



MINISTRY OF EDUCATION

Te Tāhuhu o te Mātauranga

Draft Case 1

Developing communities of mathematical inquiry

This is the first of a series of cases being prepared for the 2nd iteration (under development) of:

Quality Teaching for Diverse (All) Learners in Schooling

Best Evidence Synthesis Iteration [BES]

He Ako Reikura, He Ākonga Rerekura (Te Katoa)

Hei Kete Raukura [BES]



ITERATIVE BEST EVIDENCE SYNTHESIS PROGRAMME

HEI KETE RAUKURA

<http://educationcounts.govt.nz/goto/BES>

New Zealand

Introduction to Case 1: Developing communities of mathematical inquiry

Case 1, ‘Developing communities of mathematical inquiry’, illustrates how two teachers developed teaching practices that were highly effective for diverse learners. The case focuses on how these teachers accelerated the mathematics achievement of their year 4 to 6 students, most of whom were Māori or Pasifika. The pedagogy that supported this improvement has implications that are relevant for all students across the primary, intermediate, and lower secondary school contexts.

The teachers were participants in an intervention led by Dr Roberta Hunter, Senior Lecturer at the Massey University College of Education, Albany Campus. The intervention was carried out as part of a wider study led by Professors Glenda Anthony and Margaret Walshaw, co-directors of the Centre of Excellence for Research in Mathematics Education and co-writers of the *Effective pedagogy in mathematics/pāngarau best evidence synthesis iteration [BES]*.

New BES cases to support teachers’ work

This is the first of a series of cases being developed as part of the update of the first best evidence synthesis (BES), due for publication in 2011. The cases have been given priority in response to advice from teachers and principals. They bring the BES findings to life through real examples from schools in New Zealand and overseas. Each new case has been selected from hundreds of studies because it is outstanding in its effectiveness for diverse learners and it shows the BES findings in action. The cases are intended to support professional learning about working ‘smarter rather than harder’ in education.

Addressing areas of need

Wherever possible, the cases show teachers addressing teaching and learning issues that we know are areas of need across New Zealand. This case has been given precedence because the mathematics achievement of New Zealand students in their middle primary school years is below the international mean.¹ An international comparison revealed that students whose teachers had participated in the Numeracy Development Project (NDP) did significantly better than students whose teachers had not. However, despite most New Zealand primary school teachers having participated in the NDP, it is clear that many teachers need more support. The 2009 National Education Monitoring Project findings indicate continuing disparities for Māori, and worsening disparities for Pasifika, in mathematics. Case 1 exemplifies an approach that helps students develop all five of the key competencies, supporting teachers in implementing the New Zealand Curriculum.

This case also demonstrates how the development of a learning community countered bullying. Bullying is another issue that international research show is an area of need in New Zealand schools.² New Zealand students in their middle primary school years experience a high rate of bullying from their peers, the second highest on an international index comparing rates of student safety in their peer cultures in thirty-five countries. Māori boys and Pasifika girls and boys experience the highest rates of bullying.

Professional learning community

While teachers are the primary audience for the case, it is also intended as a resource for professional leaders and for all those working to support teachers. We are progressively releasing the new BES cases, initially in electronic form. Our intention is that schools and teacher educators should use them and provide feedback about how we could optimise the value of this resource for professional learning. Please feel free to make copies.

Feedback to inform BES development

We will draw upon your feedback as part of our iterative process as we finalise the cases for the new BES update. Please send any feedback to best.evidence@minedu.govt.nz

1 Mullis, I., Martin, M. O., & Foy, P. (2007). *TIMSS 2006 international science report: Findings from IEA’s trends in mathematics and science study at the fourth and eighth grades*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

2 Ibid, p. 367

Other resources

This case also exemplifies the findings of the *Effective pedagogy in mathematics/pāngarau best evidence synthesis iteration [BES]*. Hard copies of that BES are available free on request within New Zealand from orders@thechair.minedu.govt.nz

Summaries of this BES are available electronically in English and Māori on the UNESCO website:

- Anthony, G. & Walshaw, M. (2009). *Effective pedagogy in mathematics: Educational practices series, 19*. International Academy of Education & International Bureau of Education, UNESCO. Retrieved from www.ibe.unesco.org/en/services/publications/educational-practices.html
- Anthony, G. & Walshaw, M. (2010). *Te ako pāngarau whaihua: Educational practices series, 19*. International Academy of Education, International Bureau of Education, UNESCO. Retrieved from www.ibe.unesco.org/en/services/publications/educational-practices.html

Acknowledgments

The Ministry of Education acknowledges the work and the outstanding expertise of Dr Roberta Hunter of Massey University, Albany, who led the research and development and the teacher professional learning that is described in this case.

This work was possible also because of the larger Teaching and Learning Research Initiative, Numeracy Practices and Change Project, which was co-directed by Professors Glenda Anthony and Margaret Walshaw.

The Iterative Best Evidence Synthesis Programme pays tribute to the outstanding teaching of ‘Ava’ and ‘Moana’, whose commitment to the collaborative professional learning opportunity described in this case made possible such extraordinary gains for students. We acknowledge also the contribution of your principal and colleagues in forging such a powerful school-based professional learning community.

The development of this case has been strengthened by formative quality assurance. We acknowledge and thank:

- Dr Roberta Hunter and Professor Glenda Anthony, Massey University
- Liz Patara, Principal/Tumuaki, and Mary Lee Bogard, teacher, Clyde Quay School
- Dr Earl Irving and Dr Claire Sinnema, University of Auckland
- Associate Professor Jenny Young-Loveridge, University of Waikato
- Stephanie Greaney, Manager Evaluation and Policy, Education Review Office
- Professor Courtney Cazden, Professor Emeritus, Harvard University.

Thanks also to the team at Learning Media for your patient and iterative work in developing this case to date.

