



MINISTRY OF EDUCATION

Te Tāhuhu o te Mātauranga

Effective Pedagogy in Mathematics/Pāngarau

Best Evidence Synthesis Iteration [BES]

Glenda Anthony and Margaret Walshaw, Massey University



ITERATIVE BEST EVIDENCE SYNTHESIS PROGRAMME
<http://educationcounts.edcentre.govt.nz/goto/BES>
New Zealand

This report is one of a series of best evidence synthesis iterations (BESs) commissioned by the Ministry of Education. The Iterative Best Evidence Synthesis Programme is seeking to support collaborative knowledge building and use across policy, research and practice in education. BES draws together bodies of research evidence to explain what works and why to improve education outcomes, and to make a bigger difference for the education of all our children and young people.

Each BES is part of an iterative process that anticipates future research and development informing educational practice. This BES follows on from other BESs focused on quality teaching for diverse learners in early childhood education and schools. Its use will be informed by other BESs, focused on teacher professional learning and development and educational leadership. These documents will progressively become available at: <http://educationcounts.edcentre.govt.nz/goto/BES>

Feedback is welcome at best.evidence@minedu.govt.nz

Note: the references printed in purple refer to a list of URLs in Appendix 2. These are a selection of potentially useful sources for teachers to engage more deeply with the range of issues raised in this best evidence synthesis iteration.

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About the writers

Glenda Anthony and Margaret Walshaw, both from the School of Curriculum and Pedagogy at Massey University, bring to this Best Evidence Synthesis (BES) decades of mathematics classroom teaching and educational research experience. They are acutely aware of the challenge that educators face in constructing a democratic mathematical community with which all students can identify. For them, making a positive difference to diverse learners' outcomes is a central educational issue. At the heart of their work is a concerted effort to illuminate how this issue is best addressed. In this synthesis, they report on the outcome of their deliberations over, and search for, what makes a difference for diverse learners in mathematics/pāngarau.

Advisory Group

A core Advisory Group membership was selected to provide expertise and critique in relation to the various focuses of the BES, including Māori and Pasifika learners, early childhood, primary and secondary sectors, and teacher education. The authors wish to thank the members of this group:

- Dr Ian Christensen (Massey University and He Kupenga Hao i te Reo)
- Dr Joanna Higgins (Victoria University of Wellington)
- Roberta Hunter (Massey University)
- Garry Nathan (Auckland University)
- Dr Sally Peters (Waikato University)
- Assoc. Prof. Jenny Young-Loveridge (Waikato University)

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Ministry of Education advisory team

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External quality assurance

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Forewords

International

Even the casual visitor is struck by the dramatic changes that have occurred in New Zealand in the last 15 years. I have tuned in to local media on each of my four visits to get an initial sense of people's current concerns and issues. Based on this narrow sampling, the New Zealand of 1991 was an immensely likeable country that had seen better days and was struggling to find its place in a rapidly changing world. Although innovation and experimentation appeared to be the watchwords of the day, there seemed to be an undercurrent of apprehension and anxiety as people attempted to cope with economic disruption. Today, New Zealand continues to be an immensely likeable place, but the visitor immediately notices a quiet, understated self-assurance. It has become a largely prosperous country that, in a very real sense, has reinvented itself as a leading information economy in an increasingly globalised world. Refreshingly for the visitor from the United States, there appears to be widespread belief that government will approach problems pragmatically and is capable of solving them. If the Iterative Best Evidence Synthesis Programme is representative of New Zealand government in action, this belief would appear to be well founded.

Put quite simply, the Iterative BES Programme is the most ambitious effort I have encountered that uses rigorous scientific evidence to guide the ongoing improvement of an education system at a national level. The programme has a strong pragmatic bent and is clearly grounded in the hard-won experience of synthesising research findings to inform both policy and teachers' instructional practices. Four aspects of the programme are particularly noteworthy. The first is the overriding commitment to make the development of the best evidence syntheses transparent. This commitment takes concrete form in the exacting evaluation and feedback process that all BES reports undergo at each phase of their development, from the initial identification of relevant bodies of research literature through to the final critique and revision of the report. This is in the best traditions of science, where claims are justified in terms of the means by which they have been produced.

