



MINISTRY OF EDUCATION

Te Tāhuhu o te Mātauranga

Effective Pedagogy in Mathematics/Pāngarau

Best Evidence Synthesis Iteration [BES]

Glenda Anthony and Margaret Walshaw, Massey University



ITERATIVE BEST EVIDENCE SYNTHESIS PROGRAMME
<http://educationcounts.edcentre.govt.nz/goto/BES>
New Zealand

This report is one of a series of best evidence synthesis iterations (BESs) commissioned by the Ministry of Education. The Iterative Best Evidence Synthesis Programme is seeking to support collaborative knowledge building and use across policy, research and practice in education. BES draws together bodies of research evidence to explain what works and why to improve education outcomes, and to make a bigger difference for the education of all our children and young people.

Each BES is part of an iterative process that anticipates future research and development informing educational practice. This BES follows on from other BESs focused on quality teaching for diverse learners in early childhood education and schools. Its use will be informed by other BESs, focused on teacher professional learning and development and educational leadership. These documents will progressively become available at: <http://educationcounts.edcentre.govt.nz/goto/BES>

Feedback is welcome at best.evidence@minedu.govt.nz

Note: the references printed in purple refer to a list of URLs in Appendix 2. These are a selection of potentially useful sources for teachers to engage more deeply with the range of issues raised in this best evidence synthesis iteration.

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About the writers

Glenda Anthony and Margaret Walshaw, both from the School of Curriculum and Pedagogy at Massey University, bring to this Best Evidence Synthesis (BES) decades of mathematics classroom teaching and educational research experience. They are acutely aware of the challenge that educators face in constructing a democratic mathematical community with which all students can identify. For them, making a positive difference to diverse learners' outcomes is a central educational issue. At the heart of their work is a concerted effort to illuminate how this issue is best addressed. In this synthesis, they report on the outcome of their deliberations over, and search for, what makes a difference for diverse learners in mathematics/pāngarau.

Advisory Group

A core Advisory Group membership was selected to provide expertise and critique in relation to the various focuses of the BES, including Māori and Pasifika learners, early childhood, primary and secondary sectors, and teacher education. The authors wish to thank the members of this group:

- Dr Ian Christensen (Massey University and He Kupenga Hao i te Reo)
- Dr Joanna Higgins (Victoria University of Wellington)
- Roberta Hunter (Massey University)
- Garry Nathan (Auckland University)
- Dr Sally Peters (Waikato University)
- Assoc. Prof. Jenny Young-Loveridge (Waikato University)

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Ministry of Education advisory team

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External quality assurance

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Forewords

International

Even the casual visitor is struck by the dramatic changes that have occurred in New Zealand in the last 15 years. I have tuned in to local media on each of my four visits to get an initial sense of people's current concerns and issues. Based on this narrow sampling, the New Zealand of 1991 was an immensely likeable country that had seen better days and was struggling to find its place in a rapidly changing world. Although innovation and experimentation appeared to be the watchwords of the day, there seemed to be an undercurrent of apprehension and anxiety as people attempted to cope with economic disruption. Today, New Zealand continues to be an immensely likeable place, but the visitor immediately notices a quiet, understated self-assurance. It has become a largely prosperous country that, in a very real sense, has reinvented itself as a leading information economy in an increasingly globalised world. Refreshingly for the visitor from the United States, there appears to be widespread belief that government will approach problems pragmatically and is capable of solving them. If the Iterative Best Evidence Synthesis Programme is representative of New Zealand government in action, this belief would appear to be well founded.

Put quite simply, the Iterative BES Programme is the most ambitious effort I have encountered that uses rigorous scientific evidence to guide the ongoing improvement of an education system at a national level. The programme has a strong pragmatic bent and is clearly grounded in the hard-won experience of synthesising research findings to inform both policy and teachers' instructional practices. Four aspects of the programme are particularly noteworthy. The first is the overriding commitment to make the development of the best evidence syntheses transparent. This commitment takes concrete form in the exacting evaluation and feedback process that all BES reports undergo at each phase of their development, from the initial identification of relevant bodies of research literature through to the final critique and revision of the report. This is in the best traditions of science, where claims are justified in terms of the means by which they have been produced.

The second notable characteristic is a mature view of evidence and an emphasis on methodological and theoretical pluralism. This is important, given that attempts have been made in a number of countries, including the United States, to legislate what counts as scientific research in education on the basis of ideological adherence to a particular methodology. In taking an inclusive approach, the Iterative BES Programme acknowledges that different types of knowledge are of greatest use to teachers and to policymakers. Teachers make pedagogical decisions on the basis of a detailed understanding of specific students in particular classrooms at particular points in time. Policymakers, in contrast, typically need knowledge of trends and patterns that hold up across classrooms to make decisions that affect large numbers of students and teachers in multiple schools. Different methodologies are appropriate for developing these equally important types of knowledge.

The third noteworthy characteristic of the Iterative BES Programme is its focus on the explanatory power and coherence of theories. Priority is given to theories that give insight into learning processes and the specific means of supporting their realisation in classrooms. This pragmatic criterion is important in a field where theoretical perspectives continue to proliferate.

The final notable characteristic of the programme is its explicit attention to the issues of language and culture. This emphasis is clearly critical if New Zealand teachers and policymakers are to address the inequities inherent in the disturbingly large gaps in school achievement between children of different ethnic and racial groups. In keeping with the tenet of methodological and theoretical pluralism, the Iterative BES Programme uses group categories such as socioeconomic status, ethnicity, and culture as key variables in *assessing* efforts to achieve

equity. However, it avoids stereotyping children of particular racial, ethnic, or language groups by acknowledging the complexity of individual identity when *explaining* inequities in children's learning opportunities. Furthermore, the programme emphasises ecological models of learning that link what is happening in classrooms both to the institutional contexts in which classrooms are located and to issues of race, culture, and language. It is here that the full ambition of the programme becomes apparent: few viable models of this type currently exist in education. The BES writers are therefore charged with the task of synthesising in the true sense of the term, that is, to combine disparate and sometimes fragmented bodies of research into a single, unified whole. At the risk of understatement, this is a formidable challenge.

The writers of this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau, Drs. Glenda Anthony and Margaret Walshaw, have risen to the challenge. They were charged with the daunting task of reviewing, organising, and synthesising all mathematics education research from the early childhood years through secondary school that relates classroom processes to student learning. On my reading, the resulting synthesis of over 600 research studies is directly relevant to teachers and will be educative for policymakers. The educative value of the report stems from Anthony and Walshaw's focus on what goes on in mathematics classrooms, thereby providing a window on the complexity of effective pedagogy. The forms of pedagogical practice that they identify as effective are ambitious because they involve high expectations for *all* children's mathematical learning. The goals at which these forms of pedagogy aim are best illustrated in chapter 7, A Fraction of the Answer, in which Anthony and Walshaw pull together the key insights of the preceding chapters as they present an integrated series of cases that focus on the learning and teaching of fractions. As this chapter makes clear, the instructional goals for fractions are not limited to ensuring that children can add, subtract, multiply, and divide fractions successfully. Instead, the instructional objectives also focus on children's development of a deep understanding of fractions as amounts or quantities. At an elementary level, children who are coming to understand fractions as quantities know that $\frac{1}{6}$ is smaller than $\frac{1}{5}$ because there will be more pieces when something is divided into 6 pieces than into 5 pieces, so the pieces must be smaller. At a more advanced level, students will be able to describe real world situations that involve multiplying and dividing fractional quantities. More generally, ambitious pedagogy focuses on central mathematical ideas and principles that give meaning to computational methods and strategies.

Anthony and Walshaw's review of the relevant research indicates that central mathematical ideas and principles cannot be directly transmitted to children. However, the research also shows that discovery approaches that place children in rich environments and simply encourage them to inquire are also ineffective. Effective pedagogy is complex because it requires teachers to achieve a significant mathematical agenda by taking children's current knowledge and interests as the starting point. As Anthony and Walshaw clarify, these forms of pedagogy involve a distinctive orientation towards teaching. First and foremost, the emphasis is on building on students' existing proficiencies rather than filling gaps in students' knowledge and remediating weaknesses. As a consequence, the teacher's focus when planning for instruction is not on students' limitations but on their current mathematical competencies and interests, as these constitute resources on which the teacher can build. More generally, effective mathematical pedagogy places students' reasoning at the center of instructional decision making. As a consequence, the ongoing assessment of students' reasoning is an integral aspect of instruction, not a separate activity conducted after the fact to check whether goals for students' learning have been achieved. A key characteristic of accomplished teachers is that they continually adjust instruction, as informed by these ongoing assessments.

One of the strengths of Anthony and Walshaw's synthesis is that it provides the reader with a concrete image of what effective mathematical pedagogy looks like. Anthony and Walshaw emphasise that a respectful, non-threatening classroom atmosphere in which all students feel comfortable in making contributions is necessary but not, by itself, sufficient. As they document, the research findings indicate unequivocally that it is also essential that classroom activity

and discourse focus explicitly on central mathematical ideas and processes. The selection of instructional tasks is therefore critical. On the one hand, it is important that task contexts or scenarios are accessible to all students, regardless of cultural background. On the other, the teacher should be able to capitalise on students' solutions to support their development of increasingly sophisticated forms of mathematical reasoning. Thus, when designing and selecting tasks, the teacher has to take account both of students' current competencies and interests and their long-term learning goals. As Anthony and Walshaw discuss in chapter 5, an important way in which the teacher can build students' solutions is by introducing judiciously chosen tools and representations. A second, equally important way in which the teacher can capitalise on the potential of worthwhile mathematical tasks is to engage students in justification, abstraction, and generalisation (see chapter 4), by doing which they learn to speak the language of mathematics.

The image of effective mathematical pedagogy that emerges from Anthony and Walshaw's synthesis is of teaching as a coherent system rather than a set of discrete, interchangeable strategies. This pedagogical system encompasses:

- a non-threatening classroom atmosphere;
- instructional tasks;
- tools and representations;
- classroom discourse.

To see that these four aspects of effective pedagogy constitute a system, note that the way in which instructional tasks are realised in the classroom and experienced by students depends on the classroom atmosphere, the tools and representations available for them to use, and the nature and focus of classroom discourse. And because effective pedagogy is a system, it makes little sense to think of student learning as being *caused* by isolated teacher actions or strategies. It is for this reason that Anthony and Walshaw speak of mathematical learning being *occasioned* by teaching. In using this term, Anthony and Walshaw emphasise the teacher's proactive role in supporting students' development of increasingly sophisticated forms of mathematical reasoning.

In addition to highlighting the systemic character of effective mathematical pedagogy, Anthony and Walshaw make good on the charge to develop an ecological model of learning that links what is happening in the classroom to issues of race, culture, and language, and to the school contexts in which teachers develop and revise their instructional practices. A concern for issues of equity permeate the entire report but come to the fore in the discussion of school-home partnerships that take the diverse cultures of students and their families seriously and treat them as instructional resources.

