Effective Pedagogy in Mathematics/Pāngarau

Best Evidence Synthesis Iteration [BES]

Glenda Anthony and Margaret Walshaw, Massey University
This report is one of a series of best evidence synthesis iterations (BESs) commissioned by the Ministry of Education. The Iterative Best Evidence Synthesis Programme is seeking to support collaborative knowledge building and use across policy, research and practice in education. BES draws together bodies of research evidence to explain what works and why to improve education outcomes, and to make a bigger difference for the education of all our children and young people.

Each BES is part of an iterative process that anticipates future research and development informing educational practice. This BES follows on from other BESs focused on quality teaching for diverse learners in early childhood education and schools. Its use will be informed by other BESs, focused on teacher professional learning and development and educational leadership. These documents will progressively become available at: [http://educationcounts.edcentre.govt.nz/goto/BES](http://educationcounts.edcentre.govt.nz/goto/BES)

Feedback is welcome at best.evidence@minedu.govt.nz

Note: the references printed in purple refer to a list of URLs in Appendix 2. These are a selection of potentially useful sources for teachers to engage more deeply with the range of issues raised in this best evidence synthesis iteration.
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About the writers

Glenda Anthony and Margaret Walshaw, both from the School of Curriculum and Pedagogy at Massey University, bring to this Best Evidence Synthesis (BES) decades of mathematics classroom teaching and educational research experience. They are acutely aware of the challenge that educators face in constructing a democratic mathematical community with which all students can identify. For them, making a positive difference to diverse learners’ outcomes is a central educational issue. At the heart of their work is a concerted effort to illuminate how this issue is best addressed. In this synthesis, they report on the outcome of their deliberations over, and search for, what makes a difference for diverse learners in mathematics/pāngarau.

Advisory Group

A core Advisory Group membership was selected to provide expertise and critique in relation to the various focuses of the BES, including Màori and Pasifika learners, early childhood, primary and secondary sectors, and teacher education. The authors wish to thank the members of this group:

- Dr Ian Christensen (Massey University and He Kupenga Hao i te Reo)
- Dr Joanna Higgins (Victoria University of Wellington)
- Roberta Hunter (Massey University)
- Garry Nathan (Auckland University)
- Dr Sally Peters (Waikato University)
- Assoc. Prof. Jenny Young-Loveridge (Waikato University)

We also wish to acknowledge the supportive formative feedback received from Faith Martin (Director, Massey Child Care Centre), Brian Paewai (Runanga Kura Kaupapa Màori), Professor Anne Smith (University of Otago) and Johanna Wood (Principal, Queen Elizabeth College, Palmerston North).

Ministry of Education advisory team

The Ministry of Education, led by Dr Adrienne Alton-Lee, has guided the development of the synthesis. The team at the Ministry also gave us access to additional literature and demographic and trend data. We thank all of the team.

External quality assurance

Professor Paul Cobb from Vanderbilt University, US, has provided invaluable assistance. We would like to acknowledge his scholarly critique and thank him for his knowledgeable contribution to the synthesis.

Formative quality assurance was also provided by: Maggie Haynes (Unitec), Professor Derek Holton (University of Otago), Tamsin Meaney (EARU, University of Otago), Lynne Peterson, Tony Trinick (Auckland University), initial and ongoing Teacher Education (Victoria University of Wellington), the New Zealand Educational Institute and representation from the Post Primary Teachers’ Association (Jill Gray). We wish to thank them all for their contributions.
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Thanks also for the significant contribution made to this and other BES developments through the advice given in the development of the Guidelines for Generating a Best Evidence Synthesis Iteration by the BES Standards Reference Group; The BES Māori Educational Research Advisory Group, the BES Pasifika Educational Research Advisory Group and Associate Professor Brian Haig, University of Canterbury.
Forewords

International

Even the casual visitor is struck by the dramatic changes that have occurred in New Zealand in the last 15 years. I have tuned in to local media on each of my four visits to get an initial sense of people’s current concerns and issues. Based on this narrow sampling, the New Zealand of 1991 was an immensely likeable country that had seen better days and was struggling to find its place in a rapidly changing world. Although innovation and experimentation appeared to be the watchwords of the day, there seemed to be an undercurrent of apprehension and anxiety as people attempted to cope with economic disruption. Today, New Zealand continues to be an immensely likeable place, but the visitor immediately notices a quiet, understated self-assurance. It has become a largely prosperous country that, in a very real sense, has reinvented itself as a leading information economy in an increasingly globalised world. Refreshingly for the visitor from the United States, there appears to be widespread belief that government will approach problems pragmatically and is capable of solving them. If the Iterative Best Evidence Synthesis Programme is representative of New Zealand government in action, this belief would appear to be well founded.

Put quite simply, the Iterative BES Programme is the most ambitious effort I have encountered that uses rigorous scientific evidence to guide the ongoing improvement of an education system at a national level. The programme has a strong pragmatic bent and is clearly grounded in the hard-won experience of synthesising research findings to inform both policy and teachers’ instructional practices. Four aspects of the programme are particularly noteworthy. The first is the overriding commitment to make the development of the best evidence syntheses transparent. This commitment takes concrete form in the exacting evaluation and feedback process that all BES reports undergo at each phase of their development, from the initial identification of relevant bodies of research literature through to the final critique and revision of the report. This is in the best traditions of science, where claims are justified in terms of the means by which they have been produced.

The second notable characteristic is a mature view of evidence and an emphasis on methodological and theoretical pluralism. This is important, given that attempts have been made in a number of countries, including the United States, to legislate what counts as scientific research in education on the basis of ideological adherence to a particular methodology. In taking an inclusive approach, the Iterative BES Programme acknowledges that different types of knowledge are of greatest use to teachers and to policymakers. Teachers make pedagogical decisions on the basis of a detailed understanding of specific students in particular classrooms at particular points in time. Policymakers, in contrast, typically need knowledge of trends and patterns that hold up across classrooms to make decisions that affect large numbers of students and teachers in multiple schools. Different methodologies are appropriate for developing these equally important types of knowledge.

The third noteworthy characteristic of the Iterative BES Programme is its focus on the explanatory power and coherence of theories. Priority is given to theories that give insight into learning processes and the specific means of supporting their realisation in classrooms. This pragmatic criterion is important in a field where theoretical perspectives continue to proliferate.

The final notable characteristic of the programme is its explicit attention to the issues of language and culture. This emphasis is clearly critical if New Zealand teachers and policymakers are to address the inequities inherent in the disturbingly large gaps in school achievement between children of different ethnic and racial groups. In keeping with the tenet of methodological and theoretical pluralism, the Iterative BES Programme uses group categories such as socioeconomic status, ethnicity, and culture as key variables in assessing efforts to achieve
equity. However, it avoids stereotyping children of particular racial, ethnic, or language groups by acknowledging the complexity of individual identity when explaining inequities in children’s learning opportunities. Furthermore, the programme emphasises ecological models of learning that link what is happening in classrooms both to the institutional contexts in which classrooms are located and to issues of race, culture, and language. It is here that the full ambition of the programme becomes apparent: few viable models of this type currently exist in education. The BES writers are therefore charged with the task of synthesising in the true sense of the term, that is, to combine disparate and sometimes fragmented bodies of research into a single, unified whole. At the risk of understatement, this is a formidable challenge.

The writers of this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau, Drs. Glenda Anthony and Margaret Walshaw, have risen to the challenge. They were charged with the daunting task of reviewing, organising, and synthesising all mathematics education research from the early childhood years through secondary school that relates classroom processes to student learning. On my reading, the resulting synthesis of over 600 research studies is directly relevant to teachers and will be educative for policymakers. The educative value of the report stems from Anthony and Walshaw’s focus on what goes on in mathematics classrooms, thereby providing a window on the complexity of effective pedagogy. The forms of pedagogical practice that they identify as effective are ambitious because they involve high expectations for all children’s mathematical learning. The goals at which these forms of pedagogy aim are best illustrated in chapter 7, A Fraction of the Answer, in which Anthony and Walshaw pull together the key insights of the proceeding chapters as they present an integrated series of cases that focus on the learning and teaching of fractions. As this chapter makes clear, the instructional goals for fractions are not limited to ensuring that children can add, subtract, multiply, and divide fractions successfully. Instead, the instructional objectives also focus on children’s development of a deep understanding of fractions as amounts or quantities. At an elementary level, children who are coming to understand fractions as quantities know that $\frac{1}{6}$ is smaller than $\frac{1}{5}$ because there will be more pieces when something is divided into 6 pieces than into 5 pieces, so the pieces must be smaller. At a more advanced level, students will be able to describe real world situations that involve multiplying and dividing fractional quantities. More generally, ambitious pedagogy focuses on central mathematical ideas and principles that give meaning to computational methods and strategies.

Anthony and Walshaw’s review of the relevant research indicates that central mathematical ideas and principles cannot be directly transmitted to children. However, the research also shows that discovery approaches that place children in rich environments and simply encourage them to inquire are also ineffective. Effective pedagogy is complex because it requires teachers to achieve a significant mathematical agenda by taking children’s current knowledge and interests as the starting point. As Anthony and Walshaw clarify, these forms of pedagogy involve a distinctive orientation towards teaching. First and foremost, the emphasis is on building on students’ existing proficiencies rather than filling gaps in students’ knowledge and remediating weaknesses. As a consequence, the teacher’s focus when planning for instruction is not on students’ limitations but on their current mathematical competencies and interests, as these constitute resources on which the teacher can build. More generally, effective mathematical pedagogy places students’ reasoning at the center of instructional decision making. As a consequence, the ongoing assessment of students’ reasoning is an integral aspect of instruction, not a separate activity conducted after the fact to check whether goals for students’ learning have been achieved. A key characteristic of accomplished teachers is that they continually adjust instruction, as informed by these ongoing assessments.

One of the strengths of Anthony and Walshaw’s synthesis is that it provides the reader with a concrete image of what effective mathematical pedagogy looks like. Anthony and Walshaw emphasise that a respectful, non-threatening classroom atmosphere in which all students feel comfortable in making contributions is necessary but not, by itself, sufficient. As they document, the research findings indicate unequivocally that it is also essential that classroom activity
and discourse focus explicitly on central mathematical ideas and processes. The selection of instructional tasks is therefore critical. On the one hand, it is important that task contexts or scenarios are accessible to all students, regardless of cultural background. On the other, the teacher should be able to capitalise on students’ solutions to support their development of increasingly sophisticated forms of mathematical reasoning. Thus, when designing and selecting tasks, the teacher has to take account both of students’ current competencies and interests and their long-term learning goals. As Anthony and Walshaw discuss in chapter 5, an important way in which the teacher can build students’ solutions is by introducing judiciously chosen tools and representations. A second, equally important way in which the teacher can capitalise on the potential of worthwhile mathematical tasks is to engage students in justification, abstraction, and generalisation (see chapter 4), by doing which they learn to speak the language of mathematics.

The image of effective mathematical pedagogy that emerges from Anthony and Walshaw’s synthesis is of teaching as a coherent system rather than a set of discrete, interchangeable strategies. This pedagogical system encompasses:

- a non-threatening classroom atmosphere;
- instructional tasks;
- tools and representations;
- classroom discourse.

To see that these four aspects of effective pedagogy constitute a system, note that the way in which instructional tasks are realised in the classroom and experienced by students depends on the classroom atmosphere, the tools and representations available for them to use, and the nature and focus of classroom discourse. And because effective pedagogy is a system, it makes little sense to think of student learning as being caused by isolated teacher actions or strategies. It is for this reason that Anthony and Walshaw speak of mathematical learning being occasioned by teaching. In using this term, Anthony and Walshaw emphasise the teacher’s proactive role in supporting students’ development of increasingly sophisticated forms of mathematical reasoning.

In addition to highlighting the systemic character of effective mathematical pedagogy, Anthony and Walshaw make good on the charge to develop an ecological model of learning that links what is happening in the classroom to issues of race, culture, and language, and to the school contexts in which teachers develop and revise their instructional practices. A concern for issues of equity permeate the entire report but come to the fore in the discussion of school–home partnerships that take the diverse cultures of students and their families seriously and treat them as instructional resources.

Anthony and Walshaw make it clear that it is essential to view school contexts as settings for teachers’ ongoing learning. In a very real sense, these settings mediate the extent to which high quality teacher professional development will result in significant changes in teachers’ classroom practices. Anthony and Walshaw’s synthesis documents that mathematics instruction that places students’ reasoning at the center of instructional decision making is demanding, uncertain, and not reducible to predictable routines. The available evidence indicates that a strong network of colleagues constitutes a crucial means of support for teachers as they attempt to cope with these uncertainties and the loss of established routines. Consequently, there is every reason to expect that improvement in teachers’ instructional practices and student learning will be greater in schools where mathematics teachers participate in learning communities whose activities focus on central mathematical ideas and how to relate them to student reasoning. The value of teacher learning communities in turn foregrounds the critical role of the principal as an instructional leader.

Historically, teaching and school leadership have been loosely coupled, with the classroom being treated as the preserve of the teacher while school leaders managed around instruction. Recent research findings demonstrate the limitations of this type of school organisation
in supporting the improvement of teaching on any scale. These findings also indicate that principals can play a key role in supporting the emergence of a shared vision of what effective mathematical pedagogy looks like and in supporting teacher collaboration that focuses on challenges central to the development of effective pedagogy. This alternative type of school organisation is characterised by reciprocal accountability. Teachers are accountable to principals for developing increasingly effective pedagogical practices and principals are accountable to teachers to create opportunities for their ongoing learning. Changes of this type in the relations between teachers and school administrators are far reaching and might be viewed as too radical. It is, however, sobering to note that previous large-scale efforts to improve the quality of classroom instruction have rarely produced lasting changes in teachers’ practices. Research into educational leadership and policy indicates that this history is due in large part to the failure to take into account the institutional settings in which teachers develop and refine their instructional practices.

The broader policy and leadership literature strongly indicates that the improvement of mathematics instruction on the scale being attempted in New Zealand is not simply a matter of providing high quality teacher professional development. It also has to be framed as a problem for schools as educational organisations that structure the institutional settings in which teachers develop and revise their instructional practices. My reading of this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau is that Anthony and Walshaw have distilled valuable lessons from the available research, thereby positioning New Zealand educators to succeed where others have failed.

Paul Cobb
Professor of Mathematics Education
Vanderbilt University, Tennessee

Note: The second Hans Freudenthal Medal of the International Commission on Mathematical Instruction (ICMI) was awarded to Professor Paul Cobb in 2005, “whose work is a rare combination of theoretical developments, empirical research and practical applications. His work has had a major influence on the mathematics education community and beyond.”

Early Childhood Education

This Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau is a ‘must read’ for those in the early childhood sector who want an insight into what effective mathematical pedagogy looks like in an early childhood service. The synthesis acknowledges the vital role that quality early childhood education plays in the mathematical development of infants and young children. It also provokes early childhood teachers to reflect on practice: their mathematical awareness of the environment, the depth of their mathematical knowledge, and the importance of effective teaching and learning strategies that will support children’s optimal engagement in mathematical experiences. The extensive, wide-ranging research information is effectively balanced by vignettes which involve the reader in meaningful mathematical experiences that illustrate the possibilities for supporting mathematical learning. Effective distribution of the synthesis would enhance teaching and learning outcomes in early childhood services.

Faith Martin
Director, Massey Child Care Centre
NZEI Te Riu Roa

NZEI Te Riu Roa welcomes this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau, particularly as it takes for its starting point the assertion that “all children can learn mathematics”. This key message is at the heart of every teacher’s commitment to the mathematical learning of his or her students.

The synthesis recognises the complexity of teaching, particularly given the diverse learning needs of the students in our classrooms and centres and the necessity for specialised knowledge of mathematics. But the writers consistently underline the power that teachers have to make a difference: “It is what teachers do, think and believe (that) significantly influences student outcomes.”

A teacher’s role, whether in a school or a centre, includes the design of activities that help students to construct meaning and think for themselves. To achieve such outcomes, teachers need to appreciate the part that mathematics plays in the world around them, what the big mathematical ideas are, and how the concepts that they teach fit in with those ideas. They need to know how to teach knowledge and skills, how to match new learning with students’ prior knowledge, and which activities effectively encourage understanding and learning. Teachers also need to be conscious of developing attitudes and values. They need to create opportunities for their students to develop a critical eye and, in the context of this synthesis, a critical mathematical eye.

The primary purpose of the synthesis is to identify evidence that links pedagogical practice with effective mathematics outcomes for students. To achieve this, the writers have drawn on national and international research that contributes to our understanding of what works in mathematics education.

When reviewing the synthesis in its draft form, NZEI teachers were particularly pleased to read the chapter, Mathematics Practices Outside the Classroom, which they saw as contributing to a constructive environment and encouraging of good practice. The synthesis explores ways in which parents can contribute to their children’s mathematical development and ways in which schools can strengthen links with the home. If teachers are to successfully fulfil expectations, such links are likely to be vital. Teachers were also pleased to see the importance of school leadership recognised.

NZEI sees the Effective Pedagogy in Mathematics/Pāngarau BES as being of great benefit to teachers, teacher educators, and policymakers. The research identified in the synthesis, together with the case studies and vignettes, has the potential to stimulate much constructive professional discussion. To maximise its potential for teachers, it will need to be accompanied by professional learning opportunities and time for reflection and discussion in the school or centre setting.