Please send further feedback to: best.evidence@minedu.govt.nz

Case 1. Developing communities of mathematical inquiry

<p>Source</p>	<p>Hunter, R. (2007). <i>Teachers developing communities of mathematical inquiry</i>. Albany, Massey University: Unpublished doctoral thesis. Available from the New Zealand Education Theses database at www.educationcounts.govt.nz/goto/BES</p> <p>Hunter, R. (2008). Facilitating communities of mathematical inquiry. In M. Goos, R. Brown, & K. Makar (Eds.), <i>Navigating currents and charting directions</i> (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 31–39). Brisbane: MERGA.</p> <p>Hunter’s 2008 publication received the Beth Southwell Practical Implications Award sponsored by the Australian Association of Mathematics Teachers (AAMT) and the National Key Centre for Teaching and Research in School Science and Mathematics, Curtin University, Perth, Western Australia.</p>															
<p>Introduction</p> <p><i>‘Effect size’ is a statistical measure of the impact of an intervention on an outcome. Hattie¹ shows that the average yearly effect of teaching in New Zealand in reading, mathematics, and writing from year 4 to year 13 is $d = 0.35$. Effect sizes above 0.40 represent an improvement on business-as-usual and effect sizes of $d = 0.60$ are considered large.</i></p>	<p>This case describes how two teachers worked to develop classroom learning communities in which students learned to engage with the teacher and each other in mathematical inquiry, reasoning, and argumentation.² It traces significant changes in teacher knowledge and pedagogy and in student behaviour and mathematical practices through a collaborative, school-based, professional learning process. That process was led by the researcher over one school year. The effect sizes for the gains in both classes were very large: $d = 2.39$ for Ava’s class and $d = 2.53$ for Moana’s class. This is extraordinary progress, representing the equivalent of several years’ progress (compared with business-as-usual teaching) in just one year.</p> <p>The case dramatically illustrates the benefits of developing a genuine learning community within the peer culture. Many studies across the curriculum report high gains when using co-operative learning approaches (for example, Hattie³ found an effect size of $d = 0.59$ for co-operative learning approaches). On the other hand, studies also reveal the problems that arise when students are not effectively trained to work collaboratively or when they simply work in seating or social groupings. The source study for this case reveals how two teachers took action to transform their teaching, thereby accelerating student development in terms of a wide range of valued outcomes, including cognitive, metacognitive, and social outcomes.</p> <p>Both teachers had previously engaged in the Numeracy Development Project (NDP) but had adapted what they learned to fit with their traditional practices. Through inquiry into their own practice and the use of a smart tool, the Mathematics Communication and Participation Framework⁴ (see Appendix 1), they were able to build on what they had learned in the NDP and transform the ways in which their students interacted and participated. These changes, in turn, accelerated the students’ progress in terms of academic achievement and self-management. The case describes the wide range of instructional strategies, scaffolds, and prompts used by the two teachers and explains how these influenced student learning and behaviour.</p> <p>Case 7 describes the 2006 TIMSS⁵ findings that New Zealand students in their middle primary school years experience a high rate of bullying from their peers, the second highest on an international index comparing rates of student safety in their peer cultures in thirty-five countries. Māori boys and Pasifika girls and boys experience the highest rates of bullying. Given these findings, the magnitude of the changes achieved through teacher actions in Case 1 have important implications for policy and practice nationally. This case highlights the connections between the collaborative inquiry that the teachers engaged in and the shifts for students. It was the professional learning process and support that the teachers experienced that enabled them to achieve the changes.</p>															
<p>Learners and learning context</p>	<p>The learners in this study were two teachers and their students. Ava, a teacher of Māori and New Zealand European descent, was in her ninth year of teaching. Ava’s class included year 4 and 5 students. Moana, a Māori teacher, was in her fifth year of teaching. Moana’s class included year 4, 5, and 6 students.</p> <p>The setting was a small, suburban, multicultural decile 3 primary school. The students were mostly of Māori and Pasifika ethnicity. Tumeke School⁶ had a transient population, with around 30% of the students in each class leaving or arriving during the school year.</p> <table border="1" data-bbox="376 1688 1465 1868"> <thead> <tr> <th>Student ethnicity</th> <th>Ava’s class</th> <th>Moana’s class</th> </tr> </thead> <tbody> <tr> <td>Māori</td> <td>47%</td> <td>40%</td> </tr> <tr> <td>Pasifika</td> <td>25%</td> <td>58%</td> </tr> <tr> <td>NZ European</td> <td>24%</td> <td>2%</td> </tr> <tr> <td>Other</td> <td>4%</td> <td></td> </tr> </tbody> </table> <p>The professional development and research project described in this case was led by the study’s author, doctoral student Roberta Hunter. The project was funded from the larger Numeracy Practices and Change Project. This project was a Teaching and Learning Research Initiative (TLRI) project, co-directed by Professor Glenda Anthony and Associate Professor Margaret Walshaw.</p>	Student ethnicity	Ava’s class	Moana’s class	Māori	47%	40%	Pasifika	25%	58%	NZ European	24%	2%	Other	4%	
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Outcomes

The students in these classrooms experienced a variety of ways of engaging in mathematics. Working through a collaborative process of mathematical inquiry supported the students' well-being, enhanced their understanding of mathematics, and developed their mathematical proficiency.

The qualitative data reported in this case shows that the collaborative inquiry process supported the students in developing productive dispositions and mathematical identities. The students grew in their confidence as mathematicians. They could see the value of mathematics, wanted to learn mathematics, and believed that they could succeed if they applied themselves. These dispositions are not just a desirable end product of mathematics education; they are the means by which students learn and do mathematics.

The following graphs show levels of student achievement in mathematics on the number framework, as revealed by the NumPA diagnostic interview. It compares the students' achievement at the start of the year, before the process began, to their achievement at the end of the year, after the development of communities of mathematical inquiry. The teachers and the researcher checked the data independently.

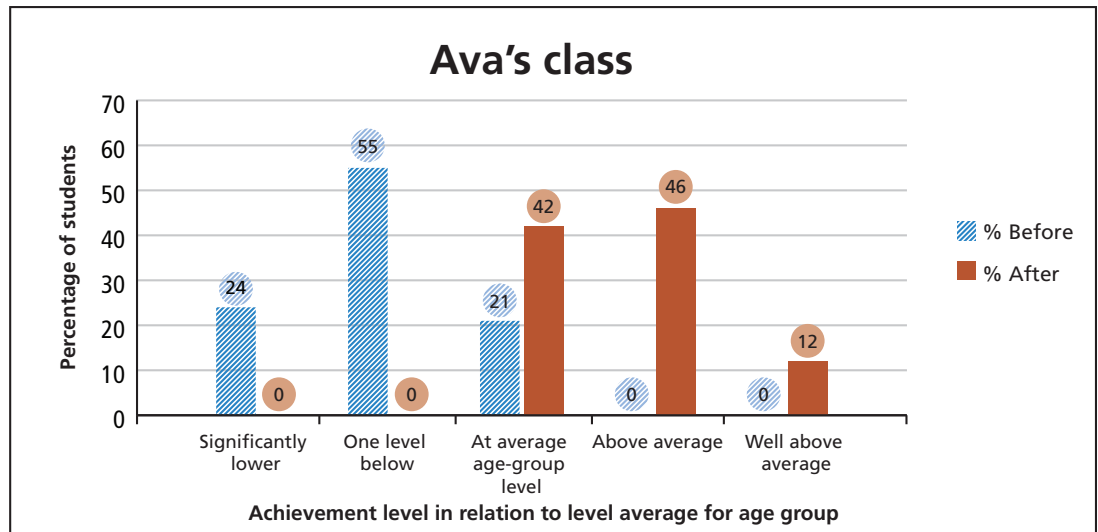


Figure 1. Shifts in mathematics achievement in Ava's class after collaborative professional development underpinned by research and development