Anthony and Walshaw make it clear that it is essential to view school contexts as settings for teachers' ongoing learning. In a very real sense, these settings mediate the extent to which high quality teacher professional development will result in significant changes in teachers' classroom practices. Anthony and Walshaw's synthesis documents that mathematics instruction that places students' reasoning at the center of instructional decision making is demanding, uncertain, and not reducible to predictable routines. The available evidence indicates that a strong network of colleagues constitutes a crucial means of support for teachers as they attempt to cope with these uncertainties and the loss of established routines. Consequently, there is every reason to expect that improvement in teachers' instructional practices and student learning will be greater in schools where mathematics teachers participate in learning communities whose activities focus on central mathematical ideas and how to relate them to student reasoning. The value of teacher learning communities in turn foregrounds the critical role of the principal as an instructional leader.

Historically, teaching and school leadership have been loosely coupled, with the classroom being treated as the preserve of the teacher while school leaders managed around instruction. Recent research findings demonstrate the limitations of this type of school organisation

in supporting the improvement of teaching on any scale. These findings also indicate that principals can play a key role in supporting the emergence of a shared vision of what effective mathematical pedagogy looks like and in supporting teacher collaboration that focuses on challenges central to the development of effective pedagogy. This alternative type of school organisation is characterised by reciprocal accountability. Teachers are accountable to principals for developing increasingly effective pedagogical practices and principals are accountable to teachers to create opportunities for their ongoing learning. Changes of this type in the relations between teachers and school administrators are far reaching and might be viewed as too radical. It is, however, sobering to note that previous large-scale efforts to improve the quality of classroom instruction have rarely produced lasting changes in teachers' practices. Research into educational leadership and policy indicates that this history is due in large part to the failure to take into account the institutional settings in which teachers develop and refine their instructional practices.

The broader policy and leadership literature strongly indicates that the improvement of mathematics instruction on the scale being attempted in New Zealand is not simply a matter of providing high quality teacher professional development. It also has to be framed as a problem for schools as educational organisations that structure the institutional settings in which teachers develop and revise their instructional practices. My reading of this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau is that Anthony and Walshaw have distilled valuable lessons from the available research, thereby positioning New Zealand educators to succeed where others have failed.

Paul Cobb

Professor of Mathematics Education
Vanderbilt University, Tennessee

Note: The second Hans Freudenthal Medal of the International Commission on Mathematical Instruction (ICMI) was awarded to Professor Paul Cobb in 2005, "whose work is a rare combination of theoretical developments, empirical research and practical applications. His work has had a major influence on the mathematics education community and beyond."

Early Childhood Education

This Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau is a 'must read' for those in the early childhood sector who want an insight into what effective mathematical pedagogy looks like in an early childhood service. The synthesis acknowledges the vital role that quality early childhood education plays in the mathematical development of infants and young children. It also provokes early childhood teachers to reflect on practice: their mathematical awareness of the environment, the depth of their mathematical knowledge, and the importance of effective teaching and learning strategies that will support children's optimal engagement in mathematical experiences. The extensive, wide-ranging research information is effectively balanced by vignettes which involve the reader in meaningful mathematical experiences that illustrate the possibilities for supporting mathematical learning. Effective distribution of the synthesis would enhance teaching and learning outcomes in early childhood services.

Faith Martin
Director, Massey Child Care Centre

NZEI Te Riu Roa

NZEI Te Riu Roa welcomes this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau, particularly as it takes for its starting point the assertion that “all children can learn mathematics”. This key message is at the heart of every teacher’s commitment to the mathematical learning of his or her students.

The synthesis recognises the complexity of teaching, particularly given the diverse learning needs of the students in our classrooms and centres and the necessity for specialised knowledge of mathematics. But the writers consistently underline the power that teachers have to make a difference: “It is what teachers do, think and believe (that) significantly influences student outcomes.”

A teacher’s role, whether in a school or a centre, includes the design of activities that help students to construct meaning and think for themselves. To achieve such outcomes, teachers need to appreciate the part that mathematics plays in the world around them, what the big mathematical ideas are, and how the concepts that they teach fit in with those ideas. They need to know how to teach knowledge and skills, how to match new learning with students’ prior knowledge, and which activities effectively encourage understanding and learning. Teachers also need to be conscious of developing attitudes and values. They need to create opportunities for their students to develop a critical eye and, in the context of this synthesis, a critical mathematical eye.

The primary purpose of the synthesis is to identify evidence that links pedagogical practice with effective mathematics outcomes for students. To achieve this, the writers have drawn on national and international research that contributes to our understanding of what works in mathematics education.

When reviewing the synthesis in its draft form, NZEI teachers were particularly pleased to read the chapter, *Mathematics Practices Outside the Classroom*, which they saw as contributing to a constructive environment and encouraging of good practice. The synthesis explores ways in which parents can contribute to their children’s mathematical development and ways in which schools can strengthen links with the home. If teachers are to successfully fulfil expectations, such links are likely to be vital. Teachers were also pleased to see the importance of school leadership recognised.

NZEI sees the *Effective Pedagogy in Mathematics/Pāngarau BES* as being of great benefit to teachers, teacher educators, and policymakers. The research identified in the synthesis, together with the case studies and vignettes, has the potential to stimulate much constructive professional discussion. To maximise its potential for teachers, it will need to be accompanied by professional learning opportunities and time for reflection and discussion in the school or centre setting.

Irene Cooper
National President
Te Manukura
NZEI Te Riu Roa

Post Primary Teachers' Association

Tēnā koutou, tēnā koutou, tēnā tatou katoa.

PPTA welcomes this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau. It is the result of a very thorough process, inclusive of the expertise of practitioners. The final report reflects and caters to their realities, and provides some very interesting and thought-provoking reading for teachers themselves, and for those involved in the pre-service and in-service education of mathematics teachers. At the same time, the research highlights the shortage of outcomes-linked research evidence specific to secondary school mathematics teaching and we hope that as a result of this BES, New Zealand researchers will step up to fill this gap.

Debbie Te Whaiti
President
New Zealand Post Primary Teachers' Association

Teacher Educators

The Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau succeeds in providing a systematic treatment of relevant outcomes-based evidence for what works for diverse learners in the New Zealand education system. One of the strengths of the document is the central positioning given by its authors to a broad notion of diversity.

Teacher educators, both initial and ongoing, will find that the BES is an invitation to engage—as teachers and as researchers—with a wide range of national and international studies. The document succeeds in preserving the complexity of pedagogical approaches through careful structuring and presentation. Well chosen classroom vignettes capture the essence of pedagogical issues for use in initial and ongoing teacher education. The CASEs are likely to prove particularly valuable for teachers by demonstrating how research can inform classroom practice.

The BES also presents a challenge to New Zealand researchers by identifying areas in which there is a paucity of outcomes-based evidence. Such evidence is scarce for Māori-medium mathematics classrooms. The senior secondary area is generally not well represented and a wider range of early childhood contexts needs to be investigated. The CASEs highlight for teacher educators the possibilities of writing up research projects undertaken as part of ongoing teacher education initiatives, and encourage them to gather further evidence to support practice.

The importance to mathematics education of the outcomes-based research evidence represented in this synthesis cannot be overstated. It is to be hoped that the value of the Iterative BES programme is widely recognised, and that it has the impact on policy and practice that it ought.

Joanna Higgins
Director, Mathematics Education Unit and Associate Director,
Jessie Hetherington Centre for Educational Research
Victoria University of Wellington

Māori-medium Mathematics

E nga mana, e nga reo, tēnā koutou katoa.

For the last 20 years, the teaching of pāngarau (mathematics) has played a significant role in the revitalisation of te reo Māori. The Effective Pedagogy in Mathematics/Pāngarau BES recognises the close relationship that exists between language and the learning and teaching of mathematics.

The BES identifies a range of major considerations and challenges for teachers and all those involved in Māori-medium education. The research makes it clear that mathematical outcomes for students are affected by a complex network of interrelated factors and environments, not just individual preferences or the language of instruction. By identifying the key elements in this network and discussing the relevant research, the writers have created what should prove a very useful resource.

The BES highlights the paucity of research into Māori-medium mathematics education, particularly in the area of teacher practice.

Tony Trinick
Māori-medium mathematics educator
Faculty of Education
The University of Auckland

Pasifika

E rima te'arapaki, te aro'a, te ko'uko'u te utuutu, 'iaku nei.

Under the protection of caring hands there's feeling of love and affection.

The Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau has drawn together a comprehensive synthesis of evidence that relates to quality mathematics pedagogical practices. Its particular strength is that it provides stimulating and thought-provoking reading for a range of stakeholders and at the same time affirms that there is no one, specific, 'quality' pedagogical approach. Rather, it directs attention to many effective approaches which make a difference for all mathematics learners. The vignettes are an added strength; they make the theoretical structures they illustrate accessible to a wider audience.

The synthesis highlights the shortage of outcomes-linked research evidence concerning quality teaching and learning for Pasifika students at all levels of schooling. It also highlights the importance of a culture of care. How this translates into quality outcomes for Pasifika students requires the attention of New Zealand researchers.

Roberta Hunter
Senior Lecturer
School of Education Studies
Massey University, Albany Campus

Chief Education Adviser, BES

The Effective Pedagogy in Mathematics/Pāngarau BES sets out to uncover and explain the links between what we do in mathematics education and what the outcomes are for learners. The result is a valuable resource that can be used to enhance a wide range of outcomes for diverse learners. These include the ability to think creatively, critically, strategically and logically; mathematical knowledge; enjoyment of intellectual challenge; self-regulatory, collaborative and problem-solving skills; and the disposition to use, enjoy and build upon that knowledge throughout life.

The BES reflects the outstanding scholarly work and professional leadership of co-authors Drs Glenda Anthony and Margaret Walshaw of Massey University. They are the first to use the new *Guidelines for Generating a Best Evidence Synthesis* and follow the collaborative development process that is central to the Iterative BES Programme. They have consulted tirelessly and responsively with a wide range of early childhood teachers, primary and secondary teachers, principals, advisers, researchers, policy workers and teacher educator colleagues from across New Zealand, and with international colleagues. The Ministry of Education acknowledges and values all these contributions—and those of the formative quality assurers, whose affirmations and challenges have been so helpful in optimising the quality and potential usefulness of this BES.

The BES celebrates and returns to early childhood educators, teachers, teacher educators and researchers a record of their professional work, highlighting the complexity of that work, and suggesting how research evidence can be a valuable resource to inform their ongoing work and that of their colleagues. From the first vignette explaining how mathematical learning can be embedded in waiata (Māori song) and dance, the vignettes bring children's learning in mathematics to life. The underlying explanations and theoretical findings have the power to inform practice in ways that are relevant and responsive to the learners in any particular centre or classroom.