Irene Cooper
National President
Te Manukura
NZEI Te Riu Roa
Post Primary Teachers’ Association

Tēnā koutou, tēnā koutou, tēnā tatou katoa.

PPTA welcomes this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau. It is the result of a very thorough process, inclusive of the expertise of practitioners. The final report reflects and caters to their realities, and provides some very interesting and thought-provoking reading for teachers themselves, and for those involved in the pre-service and in-service education of mathematics teachers. At the same time, the research highlights the shortage of outcomes-linked research evidence specific to secondary school mathematics teaching and we hope that as a result of this BES, New Zealand researchers will step up to fill this gap.

Debbie Te Whaiti
President
New Zealand Post Primary Teachers’ Association

Teacher Educators

The Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau succeeds in providing a systematic treatment of relevant outcomes-based evidence for what works for diverse learners in the New Zealand education system. One of the strengths of the document is the central positioning given by its authors to a broad notion of diversity.

Teacher educators, both initial and ongoing, will find that the BES is an invitation to engage—as teachers and as researchers—with a wide range of national and international studies. The document succeeds in preserving the complexity of pedagogical approaches through careful structuring and presentation. Well chosen classroom vignettes capture the essence of pedagogical issues for use in initial and ongoing teacher education. The CASEs are likely to prove particularly valuable for teachers by demonstrating how research can inform classroom practice.

The BES also presents a challenge to New Zealand researchers by identifying areas in which there is a paucity of outcomes-based evidence. Such evidence is scarce for Māori-medium mathematics classrooms. The senior secondary area is generally not well represented and a wider range of early childhood contexts needs to be investigated. The CASEs highlight for teacher educators the possibilities of writing up research projects undertaken as part of ongoing teacher education initiatives, and encourage them to gather further evidence to support practice.

The importance to mathematics education of the outcomes-based research evidence represented in this synthesis cannot be overstated. It is to be hoped that the value of the Iterative BES programme is widely recognised, and that it has the impact on policy and practice that it ought.

Joanna Higgins
Director, Mathematics Education Unit and Associate Director,
Jessie Hetherington Centre for Educational Research
Victoria University of Wellington
Maori-medium Mathematics

E nga mana, e nga reo, tēnā koutou katoa.

For the last 20 years, the teaching of pāngarau (mathematics) has played a significant role in the revitalisation of te reo Māori. The Effective Pedagogy in Mathematics/Pāngarau BES recognises the close relationship that exists between language and the learning and teaching of mathematics.

The BES identifies a range of major considerations and challenges for teachers and all those involved in Māori-medium education. The research makes it clear that mathematical outcomes for students are affected by a complex network of interrelated factors and environments, not just individual preferences or the language of instruction. By identifying the key elements in this network and discussing the relevant research, the writers have created what should prove a very useful resource.

The BES highlights the paucity of research into Māori-medium mathematics education, particularly in the area of teacher practice.

Tony Trinick
Māori-medium mathematics educator
Faculty of Education
The University of Auckland

Pasifika

E rima te'arapaki, te aro'a, te ko'uko'u te utuutu, 'iaku nei.
Under the protection of caring hands there's feeling of love and affection.

The Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau has drawn together a comprehensive synthesis of evidence that relates to quality mathematics pedagogical practices. Its particular strength is that it provides stimulating and thought-provoking reading for a range of stakeholders and at the same time affirms that there is no one, specific, 'quality' pedagogical approach. Rather, it directs attention to many effective approaches which make a difference for all mathematics learners. The vignettes are an added strength; they make the theoretical structures they illustrate accessible to a wider audience.

The synthesis highlights the shortage of outcomes-linked research evidence concerning quality teaching and learning for Pasifika students at all levels of schooling. It also highlights the importance of a culture of care. How this translates into quality outcomes for Pasifika students requires the attention of New Zealand researchers.

Roberta Hunter
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The Effective Pedagogy in Mathematics/Pāngarau BES sets out to uncover and explain the links between what we do in mathematics education and what the outcomes are for learners. The result is a valuable resource that can be used to enhance a wide range of outcomes for diverse learners. These include the ability to think creatively, critically, strategically and logically; mathematical knowledge; enjoyment of intellectual challenge; self-regulatory, collaborative and problem-solving skills; and the disposition to use, enjoy and build upon that knowledge throughout life.

The BES reflects the outstanding scholarly work and professional leadership of co-authors Drs Glenda Anthony and Margaret Walshaw of Massey University. They are the first to use the new Guidelines for Generating a Best Evidence Synthesis and follow the collaborative development process that is central to the Iterative BES Programme. They have consulted tirelessly and responsively with a wide range of early childhood teachers, primary and secondary teachers, principals, advisers, researchers, policy workers and teacher educator colleagues from across New Zealand, and with international colleagues. The Ministry of Education acknowledges and values all these contributions—and those of the formative quality assurers, whose affirmations and challenges have been so helpful in optimising the quality and potential usefulness of this BES.

The BES celebrates and returns to early childhood educators, teachers, teacher educators and researchers a record of their professional work, highlighting the complexity of that work, and suggesting how research evidence can be a valuable resource to inform their ongoing work and that of their colleagues. From the first vignette explaining how mathematical learning can be embedded in waiata (Māori song) and dance, the vignettes bring children’s learning in mathematics to life. The underlying explanations and theoretical findings have the power to inform practice in ways that are relevant and responsive to the learners in any particular centre or classroom.

The challenge now is for us all is to use this resource in ways that will support further systemic development in mathematics education, with strengthened outcomes for diverse learners. In many cases, the BES will affirm what is already happening, but it will be the points of challenge that take us forward. Individual teachers have already engaged with the BES in its draft form, and some report remarkable insights and developments in their practice. But it is only through the wider and systemic development of the conditions that support effective practice for diverse learners that improvements will proliferate and become self-sustaining. The findings emerging from the outcomes-linked professional learning and development BESs should be an invaluable resource in determining how to generate changed practice on such a scale.

Many teachers and early childhood educators have indicated that they want to read this BES for themselves, and to do this they need time. They need time to read, discuss and consider how they can use relevant BES findings in response to diagnostic information about the mathematical understandings of the children and young people they teach. They also need time to participate in professional learning communities. The Teacher Professional Learning and Development BES finds that such participation doesn’t guarantee better outcomes for students, but it is a consistent feature of teacher professional learning that does have a strong positive impact.

The same BES highlights the important role that external expertise with strong pedagogical content knowledge can play in facilitating and supporting changes in practice that impacts positively on student outcomes. Such expertise can be vital in engaging teachers’ theories and challenging problematic discourses. The findings do, however, caution that ‘experts’ need more than good intentions—in the worst-case scenario, teacher professional development can actually impact negatively on student achievement. This finding calls for careful and iterative evaluation of the effectiveness of all professional development.
The teacher education community in New Zealand has already made a foundational contribution to this BES with its engagement in the research and development reported in this BES, and its advice to the BES writers. As the Teacher Professional Learning and Development BES\(^4\) will show, some of our most effective professional development has been taking place as part of the Numeracy Development Projects (NDP)—with effect sizes twice those attained in England\(^5,6,7\). The primary and early childhood teachers’ union, NZEI, confirms what the evaluation reports have been saying: that teachers who have been involved in the NDP value the transformational experiences this professional learning has afforded them. Two teachers from a Hawkes Bay school explained to me recently that, as a result of professional learning undertaken through the NDP, they have changed the way they work across the curriculum—they now listen more, are more diagnostic, and they place much more emphasis on children articulating and sharing their learning strategies. The dynamic, reflective, nation-wide learning community of researchers, teacher educators, teachers, and other educators created by the NDP and its Māori-medium counterpart, Te Poutama Tau, has been inspirational for BES.

If the mathematics BES is to serve New Zealand education well, the teacher education and research communities must make it a ‘living’ BES by building on the powerful insights and exemplars it makes available, addressing the gaps, and ensuring a cumulative and increasingly dynamic shared knowledge base about what works for learners in New Zealand education. To assist in this collaborative work, the New Zealand Council for Educational Research is creating a database of relevant New Zealand education theses. It has already built a database to support this document, with live links to the electronic version so that readers can quickly access either the full text or bibliographic details for some of the most helpful articles that have informed the synthesis. These links are also listed in the print version.

It is our hope that this BES will stimulate readers to let the Iterative Best Evidence Synthesis Programme know of other/new research and development that should feature in future iterations of the synthesis. Such research needs to clearly document demonstrated or triangulated links to student outcomes (see the Guidelines for Generating a Best Evidence Synthesis Iteration, found on the BES website\(^8\)), and preferably show larger positive impacts on desired outcomes for diverse learners. We are especially seeking studies of research and development in New Zealand contexts, but we are also interested in information on overseas studies that show particularly large impacts on diverse learners. Please send details to best.evidence@minedu.govt.nz.

In the New Zealand context, where schools and centres are self managing, principals and centre leaders have a critical role to play in supporting their staff to realise the potential of this BES. The Teacher Professional Learning and Development BES indicates that, in the case of the most effective school-based interventions, principals and others in leadership roles have actively supported the development of a learning culture amongst their teachers.

For centuries, societies have required their education systems to sort children into successes and failures. Knowledge societies, such as our own, require much more. Our challenge is to ensure that all our children flourish as learners, strong in their own identities, and confident global citizens.

To achieve such goals, we need to value, build upon, and go beyond the craft practice traditions that require each teacher to ‘rediscover the wheel’. The Effective Pedagogy in Mathematics/Pāngarau BES has been designed to serve as a resource and catalyst for strengthened practice, innovation, and systemic learning. By using it, and by making learner outcomes our touchstone, we can work together to give our children a mathematics education that prepares them well for the opportunities and challenges that will be their future.

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3 Ibid.

4 Timperley et al., to be published 2007.


6 Timperley et al., to be published 2007.


8 http://educationcounts.edcentre.govt.nz/goto/BES
Authors’ Preface

What is a Best Evidence Synthesis in Mathematics?

A best evidence synthesis draws together available evidence about what pedagogical approaches work to improve student outcomes in Mathematics/Pāngarau. This synthesis is part of the Iterative Best Evidence Synthesis (BES) Programme established late in 2003 by the Ministry of Education to deepen understanding of what works in education. The programme involves policy, research, and practice in collaborative knowledge building, aimed at maximising desirable outcomes for the diverse learners in the New Zealand education system.

This best evidence synthesis in Mathematics/Pāngarau plays a key role in knowledge building for New Zealand education. As a capability tool, it identifies, evaluates, analyses, and syntheseses what the New Zealand evidence and international research tell us about quality mathematics teaching. It shows us how different contexts, systems, policies, resources, approaches, practices, and influences all impact on learners in different ways. Importantly, it illuminates what the evidence suggests can optimise outcomes for diverse mathematics learners.

The importance of dialogue

The development of this BES has been shaped by the Guidelines for Generating a Best Evidence Synthesis Iteration (Alton-Lee, 2004) and informed by dialogue amongst policy makers, educators, researchers and practitioners. Right from the very early stages of its development, the health-of-the-system perspective taken in this synthesis has ensured that we have listened to and responded to the viewpoints of a wide range of constituencies. Our interactions with these multiple communities have revealed to us the key roles that infrastructure, context, settings, and accountabilities play in a system that is functioning effectively for all its learners. Our various stakeholders have challenged us not only to produce better and more relevant educational research but to consider how this knowledge base might best be used. It is our hope that this discussion across sectors will be ongoing.

We have received a strong and positive response to the best evidence synthesis work from New Zealand’s primary and post-primary teacher associations. Both have reported on how helpful the synthesis is to their core professional work. For example, the New Zealand Educational Institute ( NZEI) writes: “In our view, the writers have drawn on national and international research which contributes to an understanding of what works in mathematics education; they have identified the significance of the context and ways in which to strengthen practice … We liked the … underpinning view that all children can learn mathematics” (p. 2). The representative for the Post Primary Teachers’ Association at the Quality Assurance Day is reported as saying: ”There are numerous wonderful ideas in the synthesis, and I found myself repeatedly jolted into possibilities for my own classroom resources.” In addition, a group of initial and ongoing mathematics teacher educators have welcomed the “sophisticated treatment of diversity” and the way in which “the complexity of pedagogical approaches is preserved” (Victoria University of Wellington College of Education, 2006, p. 1).

Writing for multiple audiences

Our task was to make the findings of the synthesis accessible to and of benefit to a range of educational stakeholders. At one level of application, it is intended to provide a strengthened basis of knowledge about mathematics pedagogical practices in New Zealand today. The evidence it produces is expected to inform teacher educators within the discipline of mathematics education about effective pedagogical practice. At another level, the synthesis attempts to make transparent to policy makers and social planners an evidential basis for quality pedagogical approaches in mathematics. At a third level, the synthesis is expected to benefit practitioners and assist them in doing the best possible job for diverse learners in their classrooms.
Our approach to the “almost overwhelming task” (Cobb, 2006) of writing with several levels of application in mind has been to draw on both formal and informal approaches. We have offset the ‘academic’ language of the BES by including a series of vignettes that expand upon broad findings. We have received feedback from a range of sources that these vignettes bring the reality of classroom life to the fore and, in particular, do not minimise the complexities of actual practice. We hope that researchers, policy makers and practitioners alike will see in the vignettes theoretical tools that have been adapted and used by actual teachers.

**The BES as a catalyst for change**

This best evidence synthesis in mathematics does more than synthesise and explain evidence about what works for diverse learners. By bringing together rigorous and useful bodies of evidence about what works in mathematics, the project plays an important function as a catalyst for change. It is designed to help strengthen education policy and educational development in ways that effectively address both the needs of diverse learners and patterns of systemic underachievement in New Zealand education. It is written with the intent of stimulating activity across practitioners, policy makers, and researchers and so to strengthen system responsiveness to educational outcomes for all students.

The writers anticipate that reflection on the findings will lead to sustainable educational development that has a positive impact on learners. It will create new insights into what makes a difference for our children and young people. Reflection on the findings will also spark new questions and renewed, fruitful engagement with mathematics education. These new questions, in turn, will render the BES a snapshot in time—provisional and subject to future change.

**Key features**

Key features of the BES are:

- Its teacher orientation. Its view is towards a strengthened basis of knowledge about instructional practices that make a difference for diverse groups of learners.
- Its cross-sectoral approach. Its scope takes in the teaching of children in early childhood centres through to the teaching of learners in senior secondary school classrooms.
- Its inclusiveness. It documents research that reveals significant educational benefits for a wide range of diverse learners. It pays particular attention to the mathematical development of Māori and Pasifika students and documents research that captures the multiple identities held by New Zealand learners.
- Its breadth of search coverage. It reports on the characteristics of effective pedagogy, following searches through multiple national databases and inventories as well as masters’ projects and theses. It provides comprehensive information about effective teaching as evidenced from small cases, large-scale explorations, and short-term and longitudinal investigations.
- Its local character. It makes explicit links between claims and bodies of evidence that have successfully translated the intentions and spirit of the Treaty of Waitangi. It identifies research relevant to the particular conditions and contexts in New Zealand, both in mathematics education in particular and in education in general, in relation to the principles and goals of Te Whāriki for early childhood settings and of The Curriculum Framework, for teachers in English or Māori-medium settings.
- Its global linkages. It connects local sources with the international literature. It identifies important Australian and international work in the area and evaluates that wide-ranging resource in relation to similarities and differences in cultures.
populations and demographics between the country of origin and New Zealand.

- Its responsiveness to concerns about democratic participation. It heeds the concern about the development of competencies that equip students for lifelong learning. This orientation coincides with the national mathematics curriculum objective of developing those knowledges, skills, and identities that will enable students to meet and respond creatively to real-life challenges.

- Its quality assurance measures. It is guided by principles of transparency, accessibility, relevance, trustworthiness, rigour, and comprehensiveness. These principles form the backdrop to the selection and systematic integration of evidence.

- Its strategic focus on policy and social planning. It uses a health-of-the-system approach to address one of the most pressing problems in education, provide a direction for future growth, and push effective teaching beyond current understandings.

- Its provisional nature. The project is an important knowledge-building tool, creating new insights from what has gone before, and will be updated in the light of findings from new studies. The findings are, above all, 'of the moment' and open to future change.

References


Executive Summary

The Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration (BES) was funded by a Ministry of Education contract awarded to Associate Professor Glenda Anthony and Dr Margaret Walshaw at Massey University. The synthesis is part of the Iterative Best Evidence Synthesis (BES) Programme, established by the Ministry of Education in New Zealand, to deepen understanding from the research literature of what is effective in education for diverse learners. The synthesis represents a systematic and credible evidence base about quality teaching in mathematics and explains the sort of pedagogical approaches that lead to improved engagement and desirable outcomes for learners from diverse social groups. It marks out the complexity of teaching and provides insight into the ways in which learners’ mathematical identities and accomplishments are occasioned by effective pedagogical practices.