The second notable characteristic is a mature view of evidence and an emphasis on methodological and theoretical pluralism. This is important, given that attempts have been made in a number of countries, including the United States, to legislate what counts as scientific research in education on the basis of ideological adherence to a particular methodology. In taking an inclusive approach, the Iterative BES Programme acknowledges that different types of knowledge are of greatest use to teachers and to policymakers. Teachers make pedagogical decisions on the basis of a detailed understanding of specific students in particular classrooms at particular points in time. Policymakers, in contrast, typically need knowledge of trends and patterns that hold up across classrooms to make decisions that affect large numbers of students and teachers in multiple schools. Different methodologies are appropriate for developing these equally important types of knowledge.

The third noteworthy characteristic of the Iterative BES Programme is its focus on the explanatory power and coherence of theories. Priority is given to theories that give insight into learning processes and the specific means of supporting their realisation in classrooms. This pragmatic criterion is important in a field where theoretical perspectives continue to proliferate.

The final notable characteristic of the programme is its explicit attention to the issues of language and culture. This emphasis is clearly critical if New Zealand teachers and policymakers are to address the inequities inherent in the disturbingly large gaps in school achievement between children of different ethnic and racial groups. In keeping with the tenet of methodological and theoretical pluralism, the Iterative BES Programme uses group categories such as socioeconomic status, ethnicity, and culture as key variables in *assessing* efforts to achieve

equity. However, it avoids stereotyping children of particular racial, ethnic, or language groups by acknowledging the complexity of individual identity when *explaining* inequities in children's learning opportunities. Furthermore, the programme emphasises ecological models of learning that link what is happening in classrooms both to the institutional contexts in which classrooms are located and to issues of race, culture, and language. It is here that the full ambition of the programme becomes apparent: few viable models of this type currently exist in education. The BES writers are therefore charged with the task of synthesising in the true sense of the term, that is, to combine disparate and sometimes fragmented bodies of research into a single, unified whole. At the risk of understatement, this is a formidable challenge.

The writers of this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau, Drs. Glenda Anthony and Margaret Walshaw, have risen to the challenge. They were charged with the daunting task of reviewing, organising, and synthesising all mathematics education research from the early childhood years through secondary school that relates classroom processes to student learning. On my reading, the resulting synthesis of over 600 research studies is directly relevant to teachers and will be educative for policymakers. The educative value of the report stems from Anthony and Walshaw's focus on what goes on in mathematics classrooms, thereby providing a window on the complexity of effective pedagogy. The forms of pedagogical practice that they identify as effective are ambitious because they involve high expectations for *all* children's mathematical learning. The goals at which these forms of pedagogy aim are best illustrated in chapter 7, A Fraction of the Answer, in which Anthony and Walshaw pull together the key insights of the preceding chapters as they present an integrated series of cases that focus on the learning and teaching of fractions. As this chapter makes clear, the instructional goals for fractions are not limited to ensuring that children can add, subtract, multiply, and divide fractions successfully. Instead, the instructional objectives also focus on children's development of a deep understanding of fractions as amounts or quantities. At an elementary level, children who are coming to understand fractions as quantities know that $\frac{1}{6}$ is smaller than $\frac{1}{5}$ because there will be more pieces when something is divided into 6 pieces than into 5 pieces, so the pieces must be smaller. At a more advanced level, students will be able to describe real world situations that involve multiplying and dividing fractional quantities. More generally, ambitious pedagogy focuses on central mathematical ideas and principles that give meaning to computational methods and strategies.