The challenge now is for us all is to use this resource in ways that will support further systemic development in mathematics education, with strengthened outcomes for diverse learners. In many cases, the BES will affirm what is already happening, but it will be the points of challenge that take us forward. Individual teachers have already engaged with the BES in its draft form, and some report remarkable insights and developments in their practice. But it is only through the wider and systemic development of the conditions that support effective practice for diverse learners that improvements will proliferate and become self-sustaining. The findings emerging from the outcomes-linked professional learning and development BESs^{1,2} should be an invaluable resource in determining how to generate changed practice on such a scale.

Many teachers and early childhood educators have indicated that they want to read this BES for themselves, and to do this they need time. They need time to read, discuss and consider how they can use relevant BES findings in response to diagnostic information about the mathematical understandings of the children and young people they teach. They also need time to participate in professional learning communities. The Teacher Professional Learning and Development BES³ finds that such participation doesn't guarantee better outcomes for students, but it is a consistent feature of teacher professional learning that does have a strong positive impact.

The same BES highlights the important role that external expertise with strong pedagogical content knowledge can play in facilitating and supporting changes in practice that impacts positively on student outcomes. Such expertise can be vital in engaging teachers' theories and challenging problematic discourses. The findings do, however, caution that 'experts' need more than good intentions—in the worst-case scenario, teacher professional development can actually impact negatively on student achievement. This finding calls for careful and iterative evaluation of the effectiveness of all professional development.

The teacher education community in New Zealand has already made a foundational contribution to this BES with its engagement in the research and development reported in this BES, and its advice to the BES writers. As the Teacher Professional Learning and Development BES⁴ will show, some of our most effective professional development has been taking place as part of the Numeracy Development Projects (NDP)—with effect sizes twice those attained in England^{5,6,7}. The primary and early childhood teachers’ union, NZEI, confirms what the evaluation reports have been saying: that teachers who have been involved in the NDP value the transformational experiences this professional learning has afforded them. Two teachers from a Hawkes Bay school explained to me recently that, as a result of professional learning undertaken through the NDP, they have changed the way they work across the curriculum—they now listen more, are more diagnostic, and they place much more emphasis on children articulating and sharing their learning strategies. The dynamic, reflective, nation-wide learning community of researchers, teacher educators, teachers, and other educators created by the NDP and its Māori-medium counterpart, Te Poutama Tau, has been inspirational for BES.

If the mathematics BES is to serve New Zealand education well, the teacher education and research communities must make it a ‘living’ BES by building on the powerful insights and exemplars it makes available, addressing the gaps, and ensuring a cumulative and increasingly dynamic shared knowledge base about what works for learners in New Zealand education. To assist in this collaborative work, the New Zealand Council for Educational Research is creating a database of relevant New Zealand education theses. It has already built a database to support this document, with live links to the electronic version so that readers can quickly access either the full text or bibliographic details for some of the most helpful articles that have informed the synthesis. These links are also listed in the print version.

It is our hope that this BES will stimulate readers to let the Iterative Best Evidence Synthesis Programme know of other/new research and development that should feature in future iterations of the synthesis. Such research needs to clearly document demonstrated or triangulated links to student outcomes (see the Guidelines for Generating a Best Evidence Synthesis Iteration, found on the BES website⁸), and preferably show larger positive impacts on desired outcomes for diverse learners. We are especially seeking studies of research and development in New Zealand contexts, but we are also interested in information on overseas studies that show particularly large impacts on diverse learners. Please send details to best.evidence@minedu.govt.nz.

In the New Zealand context, where schools and centres are self managing, principals and centre leaders have a critical role to play in supporting their staff to realise the potential of this BES. The Teacher Professional Learning and Development BES indicates that, in the case of the most effective school-based interventions, principals and others in leadership roles have actively supported the development of a learning culture amongst their teachers.

For centuries, societies have required their education systems to sort children into successes and failures. Knowledge societies, such as our own, require much more. Our challenge is to ensure that all our children flourish as learners, strong in their own identities, and confident global citizens.

To achieve such goals, we need to value, build upon, and go beyond the craft practice traditions that require each teacher to ‘rediscover the wheel’. The Effective Pedagogy in Mathematics/Pāngarau BES has been designed to serve as a resource and catalyst for strengthened practice, innovation, and systemic learning. By using it, and by making learner outcomes our touchstone, we can work together to give our children a mathematics education that prepares them well for the opportunities and challenges that will be their future.

Adrienne Alton-Lee
Chief Education Adviser
Iterative Best Evidence Synthesis Programme
New Zealand Ministry of Education

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- ² Mitchell, L., & Cubey, P. (2004). *Characteristics of Professional Development Linked to Enhanced Pedagogy and Children's Learning in Early Childhood Settings: Best Evidence Synthesis Iteration*. Wellington, NZ: Ministry of Education. educationcounts.edcentre.govt.nz/goto/BES
- ³ Ibid.
- ⁴ Timperley et al., to be published 2007.
- ⁵ Young-Loveridge, J. (2006). Patterns of Performance and progress on the Numeracy Development Project: Looking back from 2005. In *Findings from the Numeracy Development Projects 2005* (F. Ell, J. Higgins, K.C. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge). Wellington, NZ: Ministry of Education.
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- ⁷ Earl, L., Watson, N., Levin, B., Leithwood, K., Fullan, M., Torrance, N., Jantzi, D., Mascal, B., & Volante, L. (2003). *Watching and learning 3: Final report of the external evaluation of England's National Numeracy and Literacy Strategies*. London, UK: DfES
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Authors' Preface

What is a Best Evidence Synthesis in Mathematics?

A best evidence synthesis draws together available evidence about what pedagogical approaches work to improve student outcomes in Mathematics/Pāngarau. This synthesis is part of the Iterative Best Evidence Synthesis (BES) Programme established late in 2003 by the Ministry of Education to deepen understanding of what works in education. The programme involves policy, research, and practice in collaborative knowledge building, aimed at maximising desirable outcomes for the diverse learners in the New Zealand education system.

This best evidence synthesis in Mathematics/Pāngarau plays a key role in knowledge building for New Zealand education. As a capability tool, it identifies, evaluates, analyses, and synthesises what the New Zealand evidence and international research tell us about quality mathematics teaching. It shows us how different contexts, systems, policies, resources, approaches, practices, and influences all impact on learners in different ways. Importantly, it illuminates what the evidence suggests can optimise outcomes for diverse mathematics learners.

The importance of dialogue

The development of this BES has been shaped by the *Guidelines for Generating a Best Evidence Synthesis Iteration* (Alton-Lee, 2004) and informed by dialogue amongst policy makers, educators, researchers and practitioners. Right from the very early stages of its development, the health-of-the-system perspective taken in this synthesis has ensured that we have listened to and responded to the viewpoints of a wide range of constituencies. Our interactions with these multiple communities have revealed to us the key roles that infrastructure, context, settings, and accountabilities play in a system that is functioning effectively for all its learners. Our various stakeholders have challenged us not only to produce better and more relevant educational research but to consider how this knowledge base might best be used. It is our hope that this discussion across sectors will be ongoing.

We have received a strong and positive response to the best evidence synthesis work from New Zealand's primary and post-primary teacher associations. Both have reported on how helpful the synthesis is to their core professional work. For example, the New Zealand Educational Institute (NZEI) writes: "In our view, the writers have drawn on national and international research which contributes to an understanding of what works in mathematics education; they have identified the significance of the context and ways in which to strengthen practice ... We liked the ... underpinning view that all children can learn mathematics" (p. 2). The representative for the Post Primary Teachers' Association at the Quality Assurance Day is reported as saying: "There are numerous wonderful ideas in the synthesis, and I found myself repeatedly jolted into possibilities for my own classroom resources." In addition, a group of initial and ongoing mathematics teacher educators have welcomed the "sophisticated treatment of diversity" and the way in which "the complexity of pedagogical approaches is preserved" (Victoria University of Wellington College of Education, 2006, p. 1).

Writing for multiple audiences

Our task was to make the findings of the synthesis accessible to and of benefit to a range of educational stakeholders. At one level of application, it is intended to provide a strengthened basis of knowledge about mathematics pedagogical practices in New Zealand today. The evidence it produces is expected to inform teacher educators within the discipline of mathematics education about effective pedagogical practice. At another level, the synthesis attempts to make transparent to policy makers and social planners an evidential basis for quality pedagogical approaches in mathematics. At a third level, the synthesis is expected to benefit practitioners and assist them in doing the best possible job for diverse learners in their classrooms.

Our approach to the “almost overwhelming task” (Cobb, 2006) of writing with several levels of application in mind has been to draw on both formal and informal approaches. We have offset the ‘academic’ language of the BES by including a series of vignettes that expand upon broad findings. We have received feedback from a range of sources that these vignettes bring the reality of classroom life to the fore and, in particular, do not minimise the complexities of actual practice. We hope that researchers, policy makers and practitioners alike will see in the vignettes theoretical tools that have been adapted and used by actual teachers.

The BES as a catalyst for change

This best evidence synthesis in mathematics does more than synthesise and explain evidence about what works for diverse learners. By bringing together rigorous and useful bodies of evidence about what works in mathematics, the project plays an important function as a catalyst for change. It is designed to help strengthen education policy and educational development in ways that effectively address both the needs of diverse learners and patterns of systemic underachievement in New Zealand education. It is written with the intent of stimulating activity across practitioners, policy makers, and researchers and so to strengthen system responsiveness to educational outcomes for all students.

The writers anticipate that reflection on the findings will lead to sustainable educational development that has a positive impact on learners. It will create new insights into what makes a difference for our children and young people. Reflection on the findings will also spark new questions and renewed, fruitful engagement with mathematics education. These new questions, in turn, will render the BES a snapshot in time—provisional and subject to future change.

Key features

Key features of the BES are:

- Its teacher orientation. Its view is towards a strengthened basis of knowledge about instructional practices that make a difference for diverse groups of learners.
- Its cross-sectoral approach. Its scope takes in the teaching of children in early childhood centres through to the teaching of learners in senior secondary school classrooms.
- Its inclusiveness. It documents research that reveals significant educational benefits for a wide range of diverse learners. It pays particular attention to the mathematical development of Māori and Pasifika students and documents research that captures the multiple identities held by New Zealand learners.
- Its breadth of search coverage. It reports on the characteristics of effective pedagogy, following searches through multiple national databases and inventories as well as masters’ projects and theses. It provides comprehensive information about effective teaching as evidenced from small cases, large-scale explorations, and short-term and longitudinal investigations.
- Its local character. It makes explicit links between claims and bodies of evidence that have successfully translated the intentions and spirit of the Treaty of Waitangi. It identifies research relevant to the particular conditions and contexts in New Zealand, both in mathematics education in particular and in education in general, in relation to the principles and goals of *Te Whāriki* for early childhood settings and of *The Curriculum Framework*, for teachers in English or Māori-medium settings.
- Its global linkages. It connects local sources with the international literature. It identifies important Australian and international work in the area and evaluates that wide-ranging resource in relation to similarities and differences in cultures,

populations and demographics between the country of origin and New Zealand.