The search of the literature focused attention on different contexts, different communities, and multiple ways of thinking and working. Priority was given to New Zealand research into mathematics in early childhood centres and schools, both English- and Māori-medium. Personal networks enhanced the library search and facilitated access to academic journals, theses and reports, as well as other local scholarly work. The New Zealand literature was complemented by reputable work undertaken in other countries with similar population and demographic characteristics. Indices, both print and electronic, were sourced, and the search covered relevant publications within the general education literature as well as specialist educational areas. In the end, 660 pieces of research, ranging from very small, single-site studies to large scale, longitudinal, experimental studies, found their way into the report.

Key findings highlight practices that relate specifically to effective mathematics teaching and to positive learning and social outcomes in centres/kōhanga and schools/kura. The findings stress the importance of interrelationships and the development of community in the classroom. They also reveal that both the cognitive and material decisions made by teachers concerning the mathematics tasks and activities they use, significantly influence learning. The findings demonstrate the importance of children’s early mathematical experiences and stress that constituting and developing children’s mathematical identities is a joint enterprise of teacher, centre/school, and family/whānau.

Key findings

In this section, key findings are organised and presented according to five themes: the key principles underpinning effective mathematics teaching, the early years, the classroom community, the pedagogical task and activity, and educational leadership and centre–home and school–home links.

Key principles underpinning effective mathematics teaching

Teachers who enhance positive social and academic outcomes for their diverse students are committed to teaching that takes students’ mathematical thinking seriously. Their commitment to students’ thinking is underpinned by the following principles:

- an acknowledgement that all students, irrespective of age, have the capacity to become powerful mathematical learners;
- a commitment to maximise access to mathematics;
- empowerment of all to develop mathematical identities and knowledge;
- holistic development for productive citizenship through mathematics;
- relationships and the connectedness of both people and ideas;
- interpersonal respect and sensitivity;
- fairness and consistency.
The early years

Young children are powerful mathematics learners. Quality teaching guarantees the development of appropriate relationships and support as well as an awareness of children’s mathematical understanding. Research has consistently demonstrated how a wide range of children’s everyday activities, play and interests can be used to engage, challenge and extend children’s mathematical knowledge and skills. Researchers have found that effective teachers provide opportunities for children to explore mathematics through a range of imaginative and real-world learning contexts. Contexts that are rich in perceptual and social experiences support the development of problem-solving and creative-thinking skills.

There is now strong evidence that the most effective settings for young learners provide a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive, play activities. Opportunities for learning mathematics typically arise out of children’s everyday activities: counting, playing with mathematical shapes, telling time, estimating distance, sharing, cooking, and playing games. Teachers in early childhood settings need a sound understanding of mathematics to effectively capture the learning opportunities within the child’s environment and make available a range of appropriate resources and purposeful and challenging activities. Using this knowledge, effective teachers provide scaffolding that extends the child’s mathematical thinking while simultaneously valuing the child’s contribution.

The classroom community

Research has shown that opportunities to learn depend significantly on the community that is developed within centres and classrooms. Thus, people, relationships, and classroom environments are critically important. Whilst all teachers care about student engagement, research quite clearly demonstrates that pedagogy that is focused solely on the development of a trusting climate does not get to the heart of what mathematics teaching truly entails. Teachers who truly care about their students have high yet realistic expectations about enhancing students’ capacity to think, reason, communicate and reflect upon their own and others’ understandings. Research has provided conclusive evidence that effective teachers work at developing inclusive partnerships, ensure that the ideas put forward by learners are received with respect and, in time, become commensurate with mathematical convention and curricular goals.

Studies have provided conclusive evidence that teaching that is effective is able to bridge students’ intuitive understandings and the mathematical understandings sanctioned by the world at large. Language plays a central role. Mathematical language involves more than vocabulary and technical usage; it encompasses the ways that expert and novice mathematicians use language to explain and to justify concepts. The teacher who has the interests of learners at heart ensures that the home language of students in multilingual classroom environments connects with the underlying meaning of mathematical concepts and technical terms. Teachers who make a difference are focused on shaping the development of novice mathematicians who speak the precise and generalisable language of mathematics.

Mathematics teaching for diverse learners creates a space for the individual and the collective. Whilst many researchers have shown that small-group work can provide the context for social and cognitive engagement, others have cautioned that students need opportunities and time to think and work quietly away from the demands of a group. There is evidence that some students, more than others, appear to thrive in class discussion groups. Many students, including limited-English-speaking students, are reluctant to share their thinking in class discussions. Research has also shown that an over-reliance on grouping according to attainment is not necessarily productive for all students. Teachers who teach lower streamed classes tend to follow a protracted curriculum and offer less varied teaching strategies. This pedagogical
practice may have a detrimental effect on the development of a mathematical disposition and on students’ sense of their own mathematical identity.

**Pedagogical tasks and activities**

From the research, it is evident that the opportunity to learn is influenced by what is made available to learners. For all students, the ‘what’ that they do is integral to their learning. The ‘what’ is the result of sustained integration of planned and spontaneous learning opportunities made available by the teacher. The activities that teachers plan, and the sorts of mathematical inquiries that take place around those activities, are crucially important to learning. Effective teachers plan their activities with many factors in mind, including the individual student’s knowledge and experiences, and the participation norms established within the classroom. Extensive research in this area has found that effective teachers develop their planning to allow students to develop habits of mind whereby they can engage with mathematics productively and make use of appropriate tools to support their understanding.

Choice of task, tools, and activity significantly influences the development of mathematical thinking. Quality teaching at all levels ensures that mathematical tasks are not simply ‘time fillers’ and is focused instead on the solution of genuine mathematical problems. The most productive tasks and activities are those that allow students to access important mathematical concepts and relationships, to investigate mathematical structure, and to use techniques and notations appropriately. Research provides sound evidence that when teachers employ tasks for these purposes over sustained periods of time, they provide students with opportunities for success, they present an appropriate level of challenge, they increase students’ sense of control, and they enhance students’ mathematical dispositions.

Effective teaching for diverse students demands teacher knowledge. Studies exploring the impact of content and pedagogical knowledge have shown that what teachers do in classrooms is very much dependent on what they know and believe about mathematics and on what they understand about the teaching and learning of mathematics. Successful teaching of mathematics requires a teacher to have both the intention and the effect to assist pupils to make sense of mathematical topics. A teacher with the intention of developing student understanding will not necessarily produce the desired effect. Unless teachers make good sense of the mathematical ideas, they will not have the confidence to press for student understanding nor will they have the flexibility they need for spotting the entry points that will move students towards more sophisticated and mathematically grounded understandings. There is now a wealth of evidence available that shows how teachers’ knowledge can be developed with the support and encouragement of a professional community of learners.

**Educational leadership and links between centre and home/school**

Facilitating harmonious interactions between school, family, and community contributes to the enhancing of students’ aspirations, attitudes, and achievement. Research that explores practices beyond the classroom provides insight into the part that school-wide, institutional and home processes play in developing mathematical identities and capabilities. There is conclusive evidence that quality teaching is a joint enterprise involving mutual relationships and system-level processes that are shared by school personnel. Research has provided clear evidence that effective pedagogy is founded on the material, systems, human and emotional support, and resourcing provided by school leaders as well as the collaborative efforts of teachers to make a difference for all learners.

Teachers who build whānau relationships and home–community and school–centre partnerships go out of their way to facilitate harmonious interactions between the sectors. There is convincing evidence to suggest that these relationships influence students’ mathematical development. The home and community environments offer a rich source of mathematical experiences on which to build centre/school learning. Teachers who collaborate with parents, families/whānau and
community members come to understand their students better. Parents benefit too: through their purposeful involvement in school/centre activities, by assisting with homework, and in providing suitable games, music and books, they develop a greater understanding of the centre’s or school’s programme. Their involvement also provides an opportunity to scaffold the learning that takes place within the centre or school.

**Overall key findings**

This Best Evidence Synthesis examines the links between pedagogical practice and student outcomes. Consistent with recent theories of teaching and learning, it finds that quality teaching is not simply a matter of ‘knowing your subject’ or ‘being born a teacher’.

Sound subject matter knowledge and pedagogical content knowledge are prerequisites for accessing students’ conceptual understandings and for deciding where those understandings might be heading. They are also critical for accessing and adapting task, activities and resources to bring the mathematics to the fore.

The importance of building home–community and school–centre partnerships has been highlighted in a number of studies of effective teaching.

Early childhood centre researchers have provided evidence that the most effective settings offer a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive, play activities.

Within centres and classrooms, effective teachers care about their students and work at developing interrelationships that create spaces for learners to develop their mathematical and cultural identities.

Extensive research on task and activity has found that effective teachers make decisions on lesson content that provide learners with opportunities to develop their mathematical identities and their mathematical understandings.

Studies have provided conclusive evidence that teaching that is effective is able to bridge learners’ intuitive understandings and the mathematical understandings sanctioned by the world at large.

**Gaps in the literature and directions for future research**

The synthesis provides research information about effective mathematics teaching. Although the scope of researchers’ studies is broad and far-reaching, a number of gaps in the literature are apparent. Research has so far provided only limited information about effective teaching in New Zealand at the secondary school level. Additionally, there is little reported research that focuses on quality teaching for Pasifika students. Few researchers in New Zealand are exploring mathematics in early childhood centres. The New Zealand literature lacks longitudinal, large-scale studies of teaching and learning. Also missing are studies undertaken in collaboration with overseas researchers. Such research is crucially important for understanding teachers’ work and the impact of curricular change. The scholarly exchange of ideas made possible through joint projects with the international research community contributes in numerous ways to the capability of our local researchers.

It is important to keep in mind that, as a knowledge building tool, the synthesis provides insights based on what has gone before. A snapshot in time, it is subject to change as new kinds of evidence about quality teaching become available. Important mathematics initiatives are underway in New Zealand schools and centres. The Numeracy Development Projects, new assessment methods, projects involving information technology, and a greater focus on statistics in the curriculum are just three examples of changes that are currently taking place. All new initiatives require research that monitors and evaluates their introduction and ‘take up’ by centres/schools and the changes in teaching and learning that take place as a result. Such research is necessary to guide future directions in schools, educational policy, and curriculum design.
3. Early Years Mathematics Education

Introduction

An effective mathematics pedagogy is built on the premise that all children and students are powerful mathematics learners. The development of mathematical competencies begins at birth: “in the early months of life, [babies] are busy learning about mathematics as part of the explorations necessary to the process of becoming members of the community in which they live” (Pound, 1999, p. 3). Beginning at birth, the developments that occur in the child’s first five years represent a vitally important period of human development in their own right; they do not simply define a time to grow before ‘real learning’ begins in school (Ginsburg, Klein, & Starkey, 1998). Children develop holistically in the cognitive, social-emotional, and physical arenas, and mathematics plays a part in this development (Perry & Dockett, 2004).

This chapter is about young learners1 and particularly about young mathematics learners in early childhood centres. Influenced by sociocultural theorists such as Vygotsky (1986) and Rogoff (1990), a model of shared learning has emerged—shared in the sense that there is valuing of children’s control but also acknowledging a significant role for the adult (Gifford, 2005). The premise that all students can be powerful mathematics learners, irrespective of age, is underscored by the New Zealand early childhood curriculum, Te Whāriki (Ministry of Education, 1996). Consistent with a view that early childhood provisions should offer holistic learning opportunities in the cognitive, social-emotional, and physical domains, a set of core mathematical understandings, competencies, and learning dispositions is embedded within the strands2 of the document. Within the Communication – Mana Reo strand, learning outcomes for knowledge, skills, and attitudes include:

- an understanding that symbols can be ‘read’ by others and that thoughts, experiences, and ideas can be represented through words, pictures, print, numbers, sounds, shapes, models, and photographs;
- familiarity with numbers and their uses by exploring and observing the use of numbers in activities that have meaning and purpose for children;
- skill in using the counting system and mathematical symbols and concepts, such as numbers, length, weight, volume, shape, and pattern, for meaningful and increasingly complex purposes;
- the expectation that numbers can amuse, delight, illuminate, inform, and excite;
- experience with some of the technology and resources for mathematics, reading, and writing.

(Ministry of Education, 1996, p. 78)

Elsewhere in the document, there is further potential for mathematical experiences and learning, for example, “Children develop the ability to make decisions, choose their own material, and set their own problems” (Exploration – Mana Aotūroa, goal 1, p. 84).

At the outset, we want to record that there is limited empirical evidence that links quality teaching to improved educational outcomes for young children (Farquhar, 2003)—a view that is echoed by the mathematics education research community (Gifford, 2004; Perry & Dockett, 2004). To date, research appears to “have established what young children can confidently do, especially with regard to number, but we do not know much about systematically helping children to learn” (Gifford, 2004, p.100). This is an issue for educators because we now have evidence that many basic mathematical understandings are present in young children. These include enumeration, simple arithmetic, representation, problem solving, spatial skills, geometric knowledge, and some logical ability in a range of circumstances (Diezmann & Yelland, 2000; Ginsburg & Golbeck, 2004; Hughes, 1986; Kilpatrick, Swafford, & Findell, 2001; Perry & Dockett, 2002; Peters, 1992). Collectively these researchers provide evidence that young children can also demonstrate a wide range of mathematical thinking practices, including
making connections, argumentation, number sense, mental computation, algebraic reasoning, spatial and geometric reasoning, and data and probability sense.

Research studies in New Zealand and overseas suggest that while there are numerous opportunities available for children to develop mathematical ideas in early childhood education settings, these opportunities are not always utilised systematically and purposefully (Davies, 2002; Hill, 1995; Siraj-Blatchford et al., 2000; Young Loveridge, Carr, & Peters, 1995). While a radical change in mindset amongst many early childhood practitioners may be required, Gifford (2004) cautions that effective early mathematics pedagogy is also about achieving a delicate balance:

While we may want young children to start school mathematically confident, there is a danger of over-pressurising them and creating mathematics anxiety, as many adults are only too well aware. (p. 100)

With this caveat acknowledged, we provide a focused synthesis of pedagogical practices and issues related specifically to mathematics learning in early childhood education. The chapter is partitioned into sections that draw out research relevant to these key areas: task/activity pedagogical design, mathematical thinking and practices, assessment, communities of practice, teacher knowledge, and links between more formal educational settings and out-of-school settings.

The chapter concludes with a section outlining current issues around the transition from early childhood to school, in relation to mathematics pedagogy and curriculum.

**Mathematical learning experiences and activities**

**Mathematics learning experiences should be both planned and informal/spontaneous**

Young mathematics learners need opportunities and encouragement to become familiar with numbers, shapes or measuring tools before they can understand them:

They [young children] need to practise counting so that it becomes automatic before they can understand the value of numbers. When they are familiar with the shapes of building blocks, they can then use them in more varied ways and make more complex structures and patterns. Children therefore need opportunities and encouragement to become familiar and to practise if they are to investigate and generalise relationships and apply mathematics to problem solving, such as using counting to see if shares are fair. (Gifford, 2005, p. 160)

Within early childhood settings, these opportunities to learn arise from both naturally occurring, informal experiences and from planned activities. Based on findings from two large scale UK projects, Effective Provision of Pre-school Education (EPPE) and Researching Effective Pedagogy in the Early Years (REPEY), Siraj-Blatchford and Sylva (2004) concluded that the most effective settings provide both and achieve a balance between the opportunities for children to benefit from teacher-initiated group work and the provision of freely chosen, yet potentially instructive, play activities.

**Everyday activities and play situations provide a source of mathematical experiences**

Children's informal mathematical knowledge originates within the course of their typically occurring everyday activities. Infants, for example, learn about time and pattern through the use of rhymes and song and develop spatial skills and awareness as they move around their environment. Likewise, the everyday activities of telling time, sharing, cooking, playing games, completing puzzles, counting, estimating distances, and making music provide rich opportunities for young children to practise and develop mathematical competencies.
E rere Taku Poi

Royal Tangaere (1997, pp. 40–41) provides an example of how 18-month-old Rangi uses poi and an accompanying waiata to develop her sense of spatial concepts. The words of the song successfully direct her actions with the poi.

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<thead>
<tr>
<th>Original Dialogue</th>
<th>Translation</th>
<th>Nonverbal Actions</th>
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<tbody>
<tr>
<td>Toru whā</td>
<td>Three four</td>
<td>Hands on hip</td>
</tr>
<tr>
<td>E rere taku poi</td>
<td>Fly my poi</td>
<td>Swings poi in front of her and above her</td>
</tr>
<tr>
<td>E rere taku poi</td>
<td>Fly my poi above (me)</td>
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<td>E rere taku poi</td>
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<tr>
<td>E rere taku poi</td>
<td>Fly my poi</td>
<td>Brings poi back in front of her</td>
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<tr>
<td>Ki runga</td>
<td>Fly my poi</td>
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<tr>
<td>Ki runga</td>
<td>below (me)</td>
<td>Swings poi down below</td>
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<tr>
<td>E rere runga</td>
<td>Fly above (me)</td>
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<td>E rere raro</td>
<td>Fly below (me)</td>
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<td>E rere roto</td>
<td>Fly inside</td>
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<tr>
<td>E rere waho</td>
<td>Fly outside</td>
<td></td>
</tr>
<tr>
<td>E rere taku poi</td>
<td>Fly my poi</td>
<td>Swings up</td>
</tr>
<tr>
<td>E rere taku poi</td>
<td>Fly my poi</td>
<td>Swings down</td>
</tr>
<tr>
<td>Ki runga</td>
<td>Above (me)</td>
<td>Swings into body</td>
</tr>
<tr>
<td>Ki runga</td>
<td>Above (me)</td>
<td>Swings away from body</td>
</tr>
</tbody>
</table>

A number of thesis and research studies (e.g., Arakua, 2002; Craw, 2000, Davies, 2002; Haangana, 1999, Young-Loveridge, Carr, & Peters, 1995) contain documented examples of mathematics experiences in New Zealand early childhood centres and many others can be found within the early childhood and mathematics exemplars documents.