Anthony and Walshaw's review of the relevant research indicates that central mathematical ideas and principles cannot be directly transmitted to children. However, the research also shows that discovery approaches that place children in rich environments and simply encourage them to inquire are also ineffective. Effective pedagogy is complex because it requires teachers to achieve a significant mathematical agenda by taking children's current knowledge and interests as the starting point. As Anthony and Walshaw clarify, these forms of pedagogy involve a distinctive orientation towards teaching. First and foremost, the emphasis is on building on students' existing proficiencies rather than filling gaps in students' knowledge and remediating weaknesses. As a consequence, the teacher's focus when planning for instruction is not on students' limitations but on their current mathematical competencies and interests, as these constitute resources on which the teacher can build. More generally, effective mathematical pedagogy places students' reasoning at the center of instructional decision making. As a consequence, the ongoing assessment of students' reasoning is an integral aspect of instruction, not a separate activity conducted after the fact to check whether goals for students' learning have been achieved. A key characteristic of accomplished teachers is that they continually adjust instruction, as informed by these ongoing assessments.

One of the strengths of Anthony and Walshaw's synthesis is that it provides the reader with a concrete image of what effective mathematical pedagogy looks like. Anthony and Walshaw emphasise that a respectful, non-threatening classroom atmosphere in which all students feel comfortable in making contributions is necessary but not, by itself, sufficient. As they document, the research findings indicate unequivocally that it is also essential that classroom activity

and discourse focus explicitly on central mathematical ideas and processes. The selection of instructional tasks is therefore critical. On the one hand, it is important that task contexts or scenarios are accessible to all students, regardless of cultural background. On the other, the teacher should be able to capitalise on students' solutions to support their development of increasingly sophisticated forms of mathematical reasoning. Thus, when designing and selecting tasks, the teacher has to take account both of students' current competencies and interests and their long-term learning goals. As Anthony and Walshaw discuss in chapter 5, an important way in which the teacher can build students' solutions is by introducing judiciously chosen tools and representations. A second, equally important way in which the teacher can capitalise on the potential of worthwhile mathematical tasks is to engage students in justification, abstraction, and generalisation (see chapter 4), by doing which they learn to speak the language of mathematics.

The image of effective mathematical pedagogy that emerges from Anthony and Walshaw's synthesis is of teaching as a coherent system rather than a set of discrete, interchangeable strategies. This pedagogical system encompasses:

- a non-threatening classroom atmosphere;
- instructional tasks;
- tools and representations;
- classroom discourse.

To see that these four aspects of effective pedagogy constitute a system, note that the way in which instructional tasks are realised in the classroom and experienced by students depends on the classroom atmosphere, the tools and representations available for them to use, and the nature and focus of classroom discourse. And because effective pedagogy is a system, it makes little sense to think of student learning as being *caused* by isolated teacher actions or strategies. It is for this reason that Anthony and Walshaw speak of mathematical learning being *occasioned* by teaching. In using this term, Anthony and Walshaw emphasise the teacher's proactive role in supporting students' development of increasingly sophisticated forms of mathematical reasoning.

In addition to highlighting the systemic character of effective mathematical pedagogy, Anthony and Walshaw make good on the charge to develop an ecological model of learning that links what is happening in the classroom to issues of race, culture, and language, and to the school contexts in which teachers develop and revise their instructional practices. A concern for issues of equity permeate the entire report but come to the fore in the discussion of school-home partnerships that take the diverse cultures of students and their families seriously and treat them as instructional resources.

Anthony and Walshaw make it clear that it is essential to view school contexts as settings for teachers' ongoing learning. In a very real sense, these settings mediate the extent to which high quality teacher professional development will result in significant changes in teachers' classroom practices. Anthony and Walshaw's synthesis documents that mathematics instruction that places students' reasoning at the center of instructional decision making is demanding, uncertain, and not reducible to predictable routines. The available evidence indicates that a strong network of colleagues constitutes a crucial means of support for teachers as they attempt to cope with these uncertainties and the loss of established routines. Consequently, there is every reason to expect that improvement in teachers' instructional practices and student learning will be greater in schools where mathematics teachers participate in learning communities whose activities focus on central mathematical ideas and how to relate them to student reasoning. The value of teacher learning communities in turn foregrounds the critical role of the principal as an instructional leader.