- Its responsiveness to concerns about democratic participation. It heeds the concern about the development of competencies that equip students for lifelong learning. This orientation coincides with the national mathematics curriculum objective of developing those knowledges, skills, and identities that will enable students to meet and respond creatively to real-life challenges.
- Its quality assurance measures. It is guided by principles of transparency, accessibility, relevance, trustworthiness, rigour, and comprehensiveness. These principles form the backdrop to the selection and systematic integration of evidence.
- Its strategic focus on policy and social planning. It uses a health-of-the-system approach to address one of the most pressing problems in education, provide a direction for future growth, and push effective teaching beyond current understandings.
- Its provisional nature. The project is an important knowledge-building tool, creating new insights from what has gone before, and will be updated in the light of findings from new studies. The findings are, above all, 'of the moment' and open to future change.

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Executive Summary

The *Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration [BES]* was funded by a Ministry of Education contract awarded to Associate Professor Glenda Anthony and Dr Margaret Walshaw at Massey University. The synthesis is part of the Iterative Best Evidence Synthesis (BES) Programme, established by the Ministry of Education in New Zealand, to deepen understanding from the research literature of what is effective in education for diverse learners. The synthesis represents a systematic and credible evidence base about quality teaching in mathematics and explains the sort of pedagogical approaches that lead to improved engagement and desirable outcomes for learners from diverse social groups. It marks out the complexity of teaching and provides insight into the ways in which learners' mathematical identities and accomplishments are occasioned by effective pedagogical practices.

The search of the literature focused attention on different contexts, different communities, and multiple ways of thinking and working. Priority was given to New Zealand research into mathematics in early childhood centres and schools, both English- and Māori-medium. Personal networks enhanced the library search and facilitated access to academic journals, theses and reports, as well as other local scholarly work. The New Zealand literature was complemented by reputable work undertaken in other countries with similar population and demographic characteristics. Indices, both print and electronic, were sourced, and the search covered relevant publications within the general education literature as well as specialist educational areas. In the end, 660 pieces of research, ranging from very small, single-site studies to large scale, longitudinal, experimental studies, found their way into the report.

Key findings highlight practices that relate specifically to effective mathematics teaching and to positive learning and social outcomes in centres/kōhanga and schools/kura. The findings stress the importance of interrelationships and the development of community in the classroom. They also reveal that both the cognitive and material decisions made by teachers concerning the mathematics tasks and activities they use, significantly influence learning. The findings demonstrate the importance of children's early mathematical experiences and stress that constituting and developing children's mathematical identities is a joint enterprise of teacher, centre/school, and family/whānau.

Key findings

In this section, key findings are organised and presented according to five themes: the key principles underpinning effective mathematics teaching, the early years, the classroom community, the pedagogical task and activity, and educational leadership and centre–home and school–home links.

Key principles underpinning effective mathematics teaching

Teachers who enhance positive social and academic outcomes for their diverse students are committed to teaching that takes students' mathematical thinking seriously. Their commitment to students' thinking is underpinned by the following principles:

- an acknowledgement that all students, irrespective of age, have the capacity to become powerful mathematical learners;
- a commitment to maximise access to mathematics;
- empowerment of all to develop mathematical identities and knowledge;
- holistic development for productive citizenship through mathematics;
- relationships and the connectedness of both people and ideas;
- interpersonal respect and sensitivity;
- fairness and consistency.

The early years

Young children are powerful mathematics learners. Quality teaching guarantees the development of appropriate relationships and support as well as an awareness of children's mathematical understanding. Research has consistently demonstrated how a wide range of children's everyday activities, play and interests can be used to engage, challenge and extend children's mathematical knowledge and skills. Researchers have found that effective teachers provide opportunities for children to explore mathematics through a range of imaginative and real-world learning contexts. Contexts that are rich in perceptual and social experiences support the development of problem-solving and creative-thinking skills.

There is now strong evidence that the most effective settings for young learners provide a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive, play activities. Opportunities for learning mathematics typically arise out of children's everyday activities: counting, playing with mathematical shapes, telling time, estimating distance, sharing, cooking, and playing games. Teachers in early childhood settings need a sound understanding of mathematics to effectively capture the learning opportunities within the child's environment and make available a range of appropriate resources and purposeful and challenging activities. Using this knowledge, effective teachers provide scaffolding that extends the child's mathematical thinking while simultaneously valuing the child's contribution.

The classroom community

Research has shown that opportunities to learn depend significantly on the community that is developed within centres and classrooms. Thus, people, relationships, and classroom environments are critically important. Whilst all teachers care about student engagement, research quite clearly demonstrates that pedagogy that is focused solely on the development of a trusting climate does not get to the heart of what mathematics teaching truly entails. Teachers who truly care about their students have high yet realistic expectations about enhancing students' capacity to think, reason, communicate and reflect upon their own and others' understandings. Research has provided conclusive evidence that effective teachers work at developing inclusive partnerships, ensure that the ideas put forward by learners are received with respect and, in time, become commensurate with mathematical convention and curricular goals.

Studies have provided conclusive evidence that teaching that is effective is able to bridge students' intuitive understandings and the mathematical understandings sanctioned by the world at large. Language plays a central role. Mathematical language involves more than vocabulary and technical usage; it encompasses the ways that expert and novice mathematicians use language to explain and to justify concepts. The teacher who has the interests of learners at heart ensures that the home language of students in multilingual classroom environments connects with the underlying meaning of mathematical concepts and technical terms. Teachers who make a difference are focused on shaping the development of novice mathematicians who speak the precise and generalisable language of mathematics.

Mathematics teaching for diverse learners creates a space for the individual and the collective. Whilst many researchers have shown that small-group work can provide the context for social and cognitive engagement, others have cautioned that students need opportunities and time to think and work quietly away from the demands of a group. There is evidence that some students, more than others, appear to thrive in class discussion groups. Many students, including limited-English-speaking students, are reluctant to share their thinking in class discussions. Research has also shown that an over-reliance on grouping according to attainment is not necessarily productive for all students. Teachers who teach lower streamed classes tend to follow a protracted curriculum and offer less varied teaching strategies. This pedagogical

practice may have a detrimental effect on the development of a mathematical disposition and on students' sense of their own mathematical identity.

Pedagogical tasks and activities

From the research, it is evident that the opportunity to learn is influenced by what is made available to learners. For all students, the 'what' that they do is integral to their learning. The 'what' is the result of sustained integration of planned and spontaneous learning opportunities made available by the teacher. The activities that teachers plan, and the sorts of mathematical inquiries that take place around those activities, are crucially important to learning. Effective teachers plan their activities with many factors in mind, including the individual student's knowledge and experiences, and the participation norms established within the classroom. Extensive research in this area has found that effective teachers develop their planning to allow students to develop habits of mind whereby they can engage with mathematics productively and make use of appropriate tools to support their understanding.

Choice of task, tools, and activity significantly influences the development of mathematical thinking. Quality teaching at all levels ensures that mathematical tasks are not simply 'time fillers' and is focused instead on the solution of genuine mathematical problems. The most productive tasks and activities are those that allow students to access important mathematical concepts and relationships, to investigate mathematical structure, and to use techniques and notations appropriately. Research provides sound evidence that when teachers employ tasks for these purposes over sustained periods of time, they provide students with opportunities for success, they present an appropriate level of challenge, they increase students' sense of control, and they enhance students' mathematical dispositions.

Effective teaching for diverse students demands teacher knowledge. Studies exploring the impact of content and pedagogical knowledge have shown that what teachers do in classrooms is very much dependent on what they know and believe about mathematics and on what they understand about the teaching and learning of mathematics. Successful teaching of mathematics requires a teacher to have both the *intention* and the *effect* to assist pupils to make sense of mathematical topics. A teacher with the intention of developing student understanding will not necessarily produce the desired effect. Unless teachers make good sense of the mathematical ideas, they will not have the confidence to press for student understanding nor will they have the flexibility they need for spotting the entry points that will move students towards more sophisticated and mathematically grounded understandings. There is now a wealth of evidence available that shows how teachers' knowledge can be developed with the support and encouragement of a professional community of learners.

Educational leadership and links between centre and home/school

Facilitating harmonious interactions between school, family, and community contributes to the enhancing of students' aspirations, attitudes, and achievement. Research that explores practices beyond the classroom provides insight into the part that school-wide, institutional and home processes play in developing mathematical identities and capabilities. There is conclusive evidence that quality teaching is a joint enterprise involving mutual relationships and system-level processes that are shared by school personnel. Research has provided clear evidence that effective pedagogy is founded on the material, systems, human and emotional support, and resourcing provided by school leaders as well as the collaborative efforts of teachers to make a difference for all learners.

Teachers who build whānau relationships and home–community and school–centre partnerships go out of their way to facilitate harmonious interactions between the sectors. There is convincing evidence to suggest that these relationships influence students' mathematical development. The home and community environments offer a rich source of mathematical experiences on which to build centre/school learning. Teachers who collaborate with parents, families/whānau and

community members come to understand their students better. Parents benefit too: through their purposeful involvement in school/centre activities, by assisting with homework, and in providing suitable games, music and books, they develop a greater understanding of the centre's or school's programme. Their involvement also provides an opportunity to scaffold the learning that takes place within the centre or school.

Overall key findings

This Best Evidence Synthesis examines the links between pedagogical practice and student outcomes. Consistent with recent theories of teaching and learning, it finds that quality teaching is not simply a matter of 'knowing your subject' or 'being born a teacher'.

Sound subject matter knowledge and pedagogical content knowledge are prerequisites for accessing students' conceptual understandings and for deciding where those understandings might be heading. They are also critical for accessing and adapting task, activities and resources to bring the mathematics to the fore.

The importance of building home–community and school–centre partnerships has been highlighted in a number of studies of effective teaching.

Early childhood centre researchers have provided evidence that the most effective settings offer a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive, play activities.

Within centres and classrooms, effective teachers care about their students and work at developing interrelationships that create spaces for learners to develop their mathematical and cultural identities.

Extensive research on task and activity has found that effective teachers make decisions on lesson content that provide learners with opportunities to develop their mathematical identities and their mathematical understandings.

Studies have provided conclusive evidence that teaching that is effective is able to bridge learners' intuitive understandings and the mathematical understandings sanctioned by the world at large.

Gaps in the literature and directions for future research

The synthesis provides research information about effective mathematics teaching. Although the scope of researchers' studies is broad and far-reaching, a number of gaps in the literature are apparent. Research has so far provided only limited information about effective teaching in New Zealand at the secondary school level. Additionally, there is little reported research that focuses on quality teaching for Pasifika students. Few researchers in New Zealand are exploring mathematics in early childhood centres. The New Zealand literature lacks longitudinal, large-scale studies of teaching and learning. Also missing are studies undertaken in collaboration with overseas researchers. Such research is crucially important for understanding teachers' work and the impact of curricular change. The scholarly exchange of ideas made possible through joint projects with the international research community contributes in numerous ways to the capability of our local researchers.