Play—a key component of children’s experience—provides a source of spontaneous mathematical activity, language, and thought (Davies, 1999; Ginsburg & Golbeck., 2004; Irwin & Ginsburg, 2001; Parsonage, 2001). In spontaneous play, “the practice of the community creates the potential curriculum” (Macmillan, 2004, p. 37). For example, in the following episode, the child makes a connection with a home experience and uses mathematical positioning language when describing the specific details of the snail’s movements.

A four-year-old was observing a snail crawling over pieces of celery and carrot when she said: “I’m patting him. He didn’t like it [as the snail slid off the celery]. I’m putting him on the carrot. He’s trying to get off. One of these snails was on my garbage bin at home. He’s going to fall off it in a minute. He’s going down. He can get down. (Macmillan, 2004, p. 37)

While spontaneous play in an early childhood setting enables children to engage in the practice of learning mathematics independently of the teacher, research also points to the value of shared interactions, particularly those shared with an adult. The teacher from Craw’s (2000) study reports that playing games with children is “a very good basic thing for maths … we play lazy lion king … so when we count, sometimes we count 1, 2, 3, 3, … right up to 10 or in Māori up to ten … now when I play with the picture one, I count like this with them, 10, 20, 30, 40 … that’s how the new order comes in … or from 0, 5, 10” (p. 10). Findings from the REPEY study (Siraj-Blatchford & Sylva, 2004) suggest that “the achievements of settings as evidenced by their cognitive outcomes appear to be directly related to the quantity and quality of the teacher/adult planned and initiated focused group work that is provided” (p. 720).
Leaving a more in-depth discussion of the role of peer and adult scaffolding until later, the following vignettes from studies of teachers within New Zealand early childhood centres, illustrate how teacher–child interactions can support the mathematical development of young children.

**Aprons and Apples**

The teacher in the following episode takes the opportunity to integrate spatial mathematical language while assisting a young child to put on an apron for water play:

**Teacher:** You look, you wear it (plastic apron) backwards if you wear it that way. So the long piece …

**Child:** Okay, but I can't put this one.

**Teacher:** Look, the short piece and this, the long piece. The long piece goes over your front. Luke, like this and the short piece goes to your back. There! That's good—you're covered (Arakua, p. 55).

In another example, the teacher's verbal instructions for turn-taking with equipment makes explicit links with spatial concepts and the children's movement sequences: "Up on the bench, … across the ladder, … over the bench, … and come underneath. Arms out for balance" (Arakua, p. 63).

In the following episode, we see the teacher and child’s use of mathematical language within a discussion that is focused on the everyday resource of food:

"I've got a plate with two apples and oranges … three apples and cut one into half and the other one into quarters, and the other one I left like that … we were talking … some of the children are eating and some were drawing and we had a lot of language going through … I took two half ones and I said 'look, it's magic—two pieces'… I get the word out 'half and half make one' … then [child] said 'look: four pieces make one'." (Craw, p. 11)


Increasingly children are interacting with information and communication technologies (ICT) in their everyday experiences and these experiences can also be usefully linked to mathematical activity. While research into ICT use in New Zealand early childhood centres is in its early stages (e.g., Centre of Innovation project in Roskill South Kindergarten), several studies conducted in Australian contexts are available. In a study involving 58 early childhood services, Dockett, Perry, and Nanlohy (2000) found that the use of computers added value to children's learning in terms of social and cognitive gains. Interacting within an individually appropriate learning environment over which they have some control, children gain a sense of mastery, develop representational competence, and are encouraged to create and explore in a variety of ways not otherwise possible.

Internationally, one of the most ambitious ICT projects to recognise the mathematical potential of young children is The Playground project. This project allows children (aged 4 to 8) to play, design, and create their own video games. Through building their own executable representations of relationships, children are able to express mathematical relationships and ideas that would normally be reserved for much older students.

In another study involving computer software, this time as a curricular programme rather than as a tool, Sarama and Clements (2004) found that young children's use of Building Blocks resulted in significant learning gains in spatial, geometric, and numeric competencies and concepts. Sarama and Clements attribute the success of Building Blocks to the use of "activities-through-trajectories"—on and off the computer—that connect children's informal knowledge to more formal mathematics. They found that teachers who understood the learning trajectories were more effective in teaching and encouraging “informal, incidental mathematics at an appropriate and deep level” (p. 188).
However, as with other studies of interventions with a mathematics focus, the potential of ICT appears to be mediated by teacher knowledge and confidence—both in ICT and mathematics (Yelland, 2005). Dockett et al.’s research (2000), referenced earlier, noted that 67% of their sample of 179 teachers in 58 early childhood centres indicated that they had never used computers with children. Given that ICT is an increasingly significant component of our lives, concerns about differential access to computers and the Internet—at home or in early childhood settings—need to be addressed (Ginsberg, Pappas, & Seo, 2001).

**Activities with a mathematical focus**

Spontaneous free play, while potentially rich in mathematics, is not sufficient to provide mathematical experiences for young children. Evidence from observational studies suggests that children’s involvement in mathematical activities appears to be moderated by their own interest and prior knowledge. A US study by Tudge and Doucet (2004) of everyday mathematical activities engaged in by 39 three-year-olds showed that there was considerable variation in children’s engagement. From observations of mathematics engagement at home and in centres, the researchers found no evidence to support parents’ expectations that their children would more likely be engaged in mathematical activities in formal centres than at home. Davies (2002), in an intervention study involving mathematics games, also noted considerable variation in participation rates of her 10 target children. Similarly, observations of 32 case study children in the Enhancing the Mathematics of Four-Year-Olds (EMI-4s) study (Young-Loveridge et al., 1995), showed a significant variance in the mathematics focus of children.

Founded on concerns that opportunities to engage in mathematical activities may be less than optimal, early years educators recommend that centres develop an ‘orientation to numeracy’ by focusing on mathematical thinking within everyday activities and routines and by creating problem-solving opportunities. For instance, toddlers can be given playdough to encourage them to manipulate quantities through cutting and squashing the pieces (Perkins, 2003) and young children can explore sharing toys or food. In the following vignette, we see how Jake initiates a survey of bags in lockers. It appears that the activity is directly related to an earlier learning experience in which the teacher and Jake and Hugo produced a bar graph of the colour of socks worn on one day. In the Learning Story, we see how Jake attempts to collect the data in a systematic way, previously modelled by the teacher.

---

**Jake’s Learning Story**

Jake works to a plan. He is systematic and likes to complete a job. He has become very involved in surveys and likes to discriminate, sort, match, count, and record.

<table>
<thead>
<tr>
<th>Belonging Mana whenua</th>
<th>Taking an interest</th>
<th>Jake arrived, walking up the ramp, saying he would like to do a survey on bags. He came to me and we talked about how he would need to go about this. Jake thought this was a good idea for a survey as he didn’t have to ask anyone any questions!!</th>
</tr>
</thead>
</table>

---
<table>
<thead>
<tr>
<th>Well-being</th>
<th>Being involved</th>
<th>Jake had a clip board and worked on the yellow table. He drew bags and coloured them. “Look this one doesn’t have a handle”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploration</td>
<td>Persisting with difficulty</td>
<td>I asked if he was going to have multi coloured. Jake explained that there were no multi coloured crayons. I suggested he go and have a look at the sock graph to see how I had made multi colour. Came back still stumped. Finally I asked if he needed help. I showed him how I drew lines of different colours. At the table the other children discussed what made multi coloured. Two colours were two tone, so you needed three or more to be multi coloured. Jake also drew a big bag with a cross through it for ‘no bags’.</td>
</tr>
<tr>
<td>Communication</td>
<td>Expressing an idea or feeling</td>
<td>He worked through looking at the lockers. Came to get me. He was not sure if he had got them all and said there were a lot with no bags. I asked if he had started from the top and worked along. Jake looked horrified. “I started from the bottom and worked along”.</td>
</tr>
<tr>
<td>Contribution</td>
<td>Taking responsibility</td>
<td>I explained that was fine, I only meant had he worked in a line to make it easier and it didn’t matter where you started. Jake was fascinated that there were bags the same. Jake is absorbed in thinking up ideas of what he would like to survey. He seems totally in charge of this!</td>
</tr>
</tbody>
</table>

*From Ministry of Education (2007)*
The provision of more challenging and complex mathematical experiences has been investigated in a range of intervention studies. Introduced activities with a mathematical focus include games (Davies, 2002), weekly themes/projects (Sophian, 2004), problem-solving adventure stories (Casey, Kersh, & Young, 2004), imaginative kits (Macmillan, 2002), books (Young-Loveridge et al., 1995), and technology (Sarama & Clements, 2004). Results of these studies highlight the benefits of increased opportunities for students to access mathematical ideas, and increased teacher awareness of and confidence with mathematics.

In New Zealand, the landmark Enhancing the Mathematics of Four-Year-Olds (EMI-4s) intervention project (Young-Loveridge et al., 1995), focused on ways to enhance the mathematical understanding of four-year-olds. This large-scale project involved four kindergartens, one of which was a control. During a three-month period, the researchers worked collaboratively with teachers to increase children's access to mathematical experiences. The intervention within the centres was two-fold. First, teachers' awareness of mathematics in everyday contexts and of children's interests was heightened through collaboration with the researchers. Second, additional resources with a mathematical focus (e.g., books, games, dice, calculators, calendars, and measuring tools) were integrated into the centres’ programmes. In addition, efforts to increase the amount of display material involving numeracy resulted in increased talk about numbers and quantities. Improved performance, when compared with that of children attending a kindergarten without the intervention, was credited to access to additional and appropriate activities and equipment, and to a reported increase in teacher awareness of mathematics:

Much more aware. When I [teacher] am reading a story I stop when there is counting to be done, and do it slowly with them [children] and we all join in. ... I am aware of it in other areas—outside, waiting for turns, or the number of children doing something ... I am probably not aware of brand new things happening, but I think I am aware of things that have always been happening and focusing in on these. (p. 124)

In contrast to the more holistic approach of the EMI-4s study, interventions in the US typically seek to identify how specific mathematical competencies can best be taught. Griffin (2004) reports on the pre-K–2 mathematics programme, Number Worlds. Developed to teach conceptual structure for number, the programme claims to build upon children's current knowledge through the provision of multilevel activities, to utilise activities that follow a natural developmental progression, and to teach computational fluency as well as understanding. Activities include contextual problems using multiple representations of number: a group of objects, a dot-set pattern, a position on a line, a position on a scale, and a point on a dial. Evaluation reports found that the children from the intervention study made significant gains in conceptual knowledge of number and in number sense when compared with matched-control groups who received readiness training of a different sort.

Starkey, Klein, and Wakeley’s (2004) intervention programme, like Number Worlds, targeted children from economically disadvantaged families in a US context. The intervention strategy was similar to that employed in the EMI-4s project: the introduction of targeted activities with a mathematics focus, including computer-based mathematics activities, and teacher professional development. In addition, parents and children in the intervention group attended a series of three home mathematics classes where activities were presented and strategies for dyadic engagement discussed. The significant socio-economic status (SES)-related gap in mathematical knowledge found at the beginning of the pre-kindergarten year was decreased following the intervention year.

Sophian (2004) reports on another US intervention designed to address the perceived disadvantage—in terms of levels of mathematical knowledge—of children from low-income families. Implementation of an experimental mathematics curriculum developed for Head Start included teacher supports and home activities that corresponded to activities presented in the centres. However, unlike the previous interventions that focused on number, this curriculum intervention focused on the concept of unit as it applies to enumeration, measurement, and the identification of relations among geometric shapes. Measurement activities involved measuring
the same quantity using different units, an approach not typically found in other preschool curricula. For example, children made a row of prints of their own hand on strips of cloth and then used their strip to measure various objects. They then explored what happened when the teacher measured with prints of her own (larger) hand. Part–part–whole relationships, typically associated with number bonds, were developed through an examination of partitioning area. Children who received the mathematics intervention outperformed both no-intervention control children and children who had participated in a literacy rather than a mathematics intervention. As with the other intervention studies, increased teacher understandings about mathematics and increased teacher expectations about what children were able to achieve and understand in relation to mathematics were noted.

What is interesting about this study and related research projects in primary schools (e.g., Dougherty’s Measure Up Curriculum) is that it challenges the dominance of counting and number as the entry point for mathematics development. The programme differs from most other early childhood programmes in that measurement is a perspective that pervades the entire curriculum. The Head Start curriculum views mathematics learning as “primarily a matter of learning to reason effectively about quantity (particularly, for young children, tangible, manipulable, physical quantities) and only within that broader objective as a matter of learning about numbers” (Sophian, 2004, p. 76).

‘Patterning’ is another related mathematics skill that is spontaneously evoked in young children.

A two-year-old chants to herself in the bathtub, “splish, splish, oh-oh, splish, splish, oh-oh” while simultaneously squeezing the water out of a rubber toy. With each “splish” she squeezes the toy and then declares “oh-oh” when no water comes out on the third and fourth squeeze. She then submerges the toy to refill it and repeats the event. (Schwartz, 2005, p. 1)

Focusing on children’s interests and play situations, Australian researchers Papic and Mulligan (2005) developed an intervention programme built on children’s existing ideas about pattern. Evaluation of the intervention programme involving two early childhood centres (target and control) reported a sustained positive impact on the mathematical development of the intervention children (who represented a range of ability), both during the six-month period of the study and 12 months later, at the end of the first year of formal schooling. Based on strong correlations with children’s ability on patterning tasks and other numeracy assessments at the end of the first year of formal school, the researchers argue for the existence of “strong links between a child’s ability to pattern and their development of pre-algebraic and reasoning skills” (p. 615). (See also Mulligan and colleagues’ research in chapter 5.)

Appropriate challenge

A recent international discussion group on the mathematical thinking of young children (Hunting & Pearn, 2003) noted that young children are more capable than current practices suggest. The expert group called for the provision of more challenging early education programmes. This recommendation is in accord with the common finding noted in the intervention research programmes reviewed in this chapter; namely, that participating teachers were often unaware of the need to cater for children’s interests and mathematical abilities and to engage children in challenging learning experiences. Participation in the intervention programmes challenged teachers’ expectations about what mathematics young children could learn and understand.

As noted in the REPEY study of centres in the UK, many opportunities for sustained, shared cognitive engagement were missed and the cognitive challenge involved in teacher–child interactions was sometimes less than optimal. Analysis of high-cognitive challenge activities in the REPEY uncovered an interesting pattern:

In excellent settings the importance of staff members extending child-initiated episodes is very clear; just under half of child-initiated episodes observed as
high challenge involved interventions from a staff member which extended the child’s activities. The preponderance of staff extension in child-initiated activities appeared to be unique to the three highest performing (‘excellent’) settings. ... Analysis showed that the most common critical point (‘lifting the level of thinking’) occurred when a practitioner ‘extended’ a child-initiated episode by scaffolding, thematic conversation or instruction. (Siraj-Blatchford & Sylva, 2004, p. 723)

The following vignette provides an example of sustained, shared thinking (though science-based) from the REPEY data.

**Bubbles**

Boy 8:  [Who has been watching various items floating on water.] Look at the fir cone. There’s bubbles of air coming out.
Nursery Officer:  It’s spinning round. [Modelling curiosity and desire to investigate further.]
Boy 8:  That’s ‘cos it’s got air in it.
Nursery Officer:  [Picks up the fir cone and shows the children how the scales go round the fir cone in a spiral, turning the fir cone round with a winding action.] When the air comes out in bubbles it makes the fir cone spin around.
Girl 2E:  [Uses a plastic tube to blow into the water.] Look bubbles.
Nursery Officer:  What are you putting into the water to make bubbles? ... What’s coming out of the tube?
Girl 2E:  Air.

The episode illustrates the power of the skilled partner, in this case the Nursery Officer, to understand and build on what it is that Boy 8 understands or does not understand.

*From Siraj-Blatchford and Sylva (2004)*

Based on the findings of the EMI-4s study, Young-Loveridge and colleagues also concluded that effective teachers were those who were able to pick up on the children’s mathematical ideas and extend them, and, in accord with the REPEY findings, the researchers also noted frequent missed opportunities for both interaction and extension. Many of the mathematics activities were suited to children with average numeracy levels; activities more appropriate for novices and experts were noticeably missing. Given the variability of children’s entry knowledge, finding the appropriate level of challenge was seen to be a critical factor in catering for the needs of diverse students.