Historically, teaching and school leadership have been loosely coupled, with the classroom being treated as the preserve of the teacher while school leaders managed around instruction. Recent research findings demonstrate the limitations of this type of school organisation

in supporting the improvement of teaching on any scale. These findings also indicate that principals can play a key role in supporting the emergence of a shared vision of what effective mathematical pedagogy looks like and in supporting teacher collaboration that focuses on challenges central to the development of effective pedagogy. This alternative type of school organisation is characterised by reciprocal accountability. Teachers are accountable to principals for developing increasingly effective pedagogical practices and principals are accountable to teachers to create opportunities for their ongoing learning. Changes of this type in the relations between teachers and school administrators are far reaching and might be viewed as too radical. It is, however, sobering to note that previous large-scale efforts to improve the quality of classroom instruction have rarely produced lasting changes in teachers' practices. Research into educational leadership and policy indicates that this history is due in large part to the failure to take into account the institutional settings in which teachers develop and refine their instructional practices.

The broader policy and leadership literature strongly indicates that the improvement of mathematics instruction on the scale being attempted in New Zealand is not simply a matter of providing high quality teacher professional development. It also has to be framed as a problem for schools as educational organisations that structure the institutional settings in which teachers develop and revise their instructional practices. My reading of this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau is that Anthony and Walshaw have distilled valuable lessons from the available research, thereby positioning New Zealand educators to succeed where others have failed.

Paul Cobb

Professor of Mathematics Education
Vanderbilt University, Tennessee

Note: The second Hans Freudenthal Medal of the International Commission on Mathematical Instruction (ICMI) was awarded to Professor Paul Cobb in 2005, "whose work is a rare combination of theoretical developments, empirical research and practical applications. His work has had a major influence on the mathematics education community and beyond."

Early Childhood Education

This Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau is a 'must read' for those in the early childhood sector who want an insight into what effective mathematical pedagogy looks like in an early childhood service. The synthesis acknowledges the vital role that quality early childhood education plays in the mathematical development of infants and young children. It also provokes early childhood teachers to reflect on practice: their mathematical awareness of the environment, the depth of their mathematical knowledge, and the importance of effective teaching and learning strategies that will support children's optimal engagement in mathematical experiences. The extensive, wide-ranging research information is effectively balanced by vignettes which involve the reader in meaningful mathematical experiences that illustrate the possibilities for supporting mathematical learning. Effective distribution of the synthesis would enhance teaching and learning outcomes in early childhood services.

Faith Martin
Director, Massey Child Care Centre

NZEI Te Riu Roa

NZEI Te Riu Roa welcomes this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau, particularly as it takes for its starting point the assertion that “all children can learn mathematics”. This key message is at the heart of every teacher’s commitment to the mathematical learning of his or her students.

The synthesis recognises the complexity of teaching, particularly given the diverse learning needs of the students in our classrooms and centres and the necessity for specialised knowledge of mathematics. But the writers consistently underline the power that teachers have to make a difference: “It is what teachers do, think and believe (that) significantly influences student outcomes.”

A teacher’s role, whether in a school or a centre, includes the design of activities that help students to construct meaning and think for themselves. To achieve such outcomes, teachers need to appreciate the part that mathematics plays in the world around them, what the big mathematical ideas are, and how the concepts that they teach fit in with those ideas. They need to know how to teach knowledge and skills, how to match new learning with students’ prior knowledge, and which activities effectively encourage understanding and learning. Teachers also need to be conscious of developing attitudes and values. They need to create opportunities for their students to develop a critical eye and, in the context of this synthesis, a critical mathematical eye.

The primary purpose of the synthesis is to identify evidence that links pedagogical practice with effective mathematics outcomes for students. To achieve this, the writers have drawn on national and international research that contributes to our understanding of what works in mathematics education.

When reviewing the synthesis in its draft form, NZEI teachers were particularly pleased to read the chapter, *Mathematics Practices Outside the Classroom*, which they saw as contributing to a constructive environment and encouraging of good practice. The synthesis explores ways in which parents can contribute to their children’s mathematical development and ways in which schools can strengthen links with the home. If teachers are to successfully fulfil expectations, such links are likely to be vital. Teachers were also pleased to see the importance of school leadership recognised.