It is important to keep in mind that, as a knowledge building tool, the synthesis provides insights based on what has gone before. A snapshot in time, it is subject to change as new kinds of evidence about quality teaching become available. Important mathematics initiatives are underway in New Zealand schools and centres. The Numeracy Development Projects, new assessment methods, projects involving information technology, and a greater focus on statistics in the curriculum are just three examples of changes that are currently taking place. All new initiatives require research that monitors and evaluates their introduction and 'take up' by centres/schools and the changes in teaching and learning that take place as a result. Such research is necessary to guide future directions in schools, educational policy, and curriculum design.

1. Introduction

Mathematics¹ is a powerful social entity. Arguably the most international of all curriculum subjects, mathematics plays a key role in shaping how individuals deal with the various spheres of private, social and civil life. The belief that mathematics is a key engine in the economy is widely shared by politicians and social planners, the corporate sector, parents, and the general public. Mathematical competencies and identities that make a valuable contribution to society are developed from specific beliefs and practices. Marshalling evidence about the pedagogical practices in centres/kōhanga and schools/kura that allow those competencies and identities to develop is a primary educational necessity. This best evidence synthesis describes those practices and provides a critical evidence base for effective pedagogical practice.

Mathematics in New Zealand

What do we know about mathematics in New Zealand schools and early childhood centres today?

Today, just as in past decades, many students do not succeed with mathematics; they are disaffected and continually confront obstacles to engaging with the subject. The challenge for those with an interest in mathematics education is to understand what teachers might do to break this pattern. Many of the problems associated with learning mathematics have little resemblance to those encountered in other curriculum areas. Typically the problems are domain-specific—solving them is not a straightforward matter of importing more general pedagogical cures.

If we cannot point to general education for students' lack of mathematical engagement, neither can we, today, point to exclusion practices whereby, traditionally, access to mathematics was considered the prerogative of a privileged few. In our inclusive society *all* students have right of access to knowledge. Precisely how teachers can enhance all students' access to powerful mathematical ideas—irrespective of socio-economic background, home language, and out-of-school affiliations—is fundamental to this best evidence synthesis.

Research has confirmed precisely what many teachers have long appreciated: that it is the classroom teacher who has a significant influence over students' learning. For example, Rowe (2004) provides evidence that when school type and the achievement and gender of students are controlled for, class/teacher effects consistently represent, on average, 59% of the residual variance in the achievements of students. Muijs and Reynolds (2001) emphasise: "All the evidence that has been generated in the school effectiveness research community shows that classrooms are far more important than schools in determining how children perform at school" (p. vii).

To a large extent, making a difference in centres and schools rests with how teachers operationalise the core dimensions of teaching. We can be sure that those core dimensions include more than the knowledge and skill that an individual teacher brings to the task. As we shall see in this synthesis, the cognitive demands of teaching, as well as the structural, organisational, management, and domain-specific choices that teachers make, are all part of the large matrix of practice. These choices include, first and foremost, the negotiation of national mathematics curriculum policy and carry over to decisions about the human, material and technological infrastructure that allow learners to achieve mathematical and social outcomes. Such infrastructural decisions involve administrators, support staff, and parents and community; they also involve the intellectual resources of curriculum materials, assessment instruments, and computational and communications technology.

Pedagogical approaches and learner outcomes

We argue throughout this synthesis for a view of pedagogy that magnifies more than what teachers know and do in centres or classrooms to support mathematical learning. And we shall

look further than improved test scores. For us, ‘best practice’ descriptions and explanations tied to high-stakes assessment don’t tell the whole story. In this synthesis, pedagogy is tied closely to interactions between people. And these interactions cannot be separated from the axes of social and material advantage or deprivation that operate to define learners. We shall see that interactions that are productive enhance not only skill and knowledge but also identity and disposition. They also add value to life and work, to the family and to the wider community of individuals (Luke, 2005).

The term ‘pedagogical approaches’ is taken as the unit of analysis and describes the elements of practice characterised not only by regularities but also the uncertainties of practice, both inside and beyond the centre or classroom. We link those practices to achievement outcomes as well as to a range of social and cultural outcomes, including outcomes relating to affect, behaviour, communication, and participation. In addition to what the teacher knows and does, pedagogy, so defined, takes into account the ways of knowing and thinking, language, and discursive registers made available within the physical, social, cultural, historical, and economic community of practice in which the teaching is embedded. Those characteristics extend beyond the centre or classroom to tap into the complex factors associated with family and whānau partnerships as well as those associated with institutional leadership and governance.

‘Quality’ or ‘effective’ pedagogical approaches are those that achieve their purposes. The exact nature of those purposes is, invariably, the subject of debate influenced by perspectives about how things should be at a given time (Krainer, 2005). Polya (1965), for example, pressed for mathematics teachers to teach people to think: “Teaching to think means that the mathematics teachers should not merely impart information, but should try also to develop the ability of the students to use the information imparted” (p. 100). Further back in time, Ballard (1915) wrote:

“We have not yet discovered the extent to which we can trust the pupils. By adopting a general policy of mistrust, by never allowing a child to mark his own, or even another child’s exercises, by making no child responsible for anybody’s conduct or progress but his own, by retaining all corrective and coercive powers in the teacher’s hands, we gain certain advantages; we simplify matters, we minimise the likelihood of abuse of authority, and we cultivate in the pupils the virtue of obedience. But we lose much more than we gain” (p. 19).

Times have changed. Today, as far as business and industry is concerned, the goals of mathematics relate to the intellectual capacities required for future employment and citizenship in a technologically-oriented, bicultural society. Prototypically for the general public, effective teaching is that which develops in students the skills, understandings, and numerical literacy they need for dealing confidently with the mathematics of everyday life. The current academic view in New Zealand is that the mathematics taught and learned in schools and early childhood centres should provide a foundation for working, thinking and acting like mathematicians and statisticians. In that view, “[e]ffective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (National Council of Teachers of Mathematics [NCTM], 2000, p. 16).

Making a difference for all

Irrespective of their differences, the various perspectives agree that mathematics teaching should make a positive difference to the life chances of students and should enhance their participation as citizens in an information- and data-driven age (Watson, 2006). Precisely because of the “gatekeeping role that mathematics plays in students’ access to educational and economic opportunities” (Cobb & Hodge, 2002, p. 249), it should assist students to develop:

- the ability to think creatively, critically, and logically;

- the ability to structure and organise;
- the ability to process information;
- an enjoyment of intellectual challenge;
- the skills to interpret and critically evaluate statistical information in a variety of contexts;
- the skills to solve problems that help them to investigate and understand the world.

Mathematical proficiency

These are the academic outcomes that exemplify mathematical proficiency. They include more than mastery of skills and concepts: they spell out the dispositions and habits of mind that underlie what mathematicians do in their work. The National Research Council (2001) has expanded on these strands further to suggest that proficient students are those who have:

- conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- strategic competence: the ability to formulate, represent, and solve mathematical problems;
- adaptive reasoning: ability for logical thought, reflection, explanation, and justification;
- productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These are the characteristics of an apprentice user and maker of mathematics and are appropriated by the student through effective classroom processes.² They incorporate curriculum content, classroom organisational structures, instructional and assessment strategies, and classroom discourse regarding what mathematics is, how and why it is to be learned and who can learn it. These proficiency strands, which identify the mathematical learner within communities of classroom practice and beyond, are at the core of this best evidence synthesis.

Social, affective, and participatory outcomes

We would want to add to these academic outcomes a range of other outcomes that relate to affect, behaviour, communication, and participation. *Te Whāriki* and *The New Zealand Curriculum Framework* are useful guides to identifying these social, affective and participatory outcomes, relevant to particular age groups. The outcomes include:

- a sense of cultural identity and citizenship;
- a sense of belonging (mana whenua);
- contribution (mana tangata);
- well-being (mana atua);
- communication (mana reo);
- exploration (mana aotūroa);
- whānau spirit;
- commonly held values, such as respect for others, tolerance (rangimārie), fairness, caring (aroha), diligence, non-racist behaviour, and generosity (manaakitanga); and
- preparation for democratic and global citizenship.

Diversity

Recognising mathematics pedagogy as a key lever for increasing students' post-school and citizenship opportunities involves an important shift in thinking about students' access to learning. This changed focus is able to reveal how the development of mathematical proficiency over time is characterised by an enhanced, integrated relationship between teachers' intentions and actions on the one hand and learners' learning and development on the other. Such a focus is also able to signal how persistent inequities in students' mathematics education might be addressed, and this is crucially important in the light of recent analyses of international test data. These data reveal patterns of social inequity that cannot be read simply as a recent phenomenon but confirm a trend of systemic underachievement established over past decades (see Garden & Carpenter, 1987).

That trend points to pedagogical approaches that affect learners in unequal ways. Findings set out by the Ministry of Education (2004) reveal that New Zealand results, compared with the results of the 32 OECD countries participating in the Programme for International Student Assessment (PISA), are widely dispersed (see also Chamberlain, 2001; Davies, 2001; McGaw, 2004). While 15-year-old New Zealand students performed significantly above the OECD average and received a placing within the second highest group of countries, this positive information is offset by the fact that a high proportion of students are at the lower levels of proficiency. Data like these, signalling low proficiency levels amongst students, provide a sobering counterpoint to claims of equitable learning opportunities for diverse students.

Issues relating to student diversity are among the most complex and challenging issues facing mathematics education today. Deficit theories have tried to explain diversity by attributing the marginal performance of particular groups to the learners themselves or their impoverished circumstances. As such, these models have blamed the learner. We do not wish to ignore the fact that a considerable number of valuable interventions have resulted from this work. We point out, however, that in organising mathematical competence around the category of learner deficiency, and measuring against a 'natural', 'neutral' benchmark, the discipline offered simplistic explanations for mathematical proficiency. It could not explain why achievement comes to some learners through a hard and painful route.

Diversity is part of the New Zealand way of life. Over the next decades, our centres and schools will cater for increasingly diverse groups of learners, and these changing demographics will require a wider understanding of diversity. Diversity, however, is "a marginalised area of research, [and] is relatively undeveloped in mathematics education" (Cobb & Hodge, 2002, p. 250). In this synthesis, diversity discounts practices that stereotype on the basis of group affiliation. Instead, diversity tries to reconcile "the identities that [students] are invited to construct in the mathematics classroom" (ibid. p. 249) with their participation in the practices of home communities, local groups and wider communities within society. Characterised in this way, diversity encompasses learner affiliations with both local and broader communities. These affiliations are revealed through ethnicity, region, gender, socio-economic status, religion, and disability as well as identifiable learning difficulties and exceptional (including special) needs.