The intervention phase of the EMI-4s study detailed how activities could be successfully adapted to provide a suitable level of challenge. Social and physical adaptations to games included avoiding standard turn-taking by providing each child with a dice so that they could play simultaneously and the use of large-scale resources such as outdoor number tracks, instead of baseboards and counters, to reduce the need for fine motor skills. Cognitive adjustments to games to suit the skills of children who could count only to two or three included the use of dice showing patterns of only one, two and three. A suitable challenge could be provided for a more expert player by suggesting that they add or subtract the scores on two dice to determine a player’s move. It was noted by the teachers involved in the intervention phase that their increased awareness of mathematics was the key to the provision of appropriate activities and to their ability to capitalise on spontaneous mathematical interactions with children.

While provision of sufficient challenge is seen as a priority, Carr et al. (1994) note that too much challenge—determined by a combination of familiarity of context, meaningfulness of purpose, and complexity—also brings its own set of problems. If too many activities are beyond the difficulty limits of children and they are recognised as mathematical, young children may begin to avoid or ignore all mathematical tasks.
**Assessment**

Assessment in the early years is seen as an integral part of learning: "Assessment sits inside the curriculum, and assessments do not merely describe learning, they also construct and foster it" (Ministry of Education, 2004e, p. 3). The New Zealand early childhood exemplar document *Kei Tua o te Pae* describes assessment for learning as ‘noticing, recognising, and responding’: “Teachers notice a great deal as they work with children, and they recognise some of what they notice as ‘learning’. They will respond to a selection of what they recognise” (Ministry of Education, 2004e, p. 6).

Research on assessment practices in general indicates that effective pedagogy is informed by contextual knowledge of children’s learning and mathematical understanding (Alton-Lee, 2003). We know that young children are capable of building rich sources of informal and formal mathematical knowledge. We also know, however, that competence levels vary (Young-Loveridge et al., 1995) and consequently young children need learning experiences that match their current understandings (Farquhar, 2003).

In some cases, variability of children’s mathematics knowledge has been attributed to a lack of opportunity to learn rather than an inability to learn. In the EMI-4s study, Young-Loveridge and colleagues (1995) observed that some capable children who were very quiet managed to avoid bringing their expertise to the attention of their teachers. Moreover, children with low levels of expertise but who were outgoing and confident were sometimes assumed to have much greater proficiency than they actually did have. For example, a child (coded T) was confident in a range of areas but not mathematics. Prior to the intervention period of the study, his teachers assumed that his ‘mistakes’ arose from ‘just kidding around’. During the intervention phase, the combination of the teachers’ increased awareness of mathematics and T’s increased opportunities to participate in mathematical activities revealed that although he was obviously interested in number, his skills were in fact limited. Given this information, his teachers were able to support his mathematical development more effectively.

**Awareness of children’s mathematical understanding**

Early and appropriate assessment enables teachers to gain information for teaching and early intervention where necessary. Sarama and Clements (2004) contend that group interactions within early childhood settings often hide individual needs. They argue for more research that identifies effective ways of combining group work with individual assessment and teaching. Systematic observations of children, shared experiences, involving children as informants, and effective communication between teachers and families are all assessment practices that early childhood teachers have found to be effective (McNaughton, 2002).

In contrast to the more formal diagnostic testing that emphasises developmental progressions, assessment in early years mathematics needs to not only look at the individual but also acknowledge the role of the context and the child’s interactions with others (Fleer, 2002). In her case study of how one teacher in a New Zealand early childhood centre understood her children’s mathematical thinking, Craw (2000) documents the teacher’s effective practice as follows:

> In order to do this she talks about “following children around: and endeavouring to engage in “play with the children”, “talk to them” and try to interpret … “what maths” before something could be “set for the children” that would extend their learning in ways that will motivate children’s thinking to enable “eye openers in the sense that the brain is activated” and the children are able to “see things and to observe”. (p. 9)

The potential of children’s portfolios to highlight literacy and numeracy learning is advocated by Hedges (2002). She claims that a wider perspective on pedagogical documentation, including evidence of content learning, will both contribute to knowledge of children’s learning and provide a platform for children and parent communication. Similarly, Learning Stories, a formative assessment practice developed by Carr (1998), recognises the alternative strengths
of the child, and as such, offers promise for supporting personalised teaching approaches for diverse learners of mathematics (Education Review Office, 2004).

### Tom’s Learning Story

The following example of an abridged text captures an episode of learning in a New Zealand centre. The story highlights learning dispositional influences, key competencies and specific mathematical skills of measuring, understanding symbols, and communicating mathematical ideas, displayed within the context of a measuring activity.

<table>
<thead>
<tr>
<th>Belonging Mana whenua</th>
<th>Taking an interest</th>
<th>Tom held up a long piece of dough he had squeezed from the piping equipment and explained. “Look Rosie – it’s sooo long!”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well-being Mana atua</td>
<td>Being involved</td>
<td>“Yes, you’re right Tom – it sure is! Let’s get a ruler and measure it to see how long it really is,” I suggested. Tom placed his dough strip along the tape measure.</td>
</tr>
<tr>
<td>Exploration Mana aotūroa</td>
<td>Persisting with difficulty</td>
<td>“Can you see the numbers Tom – they tell you how long it is,” I explain. After studying the numbers carefully Tom announced “19 long!” “Yes, 19 centimetres,” I add.</td>
</tr>
<tr>
<td>Communication Mana reo</td>
<td>Expressing an idea or feeling</td>
<td>“I’ll make another one — but even longer this time. Look this one is … 22 cm” he continues.</td>
</tr>
<tr>
<td>Contribution Mana tangata</td>
<td>Taking responsibility</td>
<td>“Wow, can you make a strip as long as the ruler – 30 cm long Tom?” After much squeezing and slight adaptation, Tom successfully makes the strip reach from one end to the other. “Look – it’s 30 cm long now!”</td>
</tr>
</tbody>
</table>

In addition to highlighting the mathematical nature of Tom’s activity (measuring, understanding symbols, and mathematical exploration) the Learning Story portrays Tom’s interest in measuring, sustained involvement in the activities, persistence with difficulty when making the strip as long as the ruler, and verbal expression of his ideas. In terms of key competencies, Tom is involved in logical thinking and adapting ideas, and making meaning in relation to tools, symbols and language for measuring. The Learning Story is also able to capture Tom’s mathematical discussion with his teacher, and engagement with the wider culture community of mathematics users.

*From Peters (2004)*
A succession of learning stories can effectively demonstrate a child’s progress in a range of contexts, the nature of the strategies and dispositions involved, and the degree of increasing mathematical development. For example, the learning story, *Jaydon’s Towers* (see Assessment for Infants and Toddlers: He Aromatawai Kōhungahunga, Tamariki, Ministry of Education, 2004b, pp. 12–14), documents Jaydon’s spatial development within a seven-month period, based on a range of sorting and classifying and construction activities.

**Shared purpose and interests**

Building on a child’s understandings and interests requires teachers to not only assess a child’s developing mathematical knowledge but also to understand and respond to the cultural and social perspective of the learner (Hedges, 2002; Macmillan, 2004; Watego, 2005). Supporting adults need to build mathematical opportunities into those contexts that are familiar and appealing to young children—a factor that the EMI-4s study found particularly significant for lower attaining children.

In attending to children’s social viewpoint, researchers have found that young children often have their own social purpose for mathematics, especially number. Mathematical ideas that are “genuinely powerful for young children have much more to do with the processes used to interact with and do mathematics than with particular items of mathematical knowledge” (Perry & Dockett, 2002, p. 88). As such, young children rely on intuitive concepts and techniques for solving mathematical problems. For example, they may be aware of numbers and quantities but not yet use the number system and other formal mathematical tools. Having a collection of strategies to resolve situations that are relevant to them is much more important than knowing ‘correct’ mathematical terminology or being able to recite basic addition facts.

As noted in chapter 5, the resulting learning experience may sometimes be mediated by a purpose different from that intended by the teacher. Munn’s (1994, cited in Worthington & Carruthers, 2003) study of pre-school children’s counting ability noted a clear distinction between the purposes that children ascribed to counting and the purposes ascribed by adults. For example, a learning experience such as a block-based activity, designed with the aim of providing a rich problem experience, may have its purpose subverted to one of entitlement as children hoard, guard (and count) their supply of blocks. In the EMI-4s study, children’s purposes included ritualised number use (e.g., rhythm and repetition), establishing status, establishing entitlement, timing, patterns, orderliness, labelling, playing with numbers (just for fun), and exploring quantity to solve a particular problem (e.g., measuring, designing, recording, locating, and playing games).

**Differing Mathematical Purposes**

Elizabeth sees number as poetry. Her main reason for using number is for the purpose of rote-counting and for status (ages and birthdays are of great interest to her). She is an enthusiastic rote-counter, and does not appear to be interested in using number for counting things or to find out ‘how many’. Her interest in stable rote sequences extends to knowing the alphabet and to using the rhyme ‘eenie, meenie, minie, mo’ to count her sandwiches at lunch time. She enjoys repeated refrains and choruses in stories and songs.

Todd is more of a pattern maker. He enjoys drawing multiple copies of a particular object (triangles squares, aeroplanes, cars), cutting them out and rearranging them. He doesn’t use number very often, but he uses it for a wider range of purposes than does Elizabeth: for making patterns, building with blocks, keeping score at badminton. His rote-counting skills are uncertain, but when the numbers are small he can use number in problem-solving activities.

*From Carr, Peters, and Young-Loveridge (1991)*
Children’s different cultural backgrounds engage them with different experiences and expectations of different packages of mathematical purposes. For example Ha’angana (1999) provides a description of how the activity of making of a *kahoa* supported a young boy’s patterning and counting skills: “He strung together 22 pieces. He made a pattern with five colours. Each time he completed the five counts, he counted the next five pieces before he continued to string them together” (p. 34).

The usefulness of mathematics in providing an authoritative voice within contexts of sharing and entitlement is another purpose that arises in young children’s lives.

**Who Should Be Mum?**

Two young girls aged four-and-a-half years are playing in the family area of their centre. Their efforts to resolve a conflict situation are based on a mathematical logic, which while not necessarily coherent to adults, appears coherent to the children. Both girls have determined that size, as determined by height, is the critical factor in deciding who should be ‘Mum’.

Stella placed her hands on her hips and sighed. Jane adopted a similar stance and called loudly, “I’m the mother, I’m the mother.” She then moved closer to Stella, stood straight, and added “I’m the mother! See, I’m bigger than you!” “No, I’m higher,” replied Jane, “I’ll show you.” She stood right next to Stella and said, “Look! See, I’m bigger!” Stella looked, and stretched as high as she could. “And I’m big!” Jane looked again and complained, “Don’t stand on tippy toes, that’s not fair!” When Stella did not react, Jane added, “I’m gonna see my Daddy.”

_Sfard’s (2005) extended account of two four-year-old children responding to a parent’s requests for quantitative comparisons also illustrates that “if we are unable to see the children’s reasons, it is likely to be the result of our tendency to interpret their utterances the way we would interpret our own” (p. 242). Her research reiterates that children’s use of words may be dramatically different from that of adults. Awareness that children will interpret activities according to their own social and cultural experiences enables teachers to more effectively select and respond to opportunities to learn and, where appropriate, to make the purpose of mathematical learning explicit._

**Communities of practice**

Effective pedagogy assists children to access powerful mathematical ideas through the development of appropriate relationships and support. As we have seen earlier, research findings indicate that positive assessment and learning outcomes are closely associated with adult–child interactions that involve some element of sustained, shared thinking. These collaborative and responsive interactions between children and adults or more expert peers involve scaffolding and appropriation as well as providing opportunities for young children to develop metacognitive strategies and knowledge.

**Scaffolding and interactions**

A shift towards a consideration of Vygotskian principles relating to the social mediation of knowledge has prompted early childhood educators to focus not only on what it is that children are capable of on their own, but also on what they are capable of achieving with the assistance of more knowledgeable others, through scaffolding (Jordon, 2003). When working within the child’s ‘Zone of Proximal Development’ sociocultural perspectives:

emphasise that children’s higher mental processes are formed through the scaffolding of children’s developing understanding through social interactions with skilled partners. If children are to acquire knowledge about their world it is crucial that they engage in shared experiences with relevant scripts, events, and objects with adults (and peers). (Smith, 1999, p. 86)
Learning with the support of peers is one consequence of social interactions in early childhood settings and within communities. Young-Loveridge et al. (1998) provide the following account of a capable four-year-old girl, A, helping child N play the computer game.

A used a variety of methods to help N with the game. She took N’s hand and placed it on the appropriate numeral key, demonstrated how to count the objects on the screen and also translated the set on the screen into an equivalent set of fingers, perhaps intuitively sensing that it would be easier to count a set of large concrete object such as fingers, as opposed to small symbols on a screen. N’s one-to-one interactions with A, as she struggled to solve the mathematical problems posed by the computer, were the most cognitively challenging episodes in the entire observation period. (pp. 88–89)

When considering interactions with children and teachers, Siraj-Blatchford and Sylva (2004) found that effective ‘pedagogical interactions’, as distinguished from the ‘pedagogical frame’ (the behind-the-scenes aspects of pedagogy, which include planning, resources, and establishment of routines), were significant indicators of children’s performance. Effective pedagogical interactions contain elements of ‘sustained, shared thinking’ and contrast with less positive interactions involving monitoring of behaviour or engagement. Siraj-Blatchford and Sylva (2004) noted that the level of thinking was most likely to be enhanced when a practitioner “extended a child-initiated episode by scaffolding, thematic conversation or instruction” (p. 723).

**Mathematics in the Sandpit**

Outside in the sandpit, the teacher, working with two boys, simultaneously scaffolds and extends the children’s thinking.

**Teacher:** How many children have been making this? [referring to hole in the sand].

**Child:** Three.

**Teacher:** How many have been making it? One, two and yourself, three. That’s right three children have been making this and you have been digging this hole for a long time because it is a very big hole.

**Child:** Yeah! One hundred. One hundred minutes.

**Teacher:** One hundred what?

**Child:** Yeah, lots of minutes that is why we need ... take a long time to do this.

**Teacher:** One hundred minutes? That is a lot of minutes isn’t it?

*From Arakua (2002)*

For effective sustained, shared teaching episodes within a play situation, scaffolding needs to extend the child’s thinking while simultaneously valuing the child’s contribution, allowing the child to retain control of the play. Through their actions and words, adults can encourage children to persevere with a problem, think about it in different ways, and share possible solutions with peers and other adults. They can challenge children to extend their thinking or the scope of their investigations. Based on an analysis of transcripts of children and adults playing with numeracy activities in early childhood centres, Macmillan (2002, p. 4) provides the following examples of teaching strategies that are responsive to a child’s developing sense of identity as learner.

Clarifying/elaborating:

‘This one here’s number one. Can you find your number one?’

‘That goes with the tree A’s holding.’

Recognising/appreciating:

‘Wow! Look at all that matching!’

‘You’re doing a great job there. You have all the trees there.’
‘They are beautiful, aren’t they?’

Confirming:

‘There’s only one, is there?’
‘You can do it wherever you want to, A.’
‘It looks like a real one.’

Encouraging reflection by asking assisting/checking questions [link with metacognition]:

‘Do you need more?’
‘Does that match?’
‘Do we need to put them all in one dish?’

Pretending not to know the answer:

‘Truly, I can’t count!’

Creating relevance by making links with the child’s current knowledge:

‘That’s nearly as old as you.’
‘There’s a fat brick and a skinny brick.’

Modelling curiosity:

‘I wonder if mine’s a circle?’

Inviting imaginative involvement:

‘Looks like a wiggly worm.’

Inviting participation by offering choice:

‘How many do you think I’ll need?’
‘Who’s going to put a card out first?’
‘Where do you want me to put this one? In here, near the ladybug?’

Inviting participation by offering challenge:

‘Let’s count the red ones you’ve used.’
‘How many do you think I’ll need?’

Exemplifying a proactive pedagogical role, these responsive interactions contribute to an environment that stimulates exploration and offers opportunities for children to question and challenge their understanding.

**Metacognition**

Social interactions during early learning also communicate to the child messages about his or her intellectual capabilities. Learning about how to meet success and failure and how to plan for the future contribute to the early development of a child’s metacognitive knowledge base.

The *Quality Teaching for Diverse Students in Schooling: Best Evidence Synthesis* (Alton-Lee, 2003) identifies the development of metacognitive knowledge and strategies to support appropriate learning orientations and self-regulation as a significant factor for student achievement. While young children clearly have less metacognitive ability than older children, the development of self-regulation skills and related metacognitive awareness is seen as a significant goal within early childhood education. Cullen’s (1988) research demonstrates that the provision of opportunities for young children to interact with peers and to practise metacognitive-like activities such as planning, monitoring, reflecting, and directing enhances children’s development of self-regulatory skills.
Turn Taking

The following excerpt from a kōhanga reo illustrates children’s self-monitoring of their progress in a game-playing episode. Realising that they have got lost, they decide to return to the beginning in order to remember the sequence and start over.

Hinepau: Ae, timata.
Awatea: Ko taku wā.
Hinepau: Ah. (Agrees and puts back. They take a few more turns each, and then Toko has a turn out of sequence.)
Hinepau: Haere Toko. (Toko matches a pair.)
Awatea: Eeeh, ko a ia (indicating that it was Hinepau’s turn). Ka haere din, din, din (as she is pointing to the sequencing of turns).
Hinepau: (Realising she has missed a turn) Oh Ae, I haere a ia kātahi ko au. Oh, me whakahoki erā (addressing Toko). E tīka. Me haere a ia, au, koe (pointing). Terā te take—i haere koe ki terā taha, so me haere koe ki tenei taha X (indicating out-of-turn). So me whakahoki ena e rua.
Toko: All right, ka taea e koe te tiki.