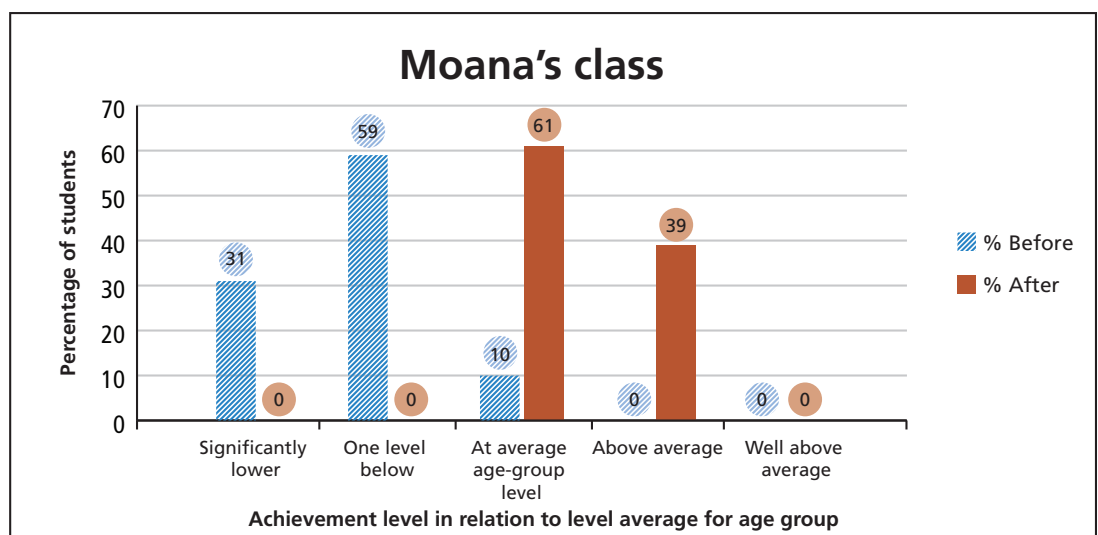


Figure 2. Shifts in mathematics achievement in Moana's class after collaborative professional development underpinned by research and development

As discussed, the effect sizes for the gains in both classes were very large: $d = 2.39$ for Ava's class and $d = 2.53$ for Moana's class. The confidence intervals about these values accord with these effect sizes. The effect sizes are also in accord (in terms of order of magnitude) with the values of Cramer's V , a measure of the strength of association of the Chi-Square tests (given categorical data). For Ava's class, $V = 0.841$ (out of a possible 1) was calculated for the achievement gain. The achievement gain for Moana's class was $V = 0.907$.

<p>Curriculum relevance</p>	<p>Studying mathematics helps students develop the ability to think creatively, critically, strategically, and logically. They learn to create models, conjecture, justify and verify, and seek patterns and generalisations. In this case, both teachers developed units that included the study of fractional numbers, statistics and measurement, and adding, subtracting, multiplying and dividing. A further extension unit focused on algebraic reasoning was developed within the project. The students actively built capability in all five key competencies: thinking, using language, symbols, and texts, managing self, relating to others, and participating and contributing.</p>
<p>The Quality Teaching Dimensions</p>	
<p>Outcomes focus</p> <p>Hua te ako, hua te ākonga</p> <p>Quality teaching is focused on valued outcomes and facilitates high standards for diverse learners.</p>	<p>Ava and Moana’s initial focus</p> <p>At the outset of the intervention, Ava was confused about the outcomes she intended to focus on. She wanted her students to learn the rules and procedures of mathematics and, in light of her previous professional learning, she also wanted them to be able to articulate, within their groups, the mathematical strategies they were using. However, she did not set out to create a mathematical discourse community (that is, a classroom community of novice mathematicians whose learning includes understanding and using the specialised language of mathematics). As each student spoke, the other students sat silently.</p> <p>Initially, Moana was focused on a limited set of low-level outcomes. She ascribed her students’ low achievement levels to their lack of prior experiences at home, their lack of interest and engagement in mathematics, and their passive approach to learning. She later reflected, “I thought I had high expectations ... I have realised I [had] low expectations.”</p> <p>Ava and Moana’s shift in focus</p> <p>Both teachers developed a comprehensive new approach that focused on valued outcomes. They set out to develop in their students the communication and participatory skills needed to advance their thinking and extend their mathematical knowledge through active participation in the classroom mathematical discourse community.</p> <p>The teachers aimed to develop their students’ abilities to explain mathematical concepts, justify their arguments, and make generalisations. They expected the students to use mathematical language and definitions and also to use generalisations and inscriptions⁸ to support their explanations and to clarify their understandings of others’ explanations. The teachers did not form these new expectations overnight. Rather, as the teachers participated in the iterative professional learning process and the changes began to be embedded, they saw how their students extended their mathematical proficiency through the new teaching and learning approach.</p> <p>Both teachers began to develop their students’ identities as mathematicians and members of a learning community in which each person had high expectations for their own outcomes. The teachers became adept at identifying and addressing barriers to progress and keeping the focus on the outcomes.</p>
<p>Teacher knowledge and inquiry</p> <p>Te mōhio o te kaiako, te tikanga uiui</p> <p>Teachers work smarter, not harder, through the use of evidence for continuous improvement.</p>	<p>Teacher knowledge and inquiry at the outset of the study: Ava</p> <p>Ava explained that she had initially liked teaching mathematics but had lost confidence in her mathematics teaching expertise after participating in the NDP. The NDP professional development had created dissonance between her prior view of mathematics teaching as focused on rules, procedures, and routines and the new focus on students developing their mathematical thinking.</p> <p>Observations revealed that Ava had tried to implement what she had learned from the NDP. But in practice, she had done what many teachers do when confronted with new curricula designed to change the nature of teaching. She had transformed the NDP lessons to fit with her old way of doing things. Her students listened to each other in silence and answered the teacher’s questions, but there was no process to build a mathematical learning community amongst students.</p> <p>Teacher knowledge and inquiry at the outset of the study: Moana</p> <p>Moana had always disliked mathematics. Her memory of her own schooling was negative: “I didn’t have the ideas that other kids had so I just never said anything. I thought they were all brighter than me.”</p> <p>Before the intervention, Moana was using a learning styles approach. She saw her Māori and Pasifika learners as kinaesthetic learners needing practical activities. (This approach has been found to have little benefit for student achievement and has even had negative effects on Māori and Pasifika student achievement.)</p> <p>In her mathematics lessons, Moana did most of the talking because she saw her role as instructing students in how to use mathematical procedures. She explained that she had adapted the NDP’s knowledge and strategy activities so that they were always at a concrete and manipulative level. She focused on students ‘learning by doing’, because she believed that they found explaining difficult.</p>