Equity

Current efforts (e.g., Cobb & Hodge, 2002) are focused on shifting from a traditional understanding of diversity towards thinking about equity. The focus on equitable pedagogical practices in this best evidence synthesis takes issue with the ill-informed belief in New Zealand society that some, but not others, are inherently equipped to learn mathematics competently. As is emphasised in the *Guidelines for Generating a Best Evidence Synthesis Iteration 2004*, equity becomes a crucially important means to redress social injustices. It is not to be confused with equality. This is because equity is about interactions between contexts and people: it is not about equal outcomes and equal approaches. Neither is it about equal access to people

and curriculum materials. Setting up equitable arrangements for learners requires different pedagogical strategies and paying attention to the different needs that result from different home environments, different mathematical identifications, and different perspectives (Clark, 2002).

Equity, as used in this synthesis, is defined not as a property of people but as a *relation* between settings and the people within those settings. Thus equity is situated rather than static and is premised on an understanding of the bicultural foundation and multicultural reality of New Zealand classrooms. Marked by fairness and justice to ‘diverse realities’ (Ministry of Education, 2004), it is responsive to the Treaty relationship held between the Crown and Māori a relationship that protects te reo (Māori language) and tikanga Māori (Māori culture) and provides assurances of same educational opportunities for Māori and non-Māori. As Cobb and Hodge (2002) argue, to understand equity, the focus needs to be not only on inequitable social structures and the ideologies that prop them up but also on how such realities play out in the everyday activity within classrooms and other cultural practices.

The synthesis is also responsive to the multiple cultural heritages increasingly brought to the centre and classroom settings and to a rapidly changing demographic profile. According to the 2001 census, by 2021, Māori enrolments at centres and schools will constitute 45% of learners and by 2050, nearly 60% of all children in New Zealand will identify as either Māori or Pasifika.³ ‘Pasifika’ is used as an umbrella term to include the cultures and ways of thinking of the Cook Islands, Fiji, Tonga, Samoa, Tuvalu, and Niue.⁴

This synthesis comes at an important point in time for mathematics education because, although the search for the characteristics of effective pedagogy in Mathematics/Pāngarau is far from new, identifying and explaining effective practice that meets the needs of *all* learners is substantially more urgent than at any previous time. Some statistics explain why this is so. Low-decile⁵ schools tend to have a greater intake of Māori and Pasifika students. In 2001, 68% of the Pasifika school population were in decile 1, 2 or 3 schools. This compares with 9.46% of the European school population. Few students from decile 1, 2, and 3 secondary schools enrol in university courses. “Middle-class students are far more likely than working-class students to experience success at school. Five times as many students with higher professional origins obtain a university entrance level bursary or better, than those from low-skilled and nonemployed families” (Nash, 1999, p. 268). These data point to the dilemmas that teachers face as they begin with the cultural and socio-economic backgrounds of their students and try to connect them to mathematics.

A few more figures drive the point home. According to Statistics New Zealand, almost one in five of all students leave school without any formal qualifications. For Māori students, the figure is one in three; for Pasifika students, it is one in four. Despite the high achievements of many Māori (see Crooks & Flockton, 2002) and Pasifika individuals, and not in any way wanting to downplay how learner performance is being raised through Māori-medium, kōhanga reo (early childhood education), kura kaupapa Māori (primary schools), and wharekura (secondary schools), the harsh reality is that average achievement, as shown in PISA and other mathematics assessments (e.g., National Education Monitoring Project), is lower for these ethnic groups.⁶

Some promising trends have been signalled in recent analyses of the 2004 Numeracy Development Project (NDP) data for 70,000 students in years 1–8 (English-medium). For all ethnic groups, achievement, as measured by the NumPA diagnostic interview, was greater than that recorded in 2003. Although the average effect size advantage for addition/subtraction was only modest (.19), the average effect sizes for the higher stages of multiplication and proportion/ratio were .4 and .43 respectively. Of particular note in the 2004 analyses was the decrease in disparities between ethnic group performances. Analysis of the 2005 achievement data in the NDP indicates that the reduction in disparities is a continuing trend (see Young-Loveridge, 2006).

To date, mathematics has been caught up in learner access to social and economic resources

and hence to future wealth and power. For mathematics education, the overriding concern is to provide equitable pedagogical access to opportunities that will develop in learners a positive mathematical disposition and enhance their life chances. Pedagogical practice that acknowledges the complexity of learners and settings allows us to move away from making gross generalisations about diverse groupings of students. Given the sociopolitical realities that shape students' constantly changing out-of-school and classroom identities, the task ahead is to change patterns of underachievement that, in the past, have been connected to a range of factors. Although there is agreement about this overarching goal, there has not been shared understanding about what pedagogy might do to achieve it.

Positioning the Best Evidence Synthesis

Against the backdrop of these statistics, the task of promoting democratic access to mathematical know-how assumes formidable proportions. How do teachers work at developing empowering approaches for learners who are polarised and disempowered by their sociocultural status? What must they do to develop an understanding of the big ideas of mathematics? How do they enhance a mathematical disposition and an appreciation of the value of mathematics in life? This synthesis, despite its best intentions, does not have ready-made practices to offer. As we shall see, the dimensions and core features of effective teaching for diverse learners are multiple. One of our most important claims is that there are no hard-and-fast rules about what methods and strategies work best. We simply do not have evidence of teaching practice that could be generalised to particular kinds of learning across all settings and across all learners. As educators, we have long known that teaching differs from one centre or classroom to another.

We caution against the tradition of identifying teacher effectiveness solely through teacher uptake of curriculum reforms or through the use of test results (Koehler & Grouws, 1992). Just as we would want to believe that reformers' visions are being realised, we have long known that teachers do not always implement them in ways that were intended by curriculum designers (Millet, Brown & Askew, 2004). We also note the limitations of craft-practice approaches to teaching that highlight teacher clarity or relative time spent on lesson components. Nor, as Alton-Lee (2005) has noted, can we say with total conviction that pedagogies customised specifically for learners with special needs produce greater achievement benefit for the learners (see Lewis & Norwich, 2000). Indeed, what we shall see is that some teaching approaches and classroom arrangements produce differential results from one setting to another. Content choices factor in too. For example, figures for NCEA results reveal that at level 2, students taking calculus recorded the lowest 'achieved' results (40%), and at level 3, 82% of statistics students recorded the highest 'achieved' results (Ministry of Education, 2006).

One thing, however, that we have gleaned from landmark studies is a set of common, underlying pedagogical principles that appear to hold good across people and settings. It is the principles upon which teachers base their practice that tend to make a difference for diverse learners. The identification of effective practice across centre and classroom settings, based on common principles, provides a rare opportunity to offer creative solutions. We do have some promising guideposts in this work. From a series of landmark best evidence syntheses (Alton-Lee, 2003; Biddulph, Biddulph, & Biddulph, 2003; Farquhar, 2003; Mitchell & Cubey, 2003), we know that practices and conditions that are respectful of the experiences of learners can make a difference to learning. We read of teachers who have provided access to learning against all odds and have done so through their belief in the rights of all learners to have access to education in a broad sense. Their work provides evidence that the effects of social disadvantage can be halted when learners encounter curricula in the classroom. Biddulph et al. (2003) have identified families/whānau and communities as key figures in bringing about academic achievement for diverse learners. Particular family attributes and processes, and community factors, as well as centre/school, family, and community partnerships, can all make a difference.

From our own discipline, we have evidence of the sorts of factors and conditions in Māori-medium schools and classrooms that raise expectations for learners' progress. Te Poutama Tau, in its responsiveness to the goals of Māori language revitalisation and empowerment, has revealed a positive effect on student achievement (Christensen, 2004). We know too that numeracy reform efforts in the UK (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997) and in this country (e.g., Thomas & Tagg, 2005; Young-Loveridge, 2005), have contributed to higher student performance.

Overview of chapters

What follows is a systematic and credible evidence base for pedagogical approaches that enhance both proficiency and equity for learners. It is drawn from research that explains the sorts of pedagogical approaches that lead to improved engagement and desirable outcomes for learners from diverse social groups. It represents a first-steps approach to providing insight into new definitions of effective mathematics teaching.

The synthesis is made up of eight chapters. Chapter 2 develops a theoretical and empirical framework for the BES. We provide an overview of seminal studies that pinpoint in unique ways how quality teaching might be characterised. These landmark investigations foreground the complexity of pedagogical practice and how difficult it is to come up with universal checklists of effective teaching. From these studies, a set of guiding principles is derived alongside the theoretical framing for our work in the body of the synthesis. In offering a theoretical basis for structuring the report, we explain the notion of 'communities of practice' and the terms that we use in the synthesis. A description of how evidence-based studies of quality teaching were located, and the standards that the studies had to meet to qualify for inclusion in this synthesis, is provided in an appendix.

Chapters 3, 4, 5, 6, and 7 form the backbone to this best evidence synthesis. Covering both the early years and school sectors, these chapters take a thematic approach and include vignettes that emphasise particular characteristics of quality teaching. In chapter 3, evidence-based practices that make a difference for young learners are illuminated. This chapter on quality mathematics education in the early years stresses the formative influences at work on young children and concludes with a section analysing the transition from centre to school. Chapter 4 focuses on people, relationships, and classroom environment and explores how teachers develop productive mathematical communities of learning. Tasks and tools are brought to centre stage in chapter 5. The activities that teachers choose and the sorts of mathematical enquiries that take place around those activities are clarified. Chapter 6 explores practices beyond the classroom. It offers insight into the roles that school-wide, institutional and home processes play in developing mathematical identities and capabilities. The principles unearthed in chapter 2 and the key themes developed in chapters 3–6 converge in chapter 7 around a discussion of teaching and learning fractions. Concluding thoughts about what makes a difference in centres and schools for New Zealand learners are set out in chapter 8.

What the authors are very keen to do is clarify how patterns of inequality *can* be countered within the mathematics classroom. These explanations are not intended to be read as prescriptions of how teachers in New Zealand centres and schools should teach mathematics. Rather, by making clear the principles and characteristics underpinning effective practice, the synthesis is intended to stimulate reflection on mathematics education within and across sectors and to generate productive critique of procedures current within the discipline. Reflection and critique will make visible a new sensibility towards the multiple dimensions of pedagogical practice.

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- ¹ Mathematics as used in this synthesis is inclusive of the statistics component in the curriculum.
- ² By 'classroom processes' is meant the learning interactions that occur within the early learning centre and mathematics classroom.
- ³ <http://www.stats.govt.nz>
- ⁴ Included are the newly arrived to New Zealand, those descended from Pasifika peoples living in New Zealand for one or more generations, and those of multiple heritages.
- ⁵ A school's decile indicates the extent to which the school draws its students from low socio-economic communities. Decile 1 schools are the 10% of schools with the highest proportion of students from low socio-economic communities, whereas decile 10 schools are the 10% of schools with the lowest proportion of these students.
- ⁶ The actual difference may well be more marked. PISA cohorts represent students aged 15 years 3 months up to 16 years 2 months. 40% of Māori students leave school before the PISA upper age limit.
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Appendix 1: Locating and Assembling BES Data

Using the ‘health-of-the-system’ approach, we sought to examine the various factors implicated in the creation of an effective learning community. We investigated a number of measures that fell naturally from the ‘what’, ‘why’, ‘how’, and ‘under what conditions’ questions concerning pedagogical approaches that facilitate learning for all students. The task was a considerable one, involving information management, the engagement of advisory and audit groups, and the seeking of contributions from the education community in general and the mathematics education community in particular. This level of engagement ensured that the Best Evidence Synthesis would be inclusive of views from across the community.