From Skerrett White (2003), p. 161

The exemplars in Kei Tua o te Pae / Assessment for Learning (Ministry of Education, 2004e) provide numerous examples of young children’s emerging metacognitive skills and awareness. In particular, the exemplars in Book 4: Children Contributing to Their Own Assessment showcase young children’s ability to set their own goals, assess their own achievements, and take on some of the responsibility for their own learning. The following episode highlights a young child’s developing metacognitive knowledge and awareness of her learning goals in conjunction with her response to making a mistake.

“Oh, No! That’s Not Right!”

The “Oh, no! That’s not right!” exemplar documents Lauren’s developing sense of what is right in terms of spatial orientation within a screen-printing activity. Lauren appears to be developing her own sense of what is right. When she aligns a second template (a basket) over an earlier print (a cat) and makes a second print, she looks at it and says, “Oh, no! That’s not right! That cat needs to be in the basket—not up there!” She tries again, and when she has aligned the basket and cat to her satisfaction and added a few more items to the picture, she comments, “I like that!”

Lauren also appears to be talking herself through the process, self-regulating her learning. The teacher
notes, "As she was drawing, she said, almost to herself, 'I'll have to concentrate!' And she did ..."

The assessment records Lauren’s apparent view that if something is not right, you either redo it or take a creative approach and readjust your goal. She sees mistakes as part of learning. Her response to an unacceptable amount of space in her second attempt to develop the original design was "Never mind, I'll put some toys in it." On this occasion, she appears to interpret mistakes as part of the process of completing a task, and she is developing strategies in response to that belief.

From Ministry of Education (2004c), pp. 8–9

With specific reference to early years mathematics, Pappas, Ginsburg, and Jiang (2003) conceptualise metacognition as involving three major components: recognition of mistakes, adaptability or selection of strategies, and awareness and expression of thought. To engage in reflective discussion requires young children to have skills both in verbal communications and in awareness and expression of thought. For instance, when developing early computational understandings, children become aware that the answer to $5 + 2$ can be found by counting on their fingers and they learn to describe this process in a reasonably coherent way. Papas and colleagues’ analysis of young children’s problem solving revealed few socio-economic status (SES) differences in their metacognitive abilities. Advantaged children in their study were, however, more facile than their less advantaged peers when it came to describing thoughts and explaining ideas. For many young children, their metacognitive development lags behind their informal mathematical competence (Blote, Klein, & Beishuizen, 2000; Ginsburg, Pappas, & Seo, 2001). For this reason, teachers should not assume that young children who can ably articulate their thinking are necessarily more mathematically competent than their less articulate peers.

Big Math for Little Kids, a US mathematics programme embedded in activities and stories, places emphasis on the development of mathematical and mathematics-related language. Research evaluation (Greenes, Ginsburg, & Balfanz, 2004) provided evidence of young children’s abilities to express their mathematical thinking through discussions involving conjectures, predictions, and verifications.

**Which Is Heavier?**

An example from the measurement strand demonstrates how children are expected to draw on their everyday knowledge to resolve a contradiction.

Children are shown a picture of an unbalanced seesaw with a frog at one end (the heavier end) and a large bear at the other end (the lighter end). Children are urged to talk about what they see, to identify what is ‘funny’ or ‘wrong with the picture’ and to tell how they know, and later, describe how they would ‘fix it.’ Typical responses demonstrate children’s understanding of heavier/lighter and of balance. For example, one child responded, "The bear is bigger. He’d be at the bottom.” Prompted to speculate about the circumstances under which this could be true, the child answered, “The bear is a balloon. So the frog is heavier!”

From Greenes, Ginsburg, and Balfanz (2004), p. 161

A study by Zur and Gelman (2004) provides evidence that three-year-olds can invoke metacognitive strategies. Using a number game context, the children were able to predict—using predictions that honoured the principles that addition increases numerosity and subtraction decreases numerosity—and then check their predictions by counting. Participation in tasks that involved predicting and then checking by counting not only had the advantage of embedding counting within a meaningful task, it also provided children with the ability to use their counting as a source of authority and sense making, adding to their metacognitive knowledge of themselves as mathematical learners.
**Centre–home links**

Children’s development of informal mathematical understandings is a result of their natural curiosity and exploration of their environment (Fuson, 1992; Tudge & Doucet, 2004). What they learn during their early years is not, however, learned in isolation. *The Best Evidence Synthesis: The Complexity of Community and Family Influences on Children’s Achievement in New Zealand* highlights the role of both home and community links. Young children’s activities are “complemented by relationships that encourage the gradual involvement of children in the skilled and valued activities of their family and the society in which they live” (Biddulph, Biddulph, & Biddulph, 2003, p. 119). Parents, families/whānau and community members have a critical role to play; they are at the centre of the social contexts in which children live and thus are well placed to provide scaffolding for them as they develop mathematical ideas.

**Home-based mathematical practices**

The young child’s environment provides a rich source of mathematical experiences. The following vignette illustrates how Freya, a six-month-old baby, initiates an interaction with her mother—an interaction based on a repetitive action related to cause and effect.

**Napping or Not?**

Freya is seated on her mother’s bed supported by a tripillow. Her mother is to her right. Toys are spread out in front of her. Her toy basket is diagonally opposite. Freya picks up a toy and then mouths it, then moves the object from her right to left hand, looking intently at the object. She leans back into the tripillow and then pulls herself forward. She leans back into the tripillow and looks up at her mother and smiles [mother smiles and kisses her], then pulls herself forward; immediately repeating action in rapid motion. Mother smiles, kisses and hugs baby. Freya repeats this action ten times, always pausing and smiling at her mother [mother responds each time with smiles, kisses and hugs].

Haynes, in interpreting the mathematical experiences present in this episode, suggests that Freya exhibits an awareness of the effect of her actions on producing a guaranteed response. Through the interactions with her mother, Freya is learning about ‘cause and effect’ relationships.

*From Haynes (2000a)*

In a New Zealand study of young Tongan children, Ha’angana (1999) provides examples of mathematics integrated into the children’s church activities (e.g., holding up a number card and exploring the significance of that number in the Bible, singing number songs). Mathematical experiences in the home were explored in an Australian study (Clarke & Robbins, 2004) that investigated how families from lower socio-economic circumstances conceptualise and enact literacy and numeracy. Parents photographed their child participating in literacy and numeracy activities in their home and community. Photographed activities collected from a total of 52 families included numerations, shape and spatial activities, cooking, game playing, money, sorting and classifying, measuring, and travel and location. From parental discussions of the photographed activities, Clarke and Robbins concluded that “children were engaged in rich and varied mathematical environments and the parents had some awareness of the range of mathematical activities in which their children were engaged” (p. 180). Involvement in the project resulted in increased parental awareness of the scope of numeracy experiences they engage in on a day-to-day basis. Parents were able to make explicit the incidental, though not accidental, numeracy opportunities: “Every time I tuned in he was actually learning, everything was literacy and numeracy” (p. 180).
More detailed studies involving researcher observations of home practices suggest variability in both the quantity and quality of mathematical interactions between adult and child. While there is some evidence that social class differences may be implicated in the extent to which children are involved in mathematical experiences, Tudge and Doucet (2004) claim that the evidence is somewhat uncertain:

For example, Starkey, Klein, and their colleagues (Starkey & Klein, 2000; Starkey et al., 1999) found that middle-class parents reported providing more mathematics activities to their children than did working-class parents, and Saxe and his colleagues (1987) found that middle-class mothers reported that their children engaged in more complex mathematical experiences more often than did working-class mothers. However, Ginsburg and Russell (1981) found no significant variation of performance for a variety of mathematical tasks for children from working- versus middle-class families, and Ginsburg et al. (1998) reported that although “many economically disadvantaged children enter school less than fully prepared to learn formal mathematics” (p. 425) the data provide little evidence that children from different socioeconomic groups have had significantly different mathematical experiences. Low-income mothers, however, tend to believe that preschool teachers are responsible for providing instruction in mathematics (Holloway et al., 1995; Starkey & Klein, 2000). (p. 23)

In their US study, Tudge and Doucet investigated naturally-occurring mathematical activities engaged in by 39 three-year-olds, evenly divided by ethnicity and social class. Each child was observed for 18 hours over the course of a single week in such a way as to cover the equivalent of a complete day in its life, including time in the home, other’s homes, the childcare centre, and public places. Overall, Tudge and Doucet estimated the number of mathematics experiences (mathematical lessons and mathematical play) that the children from their study had over an entire day to be just ten. The most commonly observed mathematics lesson involved numbering objects. Adults and children were most likely to be engaged in number-based activities and conversations when interacting with puzzles, toys, television, computer programmes, and other games. Likewise, in mathematics-related play activities, children’s most common interaction was via objects (such as toys, puzzles, books, computer programmes) that had number as a central feature.

Although considerable individual variation in the number of mathematical experiences was found, this variation was not explicable by class or by ethnicity. An additional finding was that children were just as likely to be involved in mathematical lessons in the home as in the early childhood centre setting. It was also noted that parents focused more on helping their children learn to read than on their development of mathematical understanding. Given the premise that the activities that routinely take place within the home environment are the key to understanding parents’ cultural construction of their child’s life and development (Harkness & Super, 1995), Tudge and Doucet express concern about the relative lack of importance of mathematics in young children’s lives:

One conclusion that might be drawn is that the provision of mathematical experiences to 3-year-olds is not an important cultural practice, at least with some of the parents whose children we studied. … there is certainly room for parents and other important people in children’s lives to enhance children’s opportunities for mathematical experiences. (p. 36)

Variation in New Zealand home practices was also a significant finding in the EMI-4s study. Young-Loveridge et al. (1995) found that the mothers of the ‘expert’ children encouraged their children to pursue mathematical activities related to the children’s own interests (e.g., counting the number of engines and carriages on trains, monitoring the speedometer on the car, and dealing with money), as opposed to the interests of other family members. In contrast, although the ‘novice’ children were also involved in a range of mathematical activities at home, their mothers did not comment on incorporating mathematical ideas into activities that were of interest to their children. In an earlier case study of a sample of six five-year-old children,
Young-Loveridge (1989b) found that high scorers on numeracy tasks had:

- a wide range of experiences involving numbers,
- a strong orientation towards numeracy by members of their families,
- and the opportunity to observe their mothers using numbers to solve everyday problems of their own.

The low scorers, on the other hand, had few number experiences, an orientation by their families towards literacy but not numeracy, little opportunity to observe their mother using numbers for the solution of practical problems of their own, as well as relatively low family expectations for their mastery of skills. (p. 43)

The link between the family’s ‘orientation to number’ and the mother’s attitudes to mathematics was also an issue in a recent study in England. Aubrey, Bottle, and Godfrey (2003) studied the mathematical development of two 30-month-old children, through analysis of observation and discourse in both home and out-of-home settings. Fine-grained analysis enabled the researchers to posit a relationship between the children’s engagement in mathematical experiences and the roles and relationships established with their parents and other significant adults. Quantitative analysis showed that Child L received a fairly constant input of mathematical dialogue over the observation period, whereas Child H received markedly more input with increasing age. Qualitative analysis of the interactions between the mother and child, and parent interviews, pointed to differences in the nature of the interactions and parental input.

The mother of Child H seemed to recognise that talking and communicating were important and saw mathematics as a part of everyday life. Learning for Child H arose from play activities chosen mainly by Child H herself. Her mother did not believe that she should sit down to ‘teach’ Child H in any formal way, although she did report in interview that she knew other parents did so. The mother of Child L, by contrast, saw mathematics in the home in terms of discrete activities where the goal was the acquisition of counting and arithmetical skills and in which the adult might assume a direct teaching mode. (Aubrey et al., 2003, p. 102)

These two parental styles are in accord with Walkerdine’s (1998) typifications of mother–child conversational practice in the home: for the mother of Child H, the focus was the practical accomplishment of a task and the mathematics was incidental; for the mother of Child L, the mathematics was the explicit pedagogical focus of a purposeful activity. The researchers conjecture that the existence of these “distinct parent pedagogical styles of supporting children’s early mathematical development” (p. 102), ranging from the didactic to the more genuinely participative, may create a mismatch between what some children experience in their homes and what they experience in more formal early childhood settings.

The widely-accepted features of an appropriate learning environment that is based on a view of ‘everyday’ mathematics constructed through children’s chosen investigations in their social and cultural environment may not be one to which all young preschoolers are accustomed. Fostering a positive disposition to learning mathematics where there is opportunity for ideas to be tested out and mistakes to be made will be particularly important in some cases. (Aubrey et al., 2003, p. 103)

As a consequence, they argue, a particular challenge for early childhood educators will be to familiarise young children who are less accustomed to play-oriented approaches and child-directed activity in the home setting with the approaches of the early childhood setting so that positive expectations and attitudes can be acquired for later mathematics learning.

In another investigation of home-based practices, Anderson, Anderson, and Shapiro (2005) observed shared book reading to examine how parents and their young children attended to mathematical concepts. The study involved 39 parents and their four-year-old children, all from a culturally diverse metropolitan area of Canada. Consistent with previous research, the study found that the amount of mathematical talk differed widely across families. During shared reading episodes, the illustrations were the most likely stimulus for adult–child interchange. None of the mathematical talk appeared contrived; interactions principally involved attempts
Effective Pedagogy in Mathematics/Pāngarau Best Evidence Synthesis Iteration

Facilitating centre–home links

Concerns about differential home experiences in mathematics, combined with concerns about the variability of young children’s levels of mathematical understandings (Young-Loveridge, 2004), have led some researchers to trial intervention programmes that focus on home–school links, providing support in terms of communication strategies and home resources. An evaluation of 14 experimental family numeracy programmes in Britain (The Basic Skills Agency, 1998) noted that those programmes found to be most effective had three key strands: joint and separate sessions for parents and children, a structured numeracy curriculum, and bridging activities that parents could use to develop their child's numeracy at home.

Using take-home kits containing materials familiar to the children, Macmillan (2004) describes the development of numeracy concepts among a small group of Aboriginal preschool children in an urban Koori preschool. The numeracy kits, designed to enrich the numeracy concepts of counting, ordering, and pattern, enabled the children’s families to support them in their play and develop a better understanding of their children’s numeracy potential. Through a process of mutual enculturation, the kits provided a bridge between the families and centre staff.

Including families and whānau in curriculum and assessment activities can also be beneficial in bridging the centre–home community divide. Examples in Kei Tua o te Pae / Assessment for Learning, Book 4 (Ministry of Education, 2004a) illustrate how the inclusion of mathematical experiences in construction activities (e.g., Exploring local history, pp. 10–11) and parent or ‘whānau’ stories and visitor input (e.g., Growing trees, pp. 18–19) can be linked to narrative assessment records. Field trips to local sites of interest can be recorded as wall displays and in the children’s ‘books’ for future community reflection.

While most home–school programmes involve planned action for and by parents, there is recognition of individual caregivers’ attempts to develop their own children’s numeracy. To
enhance awareness of opportunities to support mathematics exploration and learning, and to mitigate potential negative modelling of mathematics or attitudes (Ginsburg & Golbeck, 2004), parental education programmes are also advocated. The Ministry of Education has in the past funded the Feed the Mind campaign and more recently published supporting material for the parents of children involved in the Numeracy Development Project (Ministry of Education, 2004d). Supporting parents of preschool children through information workshops has also been found to be effective (Griffin & Coles, 1992).

**Teacher knowledge and beliefs**

Within early childhood education, Hedges (2002) argues that teacher beliefs, rather than teacher knowledge, have traditionally been established as the most important determinant of quality teaching and learning interactions. However, recent moves to embrace a sociocultural perspective have positioned the teacher as having a more active role to play in children's learning, a position that places greater demands of the nature and level of teacher content knowledge. If adults are to fulfil the role of the “knowing assistant and supporter” (Perry & Dockett, 2002, p. 103), they need to understand the mathematics that children are dealing with and to be aware of the many opportunities that present themselves for the learning of mathematics. Studies previously cited in this chapter clearly demonstrate that parents, whānau, and early childhood practitioners who are sensitive to the many learning opportunities that surround them are able to capitalise on those opportunities to extend children's mathematical learning in a wide range of contexts.

However, research studies have also noted that some adults are underutilising the opportunities around them. Siraj-Blatchford et al. (2002) document that, even in the most effective early childhood programmes in Britain, there were examples of teachers having inadequate knowledge and understanding of subject content. While a broad general knowledge was considered to be important, a critical level of specialist subject knowledge was regarded as essential. Content knowledge—mathematical understanding and the associated understanding of children's mathematical development—and teacher confidence as a mathematics learner appear to be at issue (Aubrey, 1997; Davies, 2003; Papic & Mulligan, 2005; Parsonage, 2001; Sarama & Clements, 2004; Young-Loveridge et al., 1998).

