDP)

Open-ended tasks (p. 106). Tasks that require students to engage in problem definition and formulation before beginning to think about a solution

Pasifika students (p. 9). Students whose families have come from Sàmoa, Tonga, the Cook Islands, Niue, Tokelau, Tuvalu, and some other, smaller Pacific nations

Pedagogical Content Knowledge (p. 199). In this context, knowledge about mathematics and how to teach it as well as knowledge about how to understand students’ thinking about mathematics

Pedagogy (p. 5). The processes and actions by which teachers engage students in learning

Poi (p. 26). A small ball, often made of woven flax, on the end of a length of string; swung rhythmically by women when performing action songs (Màori)

QUASAR (p. 95). A programme developed to help urban students develop understanding of mathematical ideas through engagement with challenging mathematical tasks

Revoicing (p. 78). The repeating, rephrasing, or expansion of student talk in order to clarify or highlight content, extend reasoning, introduce new ideas, or move discussion in another direction

Scaffolding (p. 27). Temporary, structured support designed to move learners forward in their thinking
School–home or home–school partnership (p. 160). The deliberate nurturing of relationships between the school and the home, in the interests of better supporting student learning

Sociocultural practices (p. 19). Practices relating to the social and cultural aspects of participation in the classroom

Sociocultural theory (p. 24). The theory that learning arises out of social interaction

Socio-economic status (SES) (p. 30). Categorisation of individuals or communities, based on income, family background, and qualifications

Sociomathematical norms (pp. 61–62). Shared understandings of the processes by which students and teacher contribute to a mathematical discussion

Tasks (p. 94). Defined by Doyle (1983) as “products that students are expected to produce, the operations that students are expected to use to generate those products, and the resources available to students while they are generating the products”

Te ao Māori (p. 54). The Māori world

Te Poutama Tau (p. 59). The Numeracy Project (New Zealand) as developed for implementation in Māori-medium schools

Te Whāriki (p. 24). The New Zealand early childhood curriculum (for children aged 5 or under)

Tukutuku panels (p. 115). A Māori craft form consisting of ornamental lattice-work panels woven together with strips of flax into intricate designs

Waiata (p. 26). A song (Māori)

Whānau (p. 41). Extended family (Māori)

Wharekura (p. 9). Māori-medium secondary schools, which are based on a Māori philosophy of learning

Zone of Proximal Development (ZPD) (p. 36). Vygotsky (1986) describes the ZPD as the “distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers”

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**Abbreviations**

CGI Cognitively Guided Instruction Project. .......................................................... pp. 17, 105

EAL English as an Additional Language ................................................................. p. 116

EFTPOS Electronic Funds Transfer at Point of Sale ................................................ p. 115

EMI-4s Enhancing the Mathematics of Four-Year-Olds. ........................................ p. 28

ENRP Early Numeracy Research Project ................................................................... p. 158

EPPE Effective Provision of Pre-school Education Project......................................... p. 25

ERO Education Review Office ................................................................................ p. 158

IAMP Improving Attainment in Mathematics Project ................................................ pp. 18, 99

ICME International Congress on Mathematics Education ....................................... p. 20

ICT Information and Communication Technologies ................................................. p. 27

IEA International Association for the Evaluation of Educational Achievement ........ p. 154

IMPACT Increasing the Mathematical Power of All Children and Teachers ............ p. 73

MEP Mathematics Enhancement Project .................................................................... p. 60

NCEA National Certificate of Educational Achievement ......................................... pp. 10, 66

NEMP National Education Monitoring Project ........................................................ p. 9

NNS National Numeracy Strategy ............................................................................ p. 17

PISA Program for International Student Assessment ............................................... p. 8

REPEY Researching Effective Pedagogy in the Early Years ...................................... p. 25

RME Realistic Mathematics Education ...................................................................... p. 113

TIMSS Third International Mathematics and Science Study .................................... p. 14

VAMP Values and Mathematics Project ................................................................... p. 58
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