Our initial search strategy required us to pay attention to different contexts, different communities, and multiple ways of thinking and working. With this in mind, we undertook a literature search that crossed national and international boundaries. We used a range of search engines as well as personal networks to help us find academic journals, theses, projects, and other scholarly work with a focus on mathematics in New Zealand schools and centres, and by selected authors worldwide. We searched both print indices and electronic indices, endeavouring to make our search as broad as possible within the limits of manageability. This search took into account relevant publications from the general education literature and from the literature that relates to specialist areas of education. The search covered:

- key mathematics education literature including all major mathematics education journals (e.g., *Journal for Research in Mathematics Education*, *Educational Studies in Mathematics*, *Journal of Mathematics Teacher Education*, *For the Learning of Mathematics*, *The Journal of Mathematical Behaviour*), international conference proceedings (e.g., PME, ICME), Mathematics Research Group of Australasia publications, and international handbooks of mathematics education (e.g., Bishop et al., 2003);
- relevant New Zealand-based studies, reports, and thesis databases, supported by input from the professional community and the Ministry of Education;
- education journals (e.g., *American Educational Research Journal*, *British Educational Research Journal*, *Cognition and Instruction*, *The Elementary School Journal*, *Learning and Instruction*, etc.) and New Zealand work (e.g., SAMEpapers, SET, NZJES);
- specialist journals and projects, especially those located within the wider education field (e.g., *New Zealand Research in Early Childhood Education*; *Journal of Learning Disabilities*);
- landmark international studies including TIMSS, PISA, the UK Leverhulme projects.

This search strategy led us to a large body of literature that had something to say about facilitating mathematics learning: the total number of sourced references was just over 1100. Table 1 categorises these references by source:

Table 1: Initial Database Composition by Source

Source of data	Relative frequency (n ~1100)
Mathematics education journals	24%
Mathematics education reports, books, handbooks	16%
Mathematics education conference proceedings	15%
Theses and projects	6%
General education reports, books, handbooks	10%
General education journals, reports, and proceedings	19%
Specialist journals	10%

All entries were stored and categorised using EndNote. To assist in the initial synthesis, we distinguished between ‘research’ and ‘discussion document’, and categorised entries according to (a) our ‘diversity’ descriptors (e.g., ethnicity, gender, socioeconomic), (b) centre/school level, and (c) country-of-origin of the data.

These categories and sub-divisions served as a useful starting point for overviewing the literature and allowed us to foreground our fundamental intent to be responsive to diversity. In addition, by classifying entries according to sector and country of origin, we gave ourselves a constant reminder of the need to be inclusive of all perspectives and interests. This inclusiveness gave us a body of literature comprising diverse frameworks and eclectic methodological and analytic approaches.

Selecting the evidence

Given the complexity of the teaching and learning process, it is not an easy matter to link specific outcomes with specific pedagogical approaches. In our first pass through the literature, we noted that studies could claim that student achievement was *influenced* by pedagogical practice much more readily than they could explain *how* that practice affected student achievement. Many studies offered detailed explanations of student outcomes yet failed to draw conclusive evidence about how those outcomes related to specific teaching practices. Others provided detailed explanations of pedagogical practice yet made unsubstantiated claims about, or provided only inferential evidence for, how those practices connected with student outcomes.

Granted, we were not looking for linear explanations. As Sfard (2005) points out, the complexity of the teaching–learning relationship “precludes the possibility of identifying clear-cut cause–effect relationships” (p. 407). What we were searching for were studies that were able “to offer a developing picture of what it looks like for a teacher’s practice to cultivate student [proficiency]” (Blanton & Kaput, 2005, p. 440). We were searching for studies that offered a “detailed look at how [teachers’] actions played out in the classroom and how students were involved in this” (Blanton & Kaput, 2005, p. 435) and the sorts of mathematical proficiency that resulted. Specifically, we were seeking studies that offered not just detailed descriptions of pedagogy and outcomes but rigorous explanation for close associations between pedagogical practice and particular outcomes.

Paying attention to diverse forms of research evidence required our serious consideration of the literature relating to disparate factors from different sectors and representative of different time periods. Luke and Hogan (in press) note that what is distinctive about the approach undertaken in the New Zealand Best Evidence programme “is its willingness to consider all forms of research evidence regardless of methodological paradigms and ideological rectitude, and its concern in finding contextually effective appropriate and locally powerful examples of ‘what works’... with particular populations, in particular settings, to particular educational ends” (p. 5). We have included many different kinds of evidence that take into account human volition, programme variability, cultural diversity, and multiple perspectives. Each form of evidence, characterised by its own way of looking at the world, has led to different kinds of truth claims and different ways of investigating the truth. Our pluralist stance left us free to consider the relative strengths and weaknesses of different methodological approaches.

A fundamental challenge for this BES has been to demonstrate a basis for knowledge claims. We are absolutely aware that, like data selection, assessment of evidential claims from secondary sources is a highly perspectival activity. “Even those gazing down a microscope are as capable of finding what they expect to find, or want to find, as anyone else” (Davies, 2003). In response to this challenge, studies have been reported in a way that will make the original evidence as transparent as possible. Informed by the *Guidelines for Generating a Best Evidence Synthesis Iteration 2004*, we included studies that:

- provided a description of the context, the sample, and the data;

- offered details about the particular pedagogy and the specific outcomes;
- connected research to relevant literature and theory;
- used methods that allow investigation of the pedagogy–outcome link;
- yielded findings that illuminated what did or did not work.

The *Guidelines for Generating a Best Evidence Synthesis Iteration* allowed us to deal not only with a diversity of research topics, approaches, and methods, but also to capture differences in the context, practices, and ways of thinking of researchers. The method employed in this BES for evaluating validity required us to look at the ways different pieces of data meshed together and to determine the plausibility, coherence, and trustworthiness of the interpretation offered.

Assessments about the quality of research depend to a large extent on the nature of the knowledge claims made and the degree of explanatory coherence between those claims and the evidence provided. What we were looking for was the explanatory power of the stated pedagogy–outcome link. When assessing the nature and strength of the causal relations between pedagogical approaches and learning outcomes, we were guided by Maxwell’s (2004) categorisations of two types of explanations of causality. The first type, the *regularity view of causation*, is based on observed regularities across a number of cases. The second type, *process-oriented explanations*, sees “causality as fundamentally referring to the actual causal mechanisms and processes that are involved in particular events and situations” (p. 4). Cobb argues (2006, personal communication) that regularity explanations are particularly useful for policy makers, while process-oriented explanations are relevant to teachers, who are concerned with “the mechanism through which and the conditions under which that causal relationship holds” (Shadish, Cook, & Campbell, 2002, p. 9, cited in Maxwell, 2004, p. 4). Attending to both types of explanation of causality meant including both large-scale and single-case studies. In many instances, we have found it useful to present a single case—a learner or teacher, a classroom, or a school—in the form of a vignette to exemplify the relations between learning processes and the means by which they are supported.

Research sources in this BES report

This BES report contains approximately 660 references. Included amongst these are research reports of empirical studies, ranging from very small, single-site settings (e.g., Hunter, 2002) to large-scale longitudinal studies (e.g., Balfanz, MacIver, and Byrnes, 2006). Some of the larger studies have multiple references because they include different papers/conference proceedings/book chapters or because they embrace work authored by different researchers (e.g., the New Zealand Numeracy Development Project). In addition, the references include reports containing educational statistics and policy, theoretical writings, and commentaries and reviews on multiple research findings (e.g., van Tassel-Baska, 1997) .

The *Guidelines for Generating a Best Evidence Synthesis Iteration* point to the importance of drawing on New Zealand research in order to illuminate what works in the New Zealand context. However, despite an exhaustive search for New Zealand work, it is apparent (see chapter 8 for further discussion) that the strengths and foci of New Zealand research are not evenly distributed. In some areas—for example, early years education—there are relatively few New Zealand (or Australian) researchers working with a specific focus on mathematics education (Walshaw & Anthony, 2004). Table 2 shows the country of origin of the literature included in this BES. The numbers reflect New Zealand’s relatively new positioning within the international mathematics education research community.

Table 2: Database composition according to country

Country	Relative Frequency
New Zealand	27%
Australia	17%
United Kingdom	11%
United States	49%
Other (e.g., Africa, Netherlands, Spain)	6%

Table 3 shows the proportion of the items included in the BES (both empirical studies and commentaries) that relates to each of the different sectors. Publications relating specifically to intermediate schools have been classified with the literature on primary schools.

Table 3: Database composition according to sector

Sector	Relative Frequency (n~520)
Preschool	18%
Primary school	48%
Secondary school	21%
Teacher education	13%

Synthesising the data

Our conceptual framework, outlined in chapter 2, offered a way of structuring the data. Within the community of practice frame in and beyond the classroom, we identified the following components: (a) the organisation of activities and the associated norms of participation, (b) discourse, particularly norms of mathematical argumentation, (c) the instructional tasks, and (d) the tools and resources that learners use. We began the iterative chapter-structuring process by outlining a number of key areas. These included mathematical thinking and identities, scaffolding and co-construction, tasks, activities, assessment, educational leadership, home-school/centre links, and wider school communities. Each of these served as a starting point for our exploration and was found, in the course of the investigation, to be a useful initial category for addressing questions of equity and proficiency in relation to effective mathematics teaching.

In time, we organised these categories more cohesively into groups. What we endeavoured to do was organise multiple elements, types, and levels and varying temporal conditions in line with the critical dimensions of a community of practice and the guiding principles established in chapter 2. The content of the subsequent chapters is shaped according to these dimensions and principles. Chapter 3 focuses on all three dimensions in a search for understanding of how pedagogy influences early years outcomes. Chapters 4 and 6 explore interrelationships that are centred on the joint enterprise of developing mathematical proficiency for all learners. Chapter 5 explores the role of mathematical tasks and the part that they play in enhancing students' learning.

Reminding ourselves and readers that this BES synthesis is a product of currently accessible research, we concur with Atkinson's (2000) view that "the purpose of educational research is surely not merely to provide 'answers' to the problems of the next decade or so, but to continue to inform discussion, among practitioners, researchers and policy-makers, about the nature, purpose and content of the educational enterprise" (p. 328). Rather than offering broad answers that promise much and achieve little, it is our hope that the structure we have used will foster understanding, reflection, and action concerning the characteristics of effective pedagogical approaches in mathematics.