One of the tensions for early childhood teaching is that while children demonstrate remarkable facility with many aspects of mathematics, many early childhood teachers do not have a strong mathematical background. At this stage in their development, children's mathematical potential is great and it is imperative that early childhood teachers have the competence and confidence to engage meaningfully with both the children and their mathematics (Perry & Dockett, 2002, p. 107).

Low levels of content knowledge and the resulting lack of confidence about mathematics limit teachers' ability to maximise opportunities for engaging children in the mathematics learning embedded within existing activities, as well as their ability to introduce more focused intervention activities designed to cater for diverse learners. Evidence of restricted mathematics learning occurring in early childhood settings comes from both national and international studies. In the UK, the REPEY study found that approximately 5% of four-year-olds’ time was spent doing mathematics activities (Siraj-Blatchford et al., 2002). In New Zealand, Davies (2002) and Young-Loveridge et al. (1995) found that children did not always take advantage of mathematics learning opportunities available in play, particularly with regard to number. Moreover, Davies (2003) found a mismatch between what was claimed in planning and what actually occurred in kindergarten sessions, noting the practitioners were unclear about recognising mathematical learning and expressed low confidence about their own mathematical knowledge.
Professional development implications

Research evidence suggests that professional development that aims to increase teachers’ awareness of mathematics in everyday life, to encourage greater awareness of home and cultural factors, and to improve mathematical knowledge and teaching skills provides positive outcomes for both teachers and children. White and Hosoume (1993, cited in Farquhar, 2003) report from a three-year project in the US to improve teachers’ mathematics knowledge and teaching skills that improvements in teacher knowledge and confidence are linked to increased use of hands-on activities and greater collaborative engagement. Similarly, in New Zealand, Hedges (2002) noted that as teachers gained confidence in their content knowledge, they were more likely to share this knowledge with children. They were also more likely to be aware of integrated curriculum experiences and opportunities that potentially could strengthen children’s mathematical understanding.

Within the initial teacher education sector, Haynes (2000b) demonstrates how the acquisition of knowledge of the primary school curriculum document can enhance the ability of early childhood teachers to provide mathematics learning experiences through play. The early childhood teachers in Haynes’ study, enrolled in a dual curriculum programme, reported that having knowledge of both curricula made them confident in providing children in early childhood settings with a productive start to their mathematics education (Haynes, 2000c).

An unexpected result arose from Clarke and Robbins' (2004) study of parents’ interpretation of mathematics in the home. Both the nature of the parents’ descriptions and their ability to articulate them came as a surprise to many of the teachers in the project. This is reflected in their comments:

- It is interesting because I think they are doing a lot at home, but we are not aware of it under its titles [reference to mathematical concepts such as measurement, space and shape etc.]

- (I was surprised) to see that some families were working with children in the kitchen. They pulled up a stool and did counting and measuring.

Breaking from the traditional deficit model, Clarke and Robbins (2004) aimed to change teachers’ programmes in ways that supported a diverse range of strategies for numeracy outcomes. Not only did this study demonstrate the extensive numeracy enactments taking place in many families in lower socio-economic circumstances, it also highlighted for teachers the rich constructions of mathematics that individual children were achieving and the important interpersonal relationships that actively supported and extended these constructions. For teachers to cater effectively for children’s different and diverse pathways, it is imperative that they connect with and build upon the children’s rich base of mathematical experiences in ways that acknowledge and support the family’s role.

Bridge to school

Mathematics educators are in accord with early childhood researchers (e.g., Cullen, 1998; Smith, 1996) who argue that learning and development is driven by social interactions in the context of cultural activities. Children’s developing understandings are most appropriately situated within “social and cultural contexts that make sense to the children involved” (Perry & Dockett, 2004, p. 103). Thus, to optimise young children’s mathematics learning there needs to be sufficient opportunity for them to experience and explore mathematics within everyday experiences, in both informal and formal settings.

However, while many studies demonstrate that increasing children’s access to mathematical activities and experiences results in achievement gains, early childhood educators argue that caution is needed to ensure that these gains are not at the expense of personal and social learning. The challenge of an integrated curriculum for young learners is to enhance and highlight mathematical knowledge while supporting an orientation towards learning.
The early years curriculum document, *Te Whāriki*, offers a distinctive approach with regard to the history and philosophy of the sector; one that contrasts with what is offered in school curricula. The emphasis in *Te Whāriki* is “a model of learning that weaves together intricate patterns of linked experience and meaning rather than emphasising the acquisition of discrete skills” (Ministry of Education, 1996, p. 41). One of the ways in which *Te Whāriki* sets itself apart is through its outcomes: negotiability and responsiveness to the learner substitute for the specificity and accuracy that is typically associated with more formal mathematics.

Some researchers have expressed concern that these differences may be amplified by recently adopted numeracy practices in early schooling. For example, Carr (1997) cautioned that the test results for the numeracy component of the School Entry Assessment, *Checkout/Rapua*, may become a self-fulfilling prophecy. Specifically, children identified as competent may be given more challenges, while those with low scores may be less likely to be provided with opportunities to develop “dispositions to be courageous, mindful, persistent and responsible” (p. 325). In a similar vein, Peters (2004b) claims that an emphasis on developmental progressions in number may overlook children’s competency in other areas. Because numeracy progressions are just one of many possible ways in which learning can be constructed, Peters (2004b) is concerned that assessment against developmental progressions may “lead to those who do not fit the norm being pathologised, perhaps blaming either the child or family for perceived deficits” (p. 5).

Based on her study of children in transition to school, Peters (2004a) offers a cautionary note about predetermining young children’s capabilities in mathematics. In order to facilitate the transition process, Peters (1998) argues that a more holistic picture of the child is needed. She recommends that it would be good to follow Meisel’s (1992) suggestion and “focus on a child’s current skill accomplishments, knowledge, and life experiences, and then proceed in a differentiated way to extend a child’s mastery to different and more complex levels” (p. 121).

There is considerable disparity in number competency between the most and least competent new entrants (Gilmour, 1998; Young-Loveridge, 1989b; Young-Loveridge, 1993). Diagnostic Interview (NumPA) data from the 2004 *Numeracy Development Project* (see table 3.1) suggest that entry level disparities remain a feature for year 1 students (Young-Loveridge, 2005).

<table>
<thead>
<tr>
<th>Initial Stage</th>
<th>Percentage of students (rounded to 0.1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0: Emergent</td>
<td>15.7</td>
</tr>
<tr>
<td>1: One-to-One Counting</td>
<td>29.8</td>
</tr>
<tr>
<td>2: Count All with material</td>
<td>43.7</td>
</tr>
<tr>
<td>4: Advanced Counting</td>
<td>2.2</td>
</tr>
<tr>
<td>5: Early Additive</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 3.1. Addition/Subtraction Initial Stages of NumPA for 7793 students

Earlier research studies (e.g. Young-Loveridge, 1991; Wylie, 1998) suggested that children’s numeracy skills may be extremely stable: children who were highly competent at five years maintained their higher levels several years later and the less competent continued to show fewer skills. From her EMI-5s study, Young-Loveridge found that just below 80% of children who began school in the bottom half of their cohort were still in the bottom half of the cohort after four full years at school. Correlations across the data revealed that just over half of the variability in mathematical understanding at nine years could be explained by children’s understanding of number at the age of five.

Recent data from Australia paints a different picture. Analysis by Horne (2005) of the data from a five-year longitudinal study of 572 children from 70 schools involved in the Early Numeracy Research project (ENRP) found that children’s mathematical understanding developed at
different rates and that many moved position relative to their peers. Horne reports that about a third of children who began in the lower part of the class in terms of their mathematical understanding moved into the upper part of the class within five years. This challenges the earlier findings related to stability of competency. Horne claims that:

we need to be very careful not to label children on the basis of demonstrated achievement. Children do learn at different rates ... Students who arrived at school with little knowledge in number domains made considerable gains, often moving ahead of students who had great knowledge. (p. 449)

These results challenge the belief that children who arrive at school in the lower group are condemned to remain in it.

The results also reinforce the fact that it is important for teachers to understand the diverse ways in which a child’s development can evolve. Sometimes this development can appear to be ‘splintered’ or departing from the norm, yet it is still within or close to the range of normal development (Horowitz et al., 2005). Perry and Dockett (2005) argue that the practices embedded in systemic numeracy initiatives in Australia and New Zealand support educators’ developing awareness of the knowledge and skills that young children bring to early childhood settings. Unlike findings from earlier studies, in which teachers underestimated children’s ability in mathematics and spent much of the first year teaching concepts that the children already knew (Young-Loveridge, 1989a), teachers in these numeracy projects are urged to “become familiar with children’s existing mathematical understanding as they commence school to ensure that programming is designed to meet the needs of individual students” (Board of Studies, NSW, 2002, p. 14).

In response to calls for greater harmonisation of approaches to the teaching of mathematics across the pre- and early school years (Hedge, 2003; Macmillan, 2004), researchers and educators note synergies between the mathematics embedded within the curriculum documents. In the following vignette, Perry and Dockett (2005) illustrate how mathematical practices highlighted in the school sector—mathematisation, connections, and argumentation—are also clearly accessible to young children.

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**Mud Pies**

*Bruce (4.10) and Giselle (4.11) are in the sand pit near a water trough.*

Bruce: [Tips water from a bucket into the trough. The water trough is about one-third full.] “We’ve got enough water, now let’s get back to some more sand.”

[Both children shovel sand into the water trough, counting as they go.]

Bruce: “That turns to mud, doesn’t it?”

[More shovelling of sand into the trough and counting]

Giselle: “Yeah.”

[Boy proceeds to mix the sand and water with his shovel.]

Giselle: “Don’t mix it up now. We’re still getting sand here. We’re still getting sand.”

[The children continue adding sand and water until the trough is almost full of a mixture of a consistency appropriate to make ‘mud balls’.]  

It is evident that these two children play cooperatively to work towards a solution to their problem, using mathematical ideas including early understandings of pattern and ratio. In mathematising their problem they explore the issue of required amounts of sand and water. They provide arguments about the amounts of water and sand needed, and they exhibit number and probability sense using trial and error patterns to test their assumptions about how much sand/water was needed.

*From Perry and Dockett (2005)*
In a similar manner, Haynes (2000a) provides illuminating examples of how the thinking that underlies mathematical understanding in infants’ and young children’s play situations aligns with the emergence of concepts as presented in the content strands of the school mathematics curriculum.

A move towards early years (years 0–8) teacher education programmes provides a powerful means to strengthen teachers’ understanding of the links between Te Whāriki and Mathematics in the New Zealand Curriculum. A greater awareness of the activities for the early levels of school learning can help early years educators “recognise and articulate the mathematics children encounter in early childhood, and how this relates to the student of mathematics at the formal and cultural level” (Peter & Jenks, 2000, p. 9). (See also earlier discussion on professional development.)

Collectively, the evidence-based studies presented in this chapter confirm the crucial role that quality early years education plays in the development of infants’ and young children’s mathematical proficiency. In the following chapters, we continue this journey, looking at how quality teaching in the school sector can best support the mathematics learner.

1 The term ‘young learners’ or ‘young children’ is used within this chapter to include ‘infants, toddlers and young children’.
2 The five strands in Te Whāriki are: Well-being—Mana Atua; Belonging—Mana Whenua; Contribution —Mana Tangata; Communication—Mana Reo; and Exploration—Mana Aotūroa.
3 www.ioe.ac.uk/playground
4 Parents of the target children were interviewed to ensure that the additional mathematical activities matched children’s interests.
5 www.dfes.gov.uk/research
6 Vygotsky (1986) describes the ZPD as the “distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers”.
7 Kits containing activities based on Garden Play, Puppet Play, and Pond Play were introduced to provide an additional numeracy focus within three centres.
8 Lessons, as contrasted to ‘play with academic objects’ were defined as explicit attempts to impart or elicit information involving competencies in mathematics.
9 For a child to be coded as engaged in a mathematical activity, mathematics has to be the focus of attention. For example, laying the table was included if someone pointed out to the children that four people were going to be eating and so he or she would need four forks. The researchers acknowledged that there could well be episodes of mathematical thinking that were not verbalised (e.g., counting silently while walking up steps).
10 The case study children included four with particularly high levels of numeracy (i.e., experts) and four with particularly low levels of numeracy (i.e., novices).
11 Mr McMouse and Swimmy, both authored by L. Lionni.
13 Specialist degree programmes in New Zealand that focus on Years 0-8.

References


research and practice (pp. 181-219). Mahwah: Lawrence Erlbaum Associates.
Ministry of Education (2004d). Help your child to develop numeracy ... What you do counts! Wellington: Learning Media


Appendix 1: Locating and Assembling BES Data

Using the ‘health-of-the-system’ approach, we sought to examine the various factors implicated in the creation of an effective learning community. We investigated a number of measures that fell naturally from the ‘what’, ‘why’, ‘how’, and ‘under what conditions’ questions concerning pedagogical approaches that facilitate learning for all students. The task was a considerable one, involving information management, the engagement of advisory and audit groups, and the seeking of contributions from the education community in general and the mathematics education community in particular. This level of engagement ensured that the Best Evidence Synthesis would be inclusive of views from across the community.

Our initial search strategy required us to pay attention to different contexts, different communities, and multiple ways of thinking and working. With this in mind, we undertook a literature search that crossed national and international boundaries. We used a range of search engines as well as personal networks to help us find academic journals, theses, projects, and other scholarly work with a focus on mathematics in New Zealand schools and centres, and by selected authors worldwide. We searched both print indices and electronic indices, endeavouring to make our search as broad as possible within the limits of manageability. This search took into account relevant publications from the general education literature and from the literature that relates to specialist areas of education. The search covered:

- key mathematics education literature including all major mathematics education journals (e.g., *Journal for Research in Mathematics Education, Educational Studies in Mathematics, Journal of Mathematics Teacher Education, For the Learning of Mathematics, The Journal of Mathematical Behaviour*), international conference proceedings (e.g., PME, ICME), Mathematics Research Group of Australasia publications, and international handbooks of mathematics education (e.g., Bishop et al., 2003);
- relevant New Zealand-based studies, reports, and thesis databases, supported by input from the professional community and the Ministry of Education;
- education journals (e.g., *American Educational Research Journal, British Educational Research Journal, Cognition and Instruction, The Elementary School Journal, Learning and Instruction*, etc.) and New Zealand work (e.g., SAMEpapers, SET, NZIES);
- specialist journals and projects, especially those located within the wider education field (e.g., *New Zealand Research in Early Childhood Education, Journal of Learning Disabilities*);
- landmark international studies including TIMSS, PISA, the UK Leverhulme projects.

This search strategy led us to a large body of literature that had something to say about facilitating mathematics learning: the total number of sourced references was just over 1100. Table 1 categorises these references by source:

<table>
<thead>
<tr>
<th>Source of data</th>
<th>Relative frequency (n ~1100)</th>
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<tbody>
<tr>
<td>Mathematics education journals</td>
<td>24%</td>
</tr>
<tr>
<td>Mathematics education reports, books, handbooks</td>
<td>16%</td>
</tr>
<tr>
<td>Mathematics education conference proceedings</td>
<td>15%</td>
</tr>
<tr>
<td>Theses and projects</td>
<td>6%</td>
</tr>
<tr>
<td>General education reports, books, handbooks</td>
<td>10%</td>
</tr>
<tr>
<td>General education journals, reports, and proceedings</td>
<td>19%</td>
</tr>
<tr>
<td>Specialist journals</td>
<td>10%</td>
</tr>
</tbody>
</table>

Table 1: Initial Database Composition by Source
All entries were stored and categorised using EndNote. To assist in the initial synthesis, we distinguished between ‘research’ and ‘discussion document’, and categorised entries according to (a) our ‘diversity’ descriptors (e.g., ethnicity, gender, socioeconomic), (b) centre/school level, and (c) country-of-origin of the data.

These categories and sub-divisions served as a useful starting point for overviewing the literature and allowed us to foreground our fundamental intent to be responsive to diversity. In addition, by classifying entries according to sector and country of origin, we gave ourselves a constant reminder of the need to be inclusive of all perspectives and interests. This inclusiveness gave us a body of literature comprising diverse frameworks and eclectic methodological and analytic approaches.

**Selecting the evidence**

Given the complexity of the teaching and learning process, it is not an easy matter to link specific outcomes with specific pedagogical approaches. In our first pass through the literature, we noted that studies could claim that student achievement was influenced by pedagogical practice much more readily than they could explain how that practice affected student achievement. Many studies offered detailed explanations of student outcomes yet failed to draw conclusive evidence about how those outcomes related to specific teaching practices. Others provided detailed explanations of pedagogical practice yet made unsubstantiated claims about, or provided only inferential evidence for, how those practices connected with student outcomes.

Granted, we were not looking for linear explanations. As Sfard (2005) points out, the complexity of the teaching–learning relationship “precludes the possibility of identifying clear-cut cause–effect relationships” (p. 407). What we were searching for were studies that were able “to offer a developing picture of what it looks like for a teacher’s practice to cultivate student [proficiency]” (Blanton & Kaput, 2005, p. 440). We were searching for studies that offered a “detailed look at how [teachers’] actions played out in the classroom and how students were involved in this” (Blanton & Kaput, 2005, p. 435) and the sorts of mathematical proficiency that resulted. Specifically, we were seeking studies that offered not just detailed descriptions of pedagogy and outcomes but rigorous explanation for close associations between pedagogical practice and particular outcomes.