	<p>Teacher inquiry through the study</p> <p>The professional development was delivered through a mix of whole-staff meetings and eight meetings of a teacher study group. The professional learning activities included examining research, considering how other teachers had changed the forms of mathematical talk towards inquiry and argumentation, and reading about mathematical practices used by students in inquiry environments. The researcher used a DVD from an international research study to stimulate examination of inquiry practices. The teachers re-examined the materials in the NDP and discussed which learning tasks best supported their goals, which involved changing the classroom discourse patterns. The TLRI project funded the necessary teacher relief time. This amounted to at least six days for each participating teacher.</p> <p>Initially, the researcher was in the school for two mornings a week and worked collaboratively with the teachers to provide support both in and out of class. Data gathering in the classrooms commenced from term 2 and included the researcher making video recordings of lessons, which were later transcribed. The study group used these recordings extensively. They viewed them repeatedly, collaboratively examining and identifying critical incidents and the antecedents and consequences of these incidents. The teachers recorded their lesson intentions and drew on excerpts from the transcripts to trace how their actions had influenced what students actually said and did.</p> <p>The researcher perceived that the classroom videos were key to the professional learning, especially at the start, because they illustrated “the power of who asks what in the classroom.” It was also important to provide the teachers with a small number of research articles in a timely way that addressed a specific need. The DVD had helped motivate the teachers by “pushing their vision of possibilities”.</p> <p>The collaboration between the researcher and teachers persisted beyond the study when some teachers co-presented their findings with the researcher in conference workshops. The teachers became members of a local mathematical professional group.</p>
<p>Smart tool</p> <p>Te raweke tapu ngaio</p>	<p>The researcher developed, and then worked with the teachers to refine, a tool they called the Mathematics Communication and Participation Framework (see Appendix 1 on page 15 of this case). This framework provided an outline of both communicative and participatory actions, along with examples. Teachers could use it to scaffold students’ learning about how to participate in communities of mathematical inquiry.</p>
<p>Teacher inquiry</p> <p>Te tikanga uiui</p>	<p>Teacher inquiry: Ava</p> <p>After reading relevant research and taking part in discussion, Ava was convinced of the value of the new approach and decided to take immediate action to change her teaching practices, using the Mathematics Communication and Participation Framework. Her initial focus was on students’ participation in mathematical explanations. Within a month, she decided to stop the use of ability-group streaming.</p> <p>Teacher inquiry: Moana</p> <p>Moana was quiet when the study group discussed the framework. She believed strongly in certain theories about teaching and learning, and at first she did not believe that the students would learn more through a learning community. She was focused on students getting the right answers. However, the videos stimulated Moana to build on what she had learned about how other teachers were changing their classroom practices. She decided to start by developing students’ ability to construct and present mathematical explanations. When she saw the initial changes in her students, she was convinced of the value of the new approach and whole-heartedly engaged with the change process.</p>
<p>Opportunity</p> <p>Kapohia, akona</p> <p>Opportunity to learn is effective and efficient.</p>	<p>In both teachers’ classes, the opportunity to learn included a whole-class session, small-group work, and later whole-class report-backs and sharing sessions. The use of mixed-ability groups rather than fixed-ability groups was critical to the effectiveness of this intervention. Students were not publicly labelled as low achievers.</p> <p>The teachers paid careful attention to giving students the time they needed to learn, scaffolds and learning supports (including support from peers and from visual representations), and opportunities for practice and application. The teachers developed their students’ ability to resolve cognitive conflicts through peer interaction. The case vividly illustrates the intensive and scaffolded opportunities to learn that students can access within a peer learning community where each student engages in thoughtful and respectful interaction with others. Such opportunities are far more limited when students can only rely on the teacher for feedback.</p>

Caring and inclusive learning communities

Te ako, he tohu manaaki, he piringa tangata

Pedagogical practices enable classes and other learning groups to work as caring, inclusive, and cohesive learning communities.

E rima te 'arapaki, te aro'a, te ko'uko'u te utuutu, 'iaku nei.

Under the protection of caring hands, there's a feeling of love and affection.

Ava builds a mathematical learning community

Ava began by developing new 'rules for talk'. At the beginning of each maths lesson, she discussed with her students how they were to work together in a mathematical community. During the lesson, she repeatedly reviewed with them how this was going and modelled the changes she wanted to see. For example, she used collective pronouns when she was speaking to the students:

Can you show us with your red pen what would happen? We want to know.

Moving beyond cumulative talk

When Ava and the researcher reviewed the video recordings of her class together, she saw that the student talk was mainly cumulative or disputational. 'Disputational' talk involves cyclical assertions and counter-assertions that remain unexamined as participants concentrate on defending their ideas. In 'cumulative' talk, questions and arguments are avoided as students uncritically build on each other's thinking. The students come to a collective view but not one that they have evaluated. Both kinds of talk are unproductive in terms of mathematical reasoning.

Requiring mathematical reasoning

Ava sought to develop her students' ability to engage in talk that probed mathematical ideas. She explicitly required them to listen, discuss, question, and make sense of the reasoning used by others. She also used modelling to scaffold the students into how to construct mathematical explanations that would be well-reasoned, conceptually clear, and logical:

Talk about what you are doing ... so whatever number you have chosen, don't just write them. You say, "I am going to work with ..." or "I have chosen this and this because ...".

Ava regarded building the students' capacities to act as a learning community not as a prelude to the mathematical work but as integral to developing mathematical discourse. For example, after giving the students individual time to think about how to solve a problem, Ava said:

You are going to explain how you are going to work it out in your group. They are going to listen. I want you to think about and explain what steps you are doing, each step you are doing, what maths thinking you are using. The others in the group need to listen carefully and stop you and question [at] any time or at any point where they can't track down what you are saying.

Using think time

Ava used 'think time' as a form of social nurturing for less confident members of the group. This involved halting the discussion to allow them time to reflect and making it clear that their learning mattered.

Promoting constructive argumentation

Ava explicitly directed the students to argue their ideas in a productive manner, as in the following exchange with students:

Ava: Argue your maths. Explore what other people say. Listen carefully, bit by bit, and make sense of each bit. Don't just agree. Check it all out first. Ask a lot of questions. Make sure that you can make sense – that you understand. What's another important thing in working in a group?

Alan: Share your ideas. Don't just say "I can do it myself" ...

Ava: That's right. We need to use each other's thinking ... because we are very supportive and that's the only way everyone will learn.

Ava believed that many of her students would be used to oppositional or aggressive argument and that this would have shaped negative beliefs about arguing:

Ava: I am aware that the students are growing into this behaviour now, but disagreeing can be so hard for these students, so I find I have to keep almost giving them permission to disagree or argue.

This meant that teaching the skills to argue constructively was an ongoing focus:

Ava: 'Arguing' is not a bad word ... sometimes I know that you people think to argue is ... I am talking about arguing in a good way. Please feel free to say if you do not agree with what someone else has said. You can say that as long as you say it in an OK sort of way.

Scaffolding newcomers

Importantly, given the transient nature of the student population, Ava took care to induct newcomers into the norms of behaviour that would enable them to participate in the mathematics discourse community.

Caring and inclusive learning communities

Te ako, he tohu manaaki, he piringa tangata

Pedagogical practices enable classes and other learning groups to work as caring, inclusive, and cohesive learning communities.

The transcripts show how the students learned to scaffold and support each other in managing the mathematical argumentation. For example, when Hemi's body language indicated discomfort after Hone had asked 'How can you prove it to us?', Pania took up the challenge:⁹

I can show them. This group, that's Hemi and his group, are saying that Jack is arguing that $\frac{5}{8}$ is less than $\frac{3}{4}$. [Pania draws a rectangle and divides it in eighths. She records $\frac{2}{8}$ three times as she shades two sections, and then she records $\frac{6}{8}$. She draws another rectangle and this time shades in $\frac{5}{8}$]. They are saying $\frac{3}{4}$ is an equivalent fraction to $\frac{6}{8}$. Like them, I can prove it is bigger because, look, this is only $\frac{5}{8}$, which is smaller than $\frac{6}{8}$. Is that right, you guys?

Moana builds a mathematical learning community

Moana was a caring and concerned teacher, but when she looked back on the first recordings of her practice, she realised that her students had shown fear in whole-class discussions. By demanding the right answers without providing sufficient scaffolding, she had failed to provide a genuinely caring classroom environment.

When Moana first set about creating a learning community, she presented the students with another teacher's chart of ground rules and instructed them to work collaboratively by following the rules. This did not work. Students constantly interjected and made negative comments to each other, both in the small groups and in the larger sharing sessions.

Scaffolding students in working together

After reflecting on the video recordings, Moana was concerned about the disengagement and silence of the girls, and especially their reluctance to question or argue with the boys. Together with the researcher, Moana planned how to make incremental changes through scaffolding the students' ability to work together. In a three-step process, Moana:

1. stopped using small groups and returned to the previous whole-class discussion format
2. asked the students to work in pairs and scaffolded the way they were to talk to and listen to each other
3. required the paired students to explain each other's reasoning (using materials to demonstrate this) as a report-back process.