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Appendix 2: URLs of citations

The following 22 papers/articles/chapters/books are suggested as potentially useful sources for teachers to engage more deeply with the range of issues raised in this best evidence synthesis iteration. Readers are encouraged to source and read them. Several are available online; the others can be sourced through libraries.

The full citations are hyperlinked in the online PDF. For the convenience of those using a hard copy of the text, the URLs are listed below.

Carpenter, Thomas P ; Franke, Megan L ; Jacobs, Victoria R

A longitudinal study of invention and understanding in children's multidigit addition and subtraction
<http://nzcer.org.nz/BES.php?id=BES001>

Clarke, Barbara ; Clarke, Doug

Mathematics teaching in Grades K-2: painting a picture of challenging supportive, and effective classrooms
<http://nzcer.org.nz/BES.php?id=BES002>

Cobb, Paul ; Boufi, Ada ; McClain, Kay ; Whitenack, Jor

Reflective discourse and collective reflection
<http://nzcer.org.nz/BES.php?id=BES020>

Empson, Susan B

Low performing students and teaching fractions for understanding: An interactions analysis
<http://nzcer.org.nz/BES.php?id=BES021>

Fraivillig, Judith L ; Murphy, Laren A ; Fuson, Karen C

Advancing children's mathematical thinking in everyday mathematics classrooms
<http://nzcer.org.nz/BES.php?id=BES003>

Gifford, Sue

A new mathematics pedagogy for the early years: in search of principles for practice
<http://nzcer.org.nz/BES.php?id=BES004>

Goos, Merrilyn

Learning mathematics a classroom community of inquiry
<http://nzcer.org.nz/BES.php?id=BES005>

Houssart, Jenny

Simplification and repetition of mathematical tasks: a recipe for success or failure?
<http://nzcer.org.nz/BES.php?id=BES006>

Irwin, Kathie ; Woodward, J (paper available online)

A snapshot of the discourse used in mathematics where students are mostly Pasifika (a case study in two classrooms)
<http://nzcer.org.nz/BES.php?id=BES007>

Kazemi, Elham ; Stipek, Deborah

Promoting conceptual thinking in four upper-elementary mathematics classrooms
<http://nzcer.org.nz/BES.php?id=BES008>

Latu, Viliami (paper available online)

Language factors that affect mathematics teaching and learning of Pasifika students
<http://nzcer.org.nz/BES.php?id=BES009>

O'Connor, Mary Catherine

"Can any fraction be turned into a decimal?" A case study of the mathematical group discussion
<http://nzcer.org.nz/BES.php?id=BES010>

Rietveld, Christine M.

Classroom learning experiences of mathematics by new entrant children with Down syndrome
<http://nzcer.org.nz/BES.php?id=BES011>

Savell, Jan ; Anthony, Glenda Joy

Crossing the home-school boundary in mathematics
<http://nzcer.org.nz/BES.php?id=BES049>

Sheldon, Steven B ; Epstein, Joyce L

Involvement counts: family and community partnerships and mathematics achievement

<http://nzcer.org.nz/BES.php?id=BESo12>

Smith, Margaret Schwan Smith ; Henningsen, Marjorie A

Implementing standards-based mathematics instruction: a casebook for professional development

<http://nzcer.org.nz/BES.php?id=BESo13>

Steinberg, Ruth M ; Empson, Susan B ; Carpenter, Thomas P

Inquiry into children's mathematical thinking as a means to teacher change

<http://nzcer.org.nz/BES.php?id=BESo14>

Watson, Anne ; De Geest, Els

Principled teaching for deep progress: Improving mathematical learning beyond methods and material

<http://nzcer.org.nz/BES.php?id=BESo15>

Wood, Terry (paper available online)

What does it mean to teach mathematics differently?

<http://nzcer.org.nz/BES.php?id=BESo16>

Yackel, Erna ; Cobb, Paul

Sociomathematical norms, argumentation, and autonomy in mathematics

<http://nzcer.org.nz/BES.php?id=BESo17>

Young-Loveridge, Jenny (paper available online)

Students views about mathematics learning: a case study of one school involved in Great Expectations Project

<http://nzcer.org.nz/BES.php?id=BESo18>

Zevenbergen, R

The construction of a mathematical habitus: implications of ability grouping in the middle years

<http://nzcer.org.nz/BES.php?id=BESo19>

Appendix 3: Glossary

The page reference for the first and/or most important occurrence of the term is given in brackets.

- Cognitive engagement** (p. 2). The state of being engaged in thinking
- Community of Practice** (p. 6). The complex network of relationships within which teachers teach and students learn
- Connectionist teachers** (p. 97). Teachers who consistently make connections between different aspects of mathematics
- Decile** (p. 9). In New Zealand, a 1–10 system used by the Ministry of Education to indicate the socio-economic status of the communities from which schools draw their students; low-decile schools receive a higher level of government funding
- Developmental progressions** (p. 47). Sequential learning pathways categorised as a series of steps
- Empirical evidence** (p. 24). Data that has been collected systematically for research purposes
- Equity** (p. 9). The principle based on the belief that social injustices should be redressed by allocating resources according to need, not power; in education, this may mean, amongst other things, the provision of different pedagogical approaches depending upon the needs of the learners
- Family Math** (p. 171). A US initiative designed to develop parents' skills so they can work with their children on their mathematics
- Feed the Mind** (p. 45). A media campaign funded by the New Zealand Ministry of Education and designed to support family involvement in children's learning
- High or low press for understanding** (p. 121). Differing levels of cognitive engagement demanded of students by teachers for clarification of thinking
- Kahoa** (p. 36). A festive necklace (Tongan)
- Kōhanga reo** (p. 9). Māori-medium early childhood centres
- Kura kaupapa Māori** (p. 10). Māori-medium schools (kura = school), based on a Māori philosophy of learning (see pp. 54–5)
- Manipulatives** (p. 133). Any concrete materials used by students to model mathematical relationships
- Mathematical argumentation** (p. 123). Presenting a case to support or refute a premise developed by mathematical thinking
- Mathematical identity** (p. 19). How a student sees him/herself as a learner and doer of mathematics
- Metacognition** (p. 38). The knowledge and processes involved in thinking about and regulating one's own thinking, which is essential for reflecting, self-monitoring, and planning
- Norms of participation** (p. 54). The rules, spoken or unspoken, that govern the way students behave and contribute in the classroom
- Number Framework** (p. 109). A model, structured in 8 stages, showing how students typically develop understanding of number and number operations (New Zealand, NDP)
- Number sense** (p. 98). An understanding of the relationships, patterns, and fundamental reasonableness that lie behind all mathematical operations
- Numeracy** (p. 28). The ability to use mathematics effectively, fluently, and with understanding in a wide variety of contexts
- Numeracy Development Project (NDP)** (pp. 9, 17). The central part of the New Zealand Ministry of Education's Numeracy Strategy, which has as its primary objective the raising of student achievement in numeracy through lifting teachers' professional capability
- NumPA** (p. 9). A structured, diagnostic interview used by teachers to place students on the early stages of the Number Framework (New Zealand, NDP)
- Open-ended tasks** (p. 106). Tasks that require students to engage in problem definition and formulation before beginning to think about a solution
- Pasifika students** (p. 9). Students whose families have come from Sāmoa, Tonga, the Cook Islands, Niue, Tokelau, Tuvalu, and some other, smaller Pacific nations
- Pedagogical Content Knowledge** (p. 199). In this context, knowledge about mathematics and how to teach it as well as knowledge about how to understand students' thinking about mathematics
- Pedagogy** (p. 5). The processes and actions by which teachers engage students in learning
- Poi** (p. 26). A small ball, often made of woven flax, on the end of a length of string; swung rhythmically by women when performing action songs (Māori)
- QUASAR** (p. 95). A programme developed to help urban students develop understanding of mathematical ideas through engagement with challenging mathematical tasks
- Revoicing** (p. 78). The repeating, rephrasing, or expansion of student talk in order to clarify or highlight content, extend reasoning, introduce new ideas, or move discussion in another direction
- Scaffolding** (p. 27). Temporary, structured support designed to move learners forward in their thinking

- School–home or home–school partnership** (p. 160). The deliberate nurturing of relationships between the school and the home, in the interests of better supporting student learning
- Sociocultural practices** (p. 19). Practices relating to the social and cultural aspects of participation in the classroom
- Sociocultural theory** (p. 24). The theory that learning arises out of social interaction
- Socio-economic status (SES)** (p. 30). Categorisation of individuals or communities, based on income, family background, and qualifications
- Sociomathematical norms** (pp. 61–62). Shared understandings of the processes by which students and teacher contribute to a mathematical discussion
- Tasks** (p. 94). Defined by Doyle (1983) as “products that students are expected to produce, the operations that students are expected to use to generate those products, and the resources available to students while they are generating the products”
- Te ao Māori** (p. 54). The Māori world
- Te Poutama Tau** (p. 59). The Numeracy Project (New Zealand) as developed for implementation in Māori-medium schools
- Te Whāriki** (p. 24). The New Zealand early childhood curriculum (for children aged 5 or under)
- Tukutuku panels** (p. 115). A Māori craft form consisting of ornamental lattice-work panels woven together with strips of flax into intricate designs
- Waiata** (p. 26). A song (Māori)
- Whānau** (p. 41). Extended family (Māori)
- Wharekura** (p. 9). Māori-medium secondary schools, which are based on a Māori philosophy of learning
- Zone of Proximal Development (ZPD)** (p. 36). Vygotsky (1986) describes the ZPD as the “distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers”

Abbreviations

CGI	Cognitively Guided Instruction Project	pp. 17, 105
EAL	English as an Additional Language	p. 116
EFTPOS	Electronic Funds Transfer at Point of Sale	p. 115
EMI-4s	Enhancing the Mathematics of Four-Year-Olds.	p. 28
ENRP	Early Numeracy Research Project	p. 158
EPPE	Effective Provision of Pre-school Education Project.	p. 25
ERO	Education Review Office.	p. 158
IAMP	Improving Attainment in Mathematics Project	pp. 18, 99
ICME	International Congress on Mathematics Education	p. 20
ICT	Information and Communication Technologies	p. 27
IEA	International Association for the Evaluation of Educational Achievement	p. 154
IMPACT	Increasing the Mathematical Power of All Children and Teachers	p. 73
MEP	Mathematics Enhancement Project.	p. 60
NCEA	National Certificate of Educational Achievement	pp. 10, 66
NEMP	National Education Monitoring Project	p. 9
NNS	National Numeracy Strategy	p. 17
PISA	Program for International Student Assessment.	p. 8
REPEY	Researching Effective Pedagogy in the Early Years	p. 25
RME	Realistic Mathematics Education	p. 113
TIMSS	Third International Mathematics and Science Study.	p. 14
VAMP	Values and Mathematics Project	p. 58

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