Paying attention to diverse forms of research evidence required our serious consideration of the literature relating to disparate factors from different sectors and representative of different time periods. Luke and Hogan (in press) note that what is distinctive about the approach undertaken in the New Zealand Best Evidence programme “is its willingness to consider all forms of research evidence regardless of methodological paradigms and ideological rectitude, and its concern in finding contextually effective appropriate and locally powerful examples of ‘what works’... with particular populations, in particular settings, to particular educational ends” (p. 5). We have included many different kinds of evidence that take into account human volition, programme variability, cultural diversity, and multiple perspectives. Each form of evidence, characterised by its own way of looking at the world, has led to different kinds of truth claims and different ways of investigating the truth. Our pluralist stance left us free to consider the relative strengths and weaknesses of different methodological approaches.

A fundamental challenge for this BES has been to demonstrate a basis for knowledge claims. We are absolutely aware that, like data selection, assessment of evidential claims from secondary sources is a highly perspectival activity. “Even those gazing down a microscope are as capable of finding what they expect to find, or want to find, as anyone else” (Davies, 2003). In response to this challenge, studies have been reported in a way that will make the original evidence as transparent as possible. Informed by the *Guidelines for Generating a Best Evidence Synthesis Iteration 2004*, we included studies that:

- provided a description of the context, the sample, and the data;
• offered details about the particular pedagogy and the specific outcomes;
• connected research to relevant literature and theory;
• used methods that allow investigation of the pedagogy–outcome link;
• yielded findings that illuminated what did or did not work.

The Guidelines for Generating a Best Evidence Synthesis Iteration allowed us to deal not only with a diversity of research topics, approaches, and methods, but also to capture differences in the context, practices, and ways of thinking of researchers. The method employed in this BES for evaluating validity required us to look at the ways different pieces of data meshed together and to determine the plausibility, coherence, and trustworthiness of the interpretation offered.

Assessments about the quality of research depend to a large extent on the nature of the knowledge claims made and the degree of explanatory coherence between those claims and the evidence provided. What we were looking for was the explanatory power of the stated pedagogy–outcome link. When assessing the nature and strength of the causal relations between pedagogical approaches and learning outcomes, we were guided by Maxwell’s (2004) categorisations of two types of explanations of causality. The first type, the regularity view of causation, is based on observed regularities across a number of cases. The second type, process-oriented explanations, sees “causality as fundamentally referring to the actual causal mechanisms and processes that are involved in particular events and situations” (p. 4). Cobb argues (2006, personal communication) that regularity explanations are particularly useful for policy makers, while process-oriented explanations are relevant to teachers, who are concerned with “the mechanism through which and the conditions under which that causal relationship holds” (Shadish, Cook, & Campbell, 2002, p. 9, cited in Maxwell, 2004, p. 4). Attending to both types of explanation of causality meant including both large-scale and single-case studies. In many instances, we have found it useful to present a single case—a learner or teacher, a classroom, or a school—in the form of a vignette to exemplify the relations between learning processes and the means by which they are supported.

**Research sources in this BES report**

This BES report contains approximately 660 references. Included amongst these are research reports of empirical studies, ranging from very small, single-site settings (e.g., Hunter, 2002) to large-scale longitudinal studies (e.g., Balfanz, Maclver, and Byrnes, 2006). Some of the larger studies have multiple references because they include different papers/conference proceedings/book chapters or because they embrace work authored by different researchers (e.g., the New Zealand Numeracy Development Project). In addition, the references include reports containing educational statistics and policy, theoretical writings, and commentaries and reviews on multiple research findings (e.g., van Tassel-Baska, 1997).

The Guidelines for Generating a Best Evidence Synthesis Iteration point to the importance of drawing on New Zealand research in order to illuminate what works in the New Zealand context. However, despite an exhaustive search for New Zealand work, it is apparent (see chapter 8 for further discussion) that the strengths and foci of New Zealand research are not evenly distributed. In some areas—for example, early years education—there are relatively few New Zealand (or Australian) researchers working with a specific focus on mathematics education (Walshaw & Anthony, 2004). Table 2 shows the country of origin of the literature included in this BES. The numbers reflect New Zealand’s relatively new positioning within the international mathematics education research community.
Table 2: Database composition according to country

<table>
<thead>
<tr>
<th>Country</th>
<th>Relative Frequency</th>
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<td>New Zealand</td>
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<tr>
<td>Australia</td>
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<tr>
<td>United Kingdom</td>
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<tr>
<td>United States</td>
<td>49%</td>
</tr>
<tr>
<td>Other (e.g., Africa, Netherlands, Spain)</td>
<td>6%</td>
</tr>
</tbody>
</table>

Table 3 shows the proportion of the items included in the BES (both empirical studies and commentaries) that relates to each of the different sectors. Publications relating specifically to intermediate schools have been classified with the literature on primary schools.

Table 3: Database composition according to sector

<table>
<thead>
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<th>Sector</th>
<th>Relative Frequency (n=520)</th>
</tr>
</thead>
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<tr>
<td>Preschool</td>
<td>18%</td>
</tr>
<tr>
<td>Primary school</td>
<td>48%</td>
</tr>
<tr>
<td>Secondary school</td>
<td>21%</td>
</tr>
<tr>
<td>Teacher education</td>
<td>13%</td>
</tr>
</tbody>
</table>

**Synthesising the data**

Our conceptual framework, outlined in chapter 2, offered a way of structuring the data. Within the community of practice frame in and beyond the classroom, we identified the following components: (a) the organisation of activities and the associated norms of participation, (b) discourse, particularly norms of mathematical argumentation, (c) the instructional tasks, and (d) the tools and resources that learners use. We began the iterative chapter-structuring process by outlining a number of key areas. These included mathematical thinking and identities, scaffolding and co-construction, tasks, activities, assessment, educational leadership, home–school/centre links, and wider school communities. Each of these served as a starting point for our exploration and was found, in the course of the investigation, to be a useful initial category for addressing questions of equity and proficiency in relation to effective mathematics teaching.

In time, we organised these categories more cohesively into groups. What we endeavoured to do was organise multiple elements, types, and levels and varying temporal conditions in line with the critical dimensions of a community of practice and the guiding principles established in chapter 2. The content of the subsequent chapters is shaped according to these dimensions and principles. Chapter 3 focuses on all three dimensions in a search for understanding of how pedagogy influences early years outcomes. Chapters 4 and 6 explore interrelationships that are centred on the joint enterprise of developing mathematical proficiency for all learners. Chapter 5 explores the role of mathematical tasks and the part that they play in enhancing students’ learning.

Reminding ourselves and readers that this BES synthesis is a product of currently accessible research, we concur with Atkinson’s (2000) view that “the purpose of educational research is surely not merely to provide ‘answers’ to the problems of the next decade or so, but to continue to inform discussion, among practitioners, researchers and policy-makers, about the nature, purpose and content of the educational enterprise” (p. 328). Rather than offering broad answers that promise much and achieve little, it is our hope that the structure we have used will foster understanding, reflection, and action concerning the characteristics of effective pedagogical approaches in mathematics.
References


Appendix 2: URLs of citations

The following 22 papers/articles/chapters/books are suggested as potentially useful sources for teachers to engage more deeply with the range of issues raised in this best evidence synthesis iteration. Readers are encouraged to source and read them. Several are available online; the others can be sourced through libraries.

The full citations are hyperlinked in the online PDF. For the convenience of those using a hard copy of the text, the URLs are listed below.

Carpenter, Thomas P ; Franke, Megan L ; Jacobs, Victoria R
A longitudinal study of invention and understanding in children’s multidigit addition and subtraction
http://nzcer.org.nz/BES.php?id=BES001

Clarke, Barbara ; Clarke, Doug
Mathematics teaching in Grades K-2: painting a picture of challenging supportive, and effective classrooms

Cobb, Paul ; Boufi, Ada ; McClain, Kay ; Whitenack, Jor
Reflective discourse and collective reflection
http://nzcer.org.nz/BES.php?id=BES020

Empson, Susan B
Low performing students and teaching fractions for understanding: An interactions analysis
http://nzcer.org.nz/BES.php?id=BES021

Fraivillig, Judith L ; Murphy, Laren A ; Fuson, Karen C
Advancing children’s mathematical thinking in everyday mathematics classrooms
http://nzcer.org.nz/BES.php?id=BES003

Gifford, Sue
A new mathematics pedagogy for the early years: in search of principles for practice
http://nzcer.org.nz/BES.php?id=BES004

Goos, Merrilyn
Learning mathematics a classroom community of inquiry
http://nzcer.org.nz/BES.php?id=BES005

Houssart, Jenny
Simplification and repetition of mathematical tasks: a recipe for success or failure?
http://nzcer.org.nz/BES.php?id=BES006

Irwin, Kathie ; Woodward, J (paper available online)
A snapshot of the discourse used in mathematics where students are mostly Pasifika (a case study in two classrooms)
http://nzcer.org.nz/BES.php?id=BES007

Kazemi, Elham ; Stipek, Deborah
Promoting conceptual thinking in four upper-elementary mathematics classrooms
http://nzcer.org.nz/BES.php?id=BES008

Latu, Viliami (paper available online)
Language factors that affect mathematics teaching and learning of Pasifika students
http://nzcer.org.nz/BES.php?id=BES009

O’Connor, Mary Catherine
“Can any fraction be turned into a decimal?” A case study of the mathematical group discussion
http://nzcer.org.nz/BES.php?id=BES010

Rietveld, Christine M.
Classroom learning experiences of mathematics by new entrant children with Down syndrome

Savell, Jan ; Anthony, Glenda Joy
Crossing the home-school boundary in mathematics
http://nzcer.org.nz/BES.php?id=BES049
Sheldon, Steven B; Epstein, Joyce L
Involvement counts: family and community partnerships and mathematics achievement
http://nzcer.org.nz/BES.php?id=BES012

Smith, Margaret Schwan Smith; Henningsen, Marjorie A
Implementing standards-based mathematics instruction: a casebook for professional development

Steinberg, Ruth M; Empson, Susan B; Carpenter, Thomas P
Inquiry into children’s mathematical thinking as a means to teacher change
http://nzcer.org.nz/BES.php?id=BES014

Watson, Anne; De Geest, Els
Principled teaching for deep progress: Improving mathematical learning beyond methods and material
http://nzcer.org.nz/BES.php?id=BES015

Wood, Terry (paper available online)
What does it mean to teach mathematics differently?
http://nzcer.org.nz/BES.php?id=BES016

Yackel, Erna; Cobb, Paul
Sociomathematical norms, argumentation, and autonomy in mathematics
http://nzcer.org.nz/BES.php?id=BES017

Young-Loveridge, Jenny (paper available online)
Students views about mathematics learning: a case study of one school involved in Great Expectations Project
http://nzcer.org.nz/BES.php?id=BES018

Zevenbergen, R
The construction of a mathematical habitus: implications of ability grouping in the middle years
http://nzcer.org.nz/BES.php?id=BES019
Appendix 3: Glossary

The page reference for the first and/or most important occurrence of the term is given in brackets.

Cognitive engagement (p. 2). The state of being engaged in thinking
Community of Practice (p. 6). The complex network of relationships within which teachers teach and students learn
Connectionist teachers (p. 97). Teachers who consistently make connections between different aspects of mathematics
Decile (p. 9). In New Zealand, a 1–10 system used by the Ministry of Education to indicate the socio-economic status of the communities from which schools draw their students; low-decile schools receive a higher level of government funding
Developmental progressions (p. 47). Sequential learning pathways categorised as a series of steps
Empirical evidence (p. 24). Data that has been collected systematically for research purposes
Equity (p. 9). The principle based on the belief that social injustices should be redressed by allocating resources according to need, not power; in education, this may mean, amongst other things, the provision of different pedagogical approaches depending upon the needs of the learners
Family Math (p. 171). A US initiative designed to develop parents’ skills so they can work with their children on their mathematics
Feed the Mind (p. 45). A media campaign funded by the New Zealand Ministry of Education and designed to support family involvement in children’s learning
High or low press for understanding (p. 121). Differing levels of cognitive engagement demanded of students by teachers for clarification of thinking
Kahoa (p. 36). A festive necklace (Tongan)
Kōhanga reo (p. 9). Màori-medium early childhood centres
Kura kaupapa Màori (p. 10). Màori-medium schools (kura = school), based on a Màori philosophy of learning (see pp. 54–5)
Manipulatives (p. 133). Any concrete materials used by students to model mathematical relationships
Mathematical argumentation (p. 123). Presenting a case to support or refute a premise developed by mathematical thinking
Mathematical identity (p. 19). How a student sees him/herself as a learner and doer of mathematics
Metacognition (p. 38). The knowledge and processes involved in thinking about and regulating one’s own thinking, which is essential for reflecting, self-monitoring, and planning
Norms of participation (p. 54). The rules, spoken or unspoken, that govern the way students behave and contribute in the classroom
Number Framework (p. 109). A model, structured in 8 stages, showing how students typically develop understanding of number and number operations (New Zealand, NDP)
Number sense (p. 98). An understanding of the relationships, patterns, and fundamental reasonableness that lie behind all mathematical operations
Numeracy (p. 28). The ability to use mathematics effectively, fluently, and with understanding in a wide variety of contexts
Numeracy Development Project (NDP) (pp. 9, 17). The central part of the New Zealand Ministry of Education’s Numeracy Strategy, which has as its primary objective the raising of student achievement in numeracy through lifting teachers’ professional capability
NumPA (p. 9). A structured, diagnostic interview used by teachers to place students on the early stages of the Number Framework (New Zealand, NDP)
Open-ended tasks (p. 106). Tasks that require students to engage in problem definition and formulation before beginning to think about a solution
Pasifika students (p. 9). Students whose families have come from Sàmoa, Tonga, the Cook Islands, Niue, Tokelau, Tuvalu, and some other, smaller Pacific nations
Pedagogical Content Knowledge (p. 199). In this context, knowledge about mathematics and how to teach it as well as knowledge about how to understand students’ thinking about mathematics
Pedagogy (p. 5). The processes and actions by which teachers engage students in learning
Poi (p. 26). A small ball, often made of woven flax, on the end of a length of string; swung rhythmically by women when performing action songs (Màori)
QUASAR (p. 95). A programme developed to help urban students develop understanding of mathematical ideas through engagement with challenging mathematical tasks
Revoicing (p. 78). The repeating, rephrasing, or expansion of student talk in order to clarify or highlight content, extend reasoning, introduce new ideas, or move discussion in another direction
Scaffolding (p. 27). Temporary, structured support designed to move learners forward in their thinking
School–home or home–school partnership (p. 160). The deliberate nurturing of relationships between the school and the home, in the interests of better supporting student learning

Sociocultural practices (p. 19). Practices relating to the social and cultural aspects of participation in the classroom

Sociocultural theory (p. 24). The theory that learning arises out of social interaction

Socio-economic status (SES) (p. 30). Categorisation of individuals or communities, based on income, family background, and qualifications

Sociomathematical norms (pp. 61–62). Shared understandings of the processes by which students and teacher contribute to a mathematical discussion

Tasks (p. 94). Defined by Doyle (1983) as “products that students are expected to produce, the operations that students are expected to use to generate those products, and the resources available to students while they are generating the products”

Te ao Māori (p. 54). The Māori world

Te Poutama Tau (p. 59). The Numeracy Project (New Zealand) as developed for implementation in Māori-medium schools

Te Whāriki (p. 24). The New Zealand early childhood curriculum (for children aged 5 or under)

Tukutuku panels (p. 115). A Māori craft form consisting of ornamental lattice-work panels woven together with strips of flax into intricate designs

Waiata (p. 26). A song (Māori)

Whānau (p. 41). Extended family (Māori)

Wharekura (p. 9). Māori-medium secondary schools, which are based on a Māori philosophy of learning

Zone of Proximal Development (ZPD) (p. 36). Vygotsky (1986) describes the ZPD as the “distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers”

### Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
<th>Page(s)</th>
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<tr>
<td>CGI</td>
<td>Cognitively Guided Instruction Project</td>
<td>17, 105</td>
</tr>
<tr>
<td>EAL</td>
<td>English as an Additional Language</td>
<td>116</td>
</tr>
<tr>
<td>EFTPOS</td>
<td>Electronic Funds Transfer at Point of Sale</td>
<td>115</td>
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<tr>
<td>EMI-4s</td>
<td>Enhancing the Mathematics of Four-Year-Olds</td>
<td>28</td>
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<tr>
<td>ENRP</td>
<td>Early Numeracy Research Project</td>
<td>158</td>
</tr>
<tr>
<td>EPPE</td>
<td>Effective Provision of Pre-school Education Project</td>
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<tr>
<td>ERO</td>
<td>Education Review Office</td>
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<td>IAMP</td>
<td>Improving Attainment in Mathematics Project</td>
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<td>ICME</td>
<td>International Congress on Mathematics Education</td>
<td>20</td>
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<tr>
<td>ICT</td>
<td>Information and Communication Technologies</td>
<td>27</td>
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