Ground rules for talk

At this stage, Moana again reflected on the videos and started to develop her own repertoire of talk to explain to the students what she meant by 'active participation in mathematical discourse'. She developed her own classroom-specific chart of the Ground Rules for Talk: How do we kōrero in our classroom? (See Appendix 2 on page 16 of this case.) She began each lesson by explicitly explaining how she wanted the students to work. In one lesson, Moana observed that the students had begun recording a response without discussion. She intervened:

Before you pick up a pen or touch the paper, you need to discuss it first, and I want you to show me how you can work together; listening and building on each other's thinking with those questions.

Considered pairings

Moana noticed that the Māori and Pasifika girls in particular were diffident in pairs with boys. She responded by putting students in single-sex pairs and putting particular girls together (for example, pairs of Pasifika girls) to scaffold a safe environment for talk. She set about creating an environment in which the students encouraged each other to be risk-takers and built up their confidence:

Moana: You don't have to whisper. You can talk because you want to make sure that you are heard.

At the end of the first month, when the pairs were able to construct and examine their explanations more collaboratively, Moana began to vary the number and combinations of students working together.

Feedback on aspects of the classroom community

Moana gave specific feedback on students' participatory behaviours as well as their mathematical thinking, for example:

Donald is really listening... He is not only listening to the person; he is watching when they write things down. His questions are really specific to what the person is doing.

Moana: You are saying, "Yeah" ... What are you saying "Yeah" to?

Beau: She's just been going ... making up suggestions ... dumb ones ... she's just...

Moana: Well, it's not dumb ones. I have been listening, and she's making you think because she is using the problem and making sense of it. She knows that you have to listen to each group member – listen to their thinking and make sense of what they think. [Turns and asks Anaru] So what was your way? Can you explain it, please?

	<p>Moana closely monitored the less able or less confident students in the context of the heterogeneous groups in order to draw attention to how their reasoning had contributed:</p> <p><i>Moana: Wow, Teremoana, see how you have made them think when you said that? Now they are using your thinking.</i></p> <p>As a result, the students' collaborative skills developed markedly. By terms 3 and 4, the following kinds of comments indicated a strongly functioning student learning community:</p> <p><i>Wiremu: Don't dis her, man, when she is taking a risk.</i></p> <p>This 10-year-old boy's comment exemplifies the shared responsibility students were taking for creating a peer culture that supports intellectual risk-taking. He is telling another boy to stop his disrespectful behaviour towards a female peer so she can take a risk in her public participation in a mathematical discourse. This comment illustrates the depth of the change that Moana was forging in the peer culture.</p>
<p>Connection</p> <p>Tūhono</p> <p>Teaching makes educationally powerful connections to students' knowledge, experiences, and identities.</p>	<p>The mathematical inquiry and discourse approach that the teachers developed was designed to ensure that their students were constantly making links to their existing knowledge and experience as they built their understandings. In addition, the teachers selected and designed mathematical problems that connected to the students' interests, for example, by having them divide pieces of chocolate or by linking to games such as space invaders that the students enjoyed in their lives outside school.</p> <p>Moana also explicitly drew upon her cultural knowledge to make connections for both Māori and Pasifika students between their experiences in other settings (such as membership of a kapa haka group or the Cook Islands Māori hair-cutting ceremony) and the new ways of participating in class. She talked about how she wanted them to work together as a whānau (extended family) in which everybody supports each other, with leadership shifting according to who has the greatest expertise in each situation. By allowing time for her students to talk and build upon each other's ideas, she enabled her students to tap into and create educationally powerful connections with their cultural knowledge and practices. This 'discursive approach' also supported her in building enhanced relationships with her students that build on these strengths.</p>
<p>Scaffolding</p> <p>Te ako poutama</p> <p>Pedagogy scaffolds, and provides appropriate feedforward and feedback on, learning.</p>	<p>The Mathematics Communication and Participation Framework provided an overview of the actions the teachers could scaffold students into using as they learned to engage in reasoned collective discourse. The framework describes three distinct developmental phases in this learning.</p> <p>The teachers used the framework to scaffold the students' use of mathematical language, thinking, and justification and to foster the behaviours that would help them to interact in a mathematical learning community. The excerpts below further demonstrate the scaffolding teachers provided through their interactions with students and careful design of learning tasks. The excerpts also demonstrate how the students themselves took on the responsibility of scaffolding their own and other group members' learning.</p> <p>In this excerpt, Ava uses revoicing to clarify and define mathematical terms:</p> <p><i>Jo: Isn't that just plussing three sticks not timesing it? You are not timesing, you're adding.</i></p> <p><i>Pania: Well, what she sort of means, it is like it is going up.</i></p> <p><i>Alan: Is that timesing, going up?</i></p> <p><i>Ava: When we talk about timesing, what do we actually mean?</i></p> <p><i>Jo: We mean multiplying not adding. Adding is a plus [indicates + with her fingers], that sign.</i></p> <p><i>Sandra: You mean when you add two more squares on, that is multiplying?</i></p> <p><i>Ava: Rachel was saying she is adding 3, adding another 3, so that's 3 plus 3 plus 3. So if you keep adding 3 all the time, what is another way of doing it?</i></p> <p><i>Alan: You can just times instead of adding. It won't take as long and it is more efficient.</i></p> <p><i>Ava: Yes, you are right. Did you all hear that? Alan said you can just times it, multiply by [groups of] three, because that is the same as adding on 3 each time. What word do we use instead of 'timesing'?</i></p> <p><i>Alan: Multiplication, multiplying.</i></p> <p>In this excerpt, Moana scaffolds the use of mathematical language and explanations, beginning as Aporo uses counters to model how his group solved a problem:</p> <p><i>Aporo: Two, four, six, eight, ten, twelve.</i></p> <p><i>Moana: So that's what's called skip-counting, because you are skipping across the numbers.</i></p> <p><i>Tere: We kept adding like two more. We counted in twos.</i></p> <p><i>Moana: Counting in twos. Yes, that is skip counting.</i></p> <p>...</p> <p><i>Moana: What have you actually done there, Mahine?</i></p> <p><i>Mahine: I have plussed 10 onto 47.</i></p> <p><i>Moana: So you have added 10 onto 47? Are there any questions? Questions like "Where did you get the 10 from?"</i></p>

	<p>When she saw that her students were not taking up opportunities to ask questions and to dispute, Moana introduced a non-verbal scaffold – koosh balls.¹⁰ The koosh balls were placed in the middle of the discussion circle. The students picked them up to indicate that they had a question or challenge. The researcher explained that although picking up the ball indicated to the explainer that there was a challenge, their self-esteem was protected because the non-verbal signal gave time to think and then respond.</p> <p>Teachers creating new knowledge about scaffolding</p> <p>As the teachers engaged in the collaborative professional learning process, they expanded on the Mathematics Communication and Participation Framework and created a wide range of further scaffolds to support the students' learning (see Appendix 3 on page 17 of this case). For example:</p> <ul style="list-style-type: none"> • Practise talking about the bits you agree with and be ready to say why. • Discuss the explanation and explore the bits which are more difficult to understand. • Take turns explaining the solution strategy¹¹ using a representation.
<p>Responsive-ness</p> <p>Tauaronui</p> <p>Quality teaching is responsive to all students' learning, identities, and well-being.</p>	<p>Creating a discourse community enabled both teachers to become more diagnostic about and responsive to the students' learning processes. Hattie¹² calls for teachers to make the learning process as visible as possible so that they can notice and react to feedback from students about what is needed. Both these teachers used non-verbal signals from students to inform next steps. When students had difficulties, their teachers provided time for reflection. Ava called this 'think time' and Moana called it 'rethink time'. As it became safer for the students to actively participate and take risks, the teachers were increasingly able to use the students' thinking as a resource for improving teaching and learning.</p> <p>Responsiveness in Ava's class</p> <p>The researcher's data analysis showed that when Ava gave students opportunities for think time during group work, along with an expectation of accountability ("You have a think about it while I pop over to this group so we can hear their thinking, then we will come back to you"), her students became more willing to express partial understandings or difficulties they had in understanding sections of an argument. This pause in mathematical dialogue was designed to support all students' active cognitive engagement.</p> <p>Misconceptions as a resource</p> <p>Initially, Ava sidelined erroneous thinking when it was made explicit in students' explanations. To get students and teachers more used to examining and analysing erroneous reasoning, the wider study group of teachers developed a set of problems devised to include solution strategies based on misconceptions commonly held by students of their age. (See Appendix 4 on page 18 of this case.)</p> <p>Responsiveness in Moana's class</p> <p>Moana's responsiveness to student learning processes was transformed as her understanding of the nature of mathematical learning shifted:</p> <p><i>[It's] good to see different thinking coming through, blows me away because before they never had a chance to explore ... I just thought that they understood by me talking all the time.</i></p> <p>Her ability to notice, recognise, and respond to students' learning processes was supported by her careful listening to their discourse.</p> <p>Moana used the concept of rethink time to help the students when they were confused:</p> <p><i>I can see you are confused. Me, too, [but] that's all right. We can take some time ... rethink about it.</i></p>

Thoughtful learning strategies

Takina te wānanga

Pedagogy promotes learning orientation, student self-regulation, metacognitive strategies, and thoughtful student discourse.

The whole approach to building the students' engagement in a mathematical discourse community required the teacher (and eventually the students) to press for thoughtfulness. While students were talking, the expectations for others in the group were clear. Each individual was required to:

- actively ask questions;
- follow the line of reasoning;
- take responsibility for the regulation of their own (and others') learning;
- be accountable for their own understanding.

The Mathematics Communication and Participation Framework supported the teachers to scaffold student learning strategies in ways that progressively extended the students' thinking and their repertoire of productive ways of engaging in mathematics. Both teachers repeatedly identified and overcame barriers to the students' use of effective learning strategies. The examples below highlight ways in which these two teachers built their students' ability to engage in thoughtful discourse communities.

Ava scaffolds mathematical thinking

Ava used the Mathematics Communication and Participation Framework to scaffold increasingly complex mathematical thinking. Her students moved from reporting solutions to developing an inquiry or argument culture. The teacher initiated change through modelling the processes of mathematical inquiry while building a safe learning environment. Both Ava's and her students' use of questions and prompts pushed the students to higher and more complex levels of reasoning.

Intellectual risk-taking

Intellectual risk-taking supports intellectual growth. Both teachers explicitly encouraged and supported their students to take intellectual risks, as in this exchange, which took place before Ava's students began a mathematical activity:

Ava: Remember how yesterday we talked about how, in maths learning, you go almost to the edge? So therefore I am going to move you out of your comfort zone. It's lovely being in a comfortable, cosy place. Even as an adult, we love to be there, too. But if you are already there, then it's time to move on, out a little bit ... so you go out there ... maybe a bit more ... a bit further next time and come back in again ...

Sandra: And when you are out there, you will make that your comfort zone. Then move on and make that your comfort zone.

Time and space to question and clarify mathematical explanations

Rather than jumping in when she saw that the students were struggling, Ava scaffolded the students' ability to thoughtfully evaluate their solution strategies. When one small group had not identified the problems in their reasoning, Ava gave them a new opportunity to question and clarify as they explained their solution strategy to the class:

Ava: I know for some of you it pushed you out to the edge. I can see some semi (and that means 'partly') confused looks and that's okay ... you will need to question anything you don't understand. We are going to listen, step by step, and when you finish one step, put the lid on the pen and see if anyone has got any questions ... Remember you need to talk about what your group did.

Sandra: [Records 52 on a sheet] The rule was timesing it by two and it goes up.

Ava: Right, can you stop there and wait for questions.

Pania: How did you get that?

Ava: [Prompts] How did you get what?

Pania: How did you work out it was 52 sticks?

Sandra: We went 25 and 25 equals 50. Number 12 and 12 equals 24 plus 2 would equal 25.

Ava: You people need to be really listening to what is being said here. You have got to ask yourself does it make sense to you.

Pania: I don't think it will make sense because 24 plus 2 would equal 26.

Ava: Well, tell you what. You have done really well. There are some adults who would not even know how to work through that sort of problem, so well done.

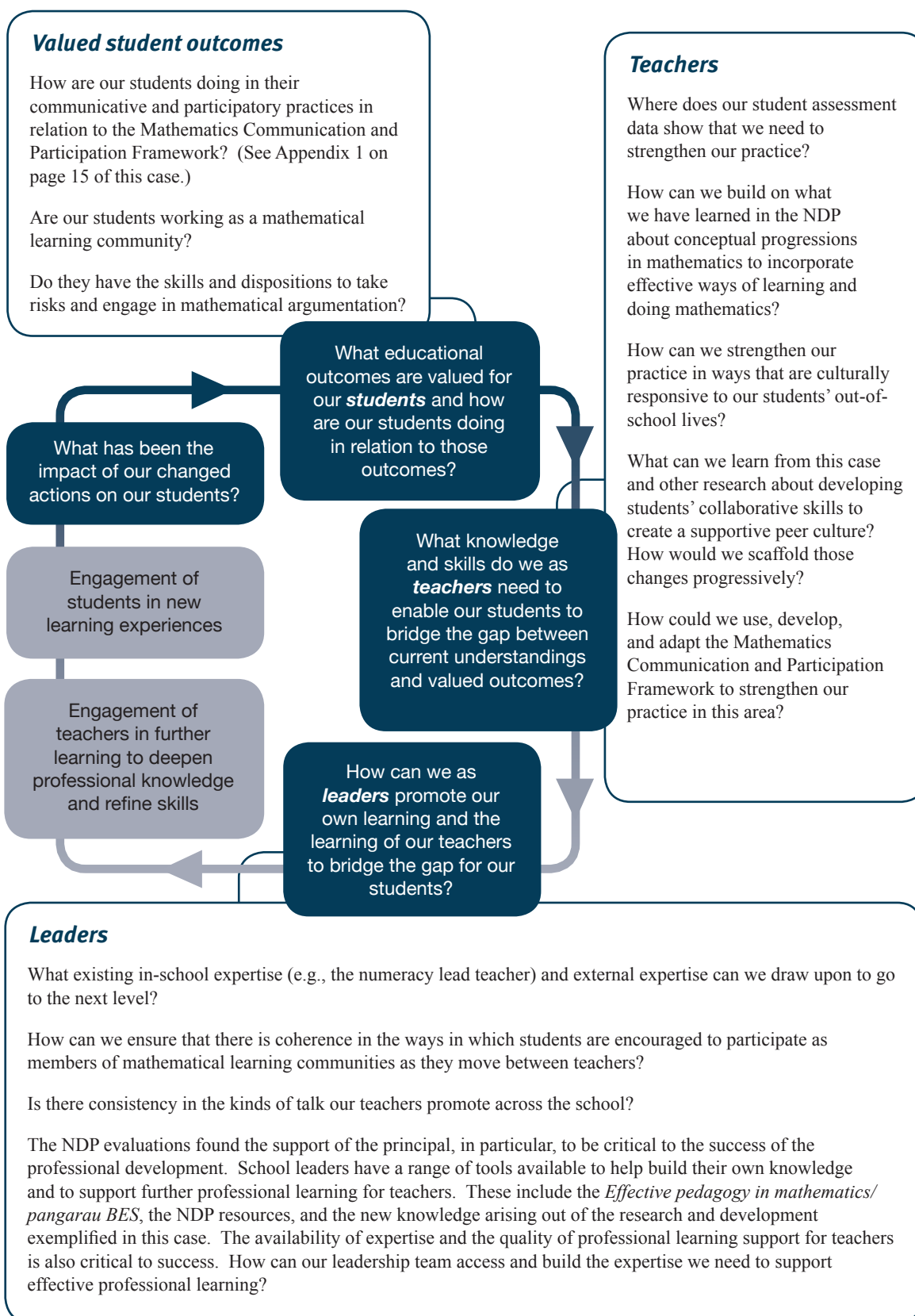
In this example, the small group extensively explored a problem and constructed multiple representations of their thinking. There was time for challenge and clarification, and there were clear expectations that 'making sense' was the goal.

	<p>Moana scaffolds mathematical thinking</p> <p>Mathematical argumentation is an essential strategy for learning mathematics. Like Ava, Moana realised that her students initially struggled with the idea that they should engage in mathematical argumentation and that this was a barrier to their thinking. She set about using incremental steps to shift their attitudes to argumentation and their ability to argue, including teaching them ‘polite’ ways to disagree and challenge.</p> <p>Positioning students as accountable for their opinions</p> <p>Moana made the students aware that she held them accountable for their own understanding. She regularly halted explanations and required students to take a stance. For example, she told them:</p> <p><i>At some point, you are going to have an opinion about it. You are going to agree with it or disagree with it.</i></p> <p>She ensured that they knew they needed a valid reason to support their stance, directing the students to think about what they were saying:</p> <p><i>Make sense of it. If you don’t agree, say so but say why. If there is anything you don’t agree with, or you would like them to explain further, or you would like to question, say so. But don’t forget that you have to have reasons. Remember it is up to you to understand.</i></p> <p>Validating autonomous mathematical reasoning</p> <p>Aroha explains a solution strategy for adding 43, 23, 13, and 3. She records 43, 23, 13, 3 and then $3 \times 4 = 12$:</p> <p><i>Aroha: I am adding 43, 23, 13, and 3, so 3 times 4 equals 12.</i></p> <p><i>Kea: Why are you trying to do that with those numbers? Where did you get the 4?</i></p> <p><i>Aroha [Points at the 3 digit on each of the four numbers] These 3s, the four 3s.</i></p> <p><i>Donald: All she is doing is, like, making it shorter by, like, doing 4 times 3.</i></p> <p><i>Hone: Because there are only the 10s left.</i></p> <p><i>Donald: Three times 4 equals 12, and she got that off all the 3s; like the 43, 23, 13, and 3. So she is just like adding the 3s all up and that equals 12.</i></p>
<p>Assessment for learning</p> <p>He Aromatawai, He Akomatawai</p> <p>Teachers and students engage constructively in goal-oriented assessment.</p>	<p>During each lesson, the teachers’ moment-by-moment assessment enabled them to decide what questions to ask, when to intervene, and how to respond to student questions. The teachers used the information gained by observing and listening to the students to focus student attention on key examples and explanations during class discussions.</p> <p>Both teachers sought to develop their classes as mathematical learning communities in which students were accountable, both individually and in their groups, for checking that their explanations and justifications were well-reasoned. The teachers scaffolded the students’ self-regulation by moving from first having them check their understanding of bits of problems to checking whole problems and from checking whether they had the most efficient solution for one problem to (eventually) checking solutions across a set of problems. They received, and progressively internalised, multiple messages from the teacher that they needed to be able to inquire into, explain, and justify their own and others’ solutions to mathematical problems. The students became increasingly autonomous learners, able to analyse and validate their own reasoning rather than placing this responsibility on the teacher. In addition, the development of strong learning community norms meant that the students progressively took responsibility not only for their own learning but also for the learning orientation of their peers. For example, when eight-year-old Pita in Moana’s class saw another student scribbling on a recording sheet during their group work, he said: “Don’t, man. You listen or ask or you aren’t even learning, man.”</p>
<p>Alignment</p> <p>Tātarite</p> <p>Curriculum goals, resources, task design, teaching, school practices, and home support are effectively aligned.</p>	<p>The support they gained through participating in the teacher study group working with the researcher was critical to the teachers’ success. This is consistent with the finding in the <i>Teacher professional learning and development BES</i> that teachers need opportunities to process their new learning with others if significant change is to occur. Moana began making changes after she saw the impact of such changes in her colleagues’ classrooms.</p> <p>The researcher also worked with the whole staff in setting up the project and had the strong support of the school principal, the board of trustees, and senior leadership. The school had an open-door policy towards including the wider community in the life of the school, including this project. The teachers communicated with the parents through their newsletters and homework sheets, explaining the types of problems the students were working with and the questions they were being encouraged to use. The teachers drew on their own knowledge of their students’ backgrounds to structure and discuss the classroom social norms in ways that were socially and culturally responsive.</p>

Other resources

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Professional learning: Starter questions



Appendix 1. The Mathematics Communication and Participation Framework¹³

An outline of the communicative and participatory actions teachers facilitate students to engage in as they (the teachers) scaffold the use of reasoned, collective, mathematical discourse.

		Phase One	Phase Two	Phase Three
Communicative actions	Making conceptual explanations	Use problem context to make explanation experientially real.	Provide alternative ways to explain solution strategies.	Revise, extend, or elaborate on sections of explanations.
	Making explanatory justification	Indicate agreement or disagreement with an explanation.	Provide mathematical reasons for agreeing or disagreeing with solution strategy. Justify using other explanations.	Validate reasoning using own means. Resolve disagreement by discussing viability of different solution strategies.
	Making generalisations	Look for patterns and connections. Compare and contrast own reasoning with that used by others.	Make comparisons and explain the differences and similarities between solution strategies. Explain number properties, relationships.	Analyse and make comparisons between explanations that are different, efficient, sophisticated. Provide further examples of number patterns, number relations, and number properties.
	Using representations and inscriptions	Discuss and use a range of representations or inscriptions to support an explanation.	Describe inscriptions used to explain and justify conceptually as actions on quantities, not manipulation of symbols.	Interpret inscriptions used by others and contrast with own. Translate across representations to clarify and justify reasoning.
	Using mathematical language and definitions	Use mathematical words to describe actions.	Use correct mathematical terms. Ask questions to clarify terms and actions.	Use mathematical words to describe actions (strategies). Reword or re-explain mathematical terms and solution strategies. Use other examples to illustrate.
Participatory actions		Active listening and questioning for more information. Collaborative support and responsibility for reasoning of all group members. Discuss, interpret, and reinterpret problems. Agree on the construction of one solution strategy that all members can explain. Indicate need to question during large-group sharing. Use questions that clarify specific sections of explanations or gain more information about an explanation.	Prepare a group explanation and justification collaboratively. Prepare ways to re-explain or justify the selected group explanation. Provide support for group members when explaining and justifying to the large group or when responding to questions and challenges. Use wait time as a think time before answering or asking questions. Indicate need to question and challenge. Use questions that challenge an explanation mathematically and draw justification. Ask clarifying questions if representation and inscriptions or mathematical terms are not clear.	Indicate need to question during and after explanations. Ask a range of questions including those that draw justification and generalised models of problem situations, number patterns, and properties. Work together collaboratively in small groups, examining and exploring all group members' reasoning. Compare and contrast and select most proficient (that all members can understand, explain, and justify).

Appendix 2. Moana's chart for the ground rules for talk

How do we kōrero in our classroom?

We make sure that we discuss things together as a whānau. We listen carefully and actively to each other.

That means:

- We ask everyone to take a turn at explaining their thinking first.
- We think about what other questions we need to ask to understand what they are explaining.
- We ask questions 'politely' as someone is explaining their thinking; we do not wait until they have completed their explanation.
- We ask for reasons why. We use 'what' and 'why' questions.
- We make sure that we are prepared to change our minds.
- We think carefully about what they have explained before we speak or question.
- We work as a whānau to reach agreement. We respect other people's ideas. We don't just use our own.
- We make sure that everyone in the group is asked and supported to talk.
- We all take responsibility for the explanation.
- We expect challenges and enjoy explaining mathematically why we might agree or disagree.
- We think about all the different ways before a decision is made about the group's strategy solution. We make sure that as we 'maths argue' we use "I think ... because ... but why ..." or we use "If you say that, then ...".

Appendix 3. Tumeke School’s expansions of sections of the Mathematics Communication and Participation Framework

This chart illustrates the way Moana further elaborated sections of the Mathematics Communication and Participation Framework. She did this in two steps. She constructed the first step after she began exploring the framework, and she put the second step in place when she observed that her students were managing the first set of expectations and would not lose confidence when expected to engage at a higher level in the mathematical practices.

<p>Making conceptual explanations</p> <p>Use problem context to make your explanation experientially real.</p>	<p>Think of a strategy solution and then explain it to the group.</p> <p>Listen carefully and make sense of each explanation step by step.</p> <p>Make a step-by-step explanation together.</p> <p>Make sure that everyone understands. Keep checking that they do.</p> <p>Take turns explaining the solution strategy using a representation.</p> <p>Use equipment, the story is the problem, a drawing or diagram, or/and numbers to explain another way or backing for the explanation.</p>	<p>Keep asking questions until every section of the explanation is understood.</p> <p>Be ready to state a lack of understanding and ask for the explanation to be explained in another way.</p> <p>Ask questions (What did you...?) of sections of the explanation.</p> <p>Discuss the explanation and explore the bits which are more difficult to understand.</p> <p>Discuss the questions the listeners might ask about the explanation.</p>
<p>Making explanatory justification</p> <p>Indicate agreement or disagreement with an explanation and have a mathematical reason for the stance.</p>	<p>Listen to each person in your group and state agreement with their explanation OR state disagreement with their explanation.</p> <p>Practise talking about the bits you agree with and be ready to say why.</p> <p>Ask questions of each other about why you agree or disagree with the explanation.</p> <p>Pick one section of an explanation and provide a mathematical reason for agreeing with it.</p> <p>Discuss the explanation or a section of the explanation and talk about the bits that the listeners might not agree with and why.</p>	<p>Provide a mathematical reason for disagreeing with the explanation or a section of the explanation.</p> <p>Think about using material or drawing pictures about the bit of the explanation that there have been a lot of questions about it in the group.</p> <p>Ask questions of each other (Why did you...? How can you say...?)</p> <p>Question until you understand and are convinced.</p> <p>Explain and use different ways to explain until you are ALL convinced.</p>

Appendix 4. Problem example using student misconceptions developed by the Tumeke School study group

Peter and Jack had a disagreement. Peter said that $\frac{5}{8}$ of a jelly snake was bigger than $\frac{3}{4}$ of a jelly snake because the numbers are bigger.

Jack said that it was the other way around, that $\frac{3}{4}$ of a jelly snake was bigger than $\frac{5}{8}$ of a jelly snake because you are talking about fractions of one jelly snake.

Who is right? When your group has decided who is correct and why, you need to work out lots of different ways to explain your answer. Remember you have to convince either Peter or Jack, and they both take a lot of convincing! Use pictures as well as numbers in your explanation.

Endnotes

- 1 Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. London: Routledge.
- 2 'Mathematical argumentation' refers to the process of resolving mathematical disagreement by examining the premises of the different positions to establish which outcome is correct. There are constructive ways of doing this, and students need teacher guidance and modelling.
- 3 Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. London: Routledge.
- 4 The case explains how the teachers used the Mathematics Communication and Participation Framework to scaffold the development of mathematical communities of inquiry amongst the students.
- 5 Mullis, I., Martin, M. O., & Foy, P. (2007). Findings from IEA's trends in mathematics and science study at the fourth and eighth grades. In collaboration with J. F. Olson, C. Preschoff, E. Erberber, A. Arora, & J. Galia (Eds.), *Index of students' perception of being safe in school (SPBSS) and trends*, (p. 367). United States: TIMSS & PIRLS International Study Center.
- 6 This is not the school's actual name.
- 7 This section draws from *The New Zealand curriculum* (Ministry of Education (2007). *The New Zealand curriculum*. Wellington: Learning Media.) and, in some cases, the parallel document, *Te Marautanga o Aotearoa* (Ministry of Education (2008). *Te marautanga o Aotearoa*. Wellington: Learning Media). It includes quotes and paraphrases from pages 8, 9, 10, and 30 of *The New Zealand curriculum*.
- 8 An 'inscription' is a tool or artefact that symbolises an idea and helps organise mathematical thinking. Examples include graphs, diagrams, and the number system itself. See pages 127–135 of the *Effective pedagogy in mathematics/pāngarau BES* for further explanation.
- 9 The students were attempting to solve the problem presented in Case 1, Appendix 4, page 15.
- 10 The koosh balls were soft balls that did not roll but fitted into the students' hands.
- 11 A 'solution strategy' is a strategy to solve a specific problem.
- 12 Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. London: Routledge.
- 13 Hunter, R. (2007). The Mathematics Communication and Participation Framework: An outline of the communicative and participatory actions teachers facilitate students to engage in to scaffold the use of reasoned collective discourse. This framework was informed by the work of: Wood, T., & McNeal, B. (2003). Complexity in teaching and children's mathematical thinking. In N. L. Pateman, B. J. Dougherty, & J. Zilliox (Eds.). *Proceedings of the 27th Annual Conference of the International Group for the Psychology of Mathematics Education, Vol. 4*, pp. 435–443. Honolulu, HI: ME.