NZEI sees the *Effective Pedagogy in Mathematics/Pāngarau BES* as being of great benefit to teachers, teacher educators, and policymakers. The research identified in the synthesis, together with the case studies and vignettes, has the potential to stimulate much constructive professional discussion. To maximise its potential for teachers, it will need to be accompanied by professional learning opportunities and time for reflection and discussion in the school or centre setting.

Irene Cooper
National President
Te Manukura
NZEI Te Riu Roa

Post Primary Teachers' Association

Tēnā koutou, tēnā koutou, tēnā tatou katoa.

PPTA welcomes this Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau. It is the result of a very thorough process, inclusive of the expertise of practitioners. The final report reflects and caters to their realities, and provides some very interesting and thought-provoking reading for teachers themselves, and for those involved in the pre-service and in-service education of mathematics teachers. At the same time, the research highlights the shortage of outcomes-linked research evidence specific to secondary school mathematics teaching and we hope that as a result of this BES, New Zealand researchers will step up to fill this gap.

Debbie Te Whaiti
President
New Zealand Post Primary Teachers' Association

Teacher Educators

The Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau succeeds in providing a systematic treatment of relevant outcomes-based evidence for what works for diverse learners in the New Zealand education system. One of the strengths of the document is the central positioning given by its authors to a broad notion of diversity.

Teacher educators, both initial and ongoing, will find that the BES is an invitation to engage—as teachers and as researchers—with a wide range of national and international studies. The document succeeds in preserving the complexity of pedagogical approaches through careful structuring and presentation. Well chosen classroom vignettes capture the essence of pedagogical issues for use in initial and ongoing teacher education. The CASEs are likely to prove particularly valuable for teachers by demonstrating how research can inform classroom practice.

The BES also presents a challenge to New Zealand researchers by identifying areas in which there is a paucity of outcomes-based evidence. Such evidence is scarce for Māori-medium mathematics classrooms. The senior secondary area is generally not well represented and a wider range of early childhood contexts needs to be investigated. The CASEs highlight for teacher educators the possibilities of writing up research projects undertaken as part of ongoing teacher education initiatives, and encourage them to gather further evidence to support practice.

The importance to mathematics education of the outcomes-based research evidence represented in this synthesis cannot be overstated. It is to be hoped that the value of the Iterative BES programme is widely recognised, and that it has the impact on policy and practice that it ought.

Joanna Higgins
Director, Mathematics Education Unit and Associate Director,
Jessie Hetherington Centre for Educational Research
Victoria University of Wellington

Māori-medium Mathematics

E nga mana, e nga reo, tēnā koutou katoa.

For the last 20 years, the teaching of pāngarau (mathematics) has played a significant role in the revitalisation of te reo Māori. The Effective Pedagogy in Mathematics/Pāngarau BES recognises the close relationship that exists between language and the learning and teaching of mathematics.

The BES identifies a range of major considerations and challenges for teachers and all those involved in Māori-medium education. The research makes it clear that mathematical outcomes for students are affected by a complex network of interrelated factors and environments, not just individual preferences or the language of instruction. By identifying the key elements in this network and discussing the relevant research, the writers have created what should prove a very useful resource.

The BES highlights the paucity of research into Māori-medium mathematics education, particularly in the area of teacher practice.

Tony Trinick
Māori-medium mathematics educator
Faculty of Education
The University of Auckland

Pasifika

E rima te'arapaki, te aro'a, te ko'uko'u te utuutu, 'iaku nei.

Under the protection of caring hands there's feeling of love and affection.

The Best Evidence Synthesis of Effective Pedagogy in Mathematics/Pāngarau has drawn together a comprehensive synthesis of evidence that relates to quality mathematics pedagogical practices. Its particular strength is that it provides stimulating and thought-provoking reading for a range of stakeholders and at the same time affirms that there is no one, specific, 'quality' pedagogical approach. Rather, it directs attention to many effective approaches which make a difference for all mathematics learners. The vignettes are an added strength; they make the theoretical structures they illustrate accessible to a wider audience.

The synthesis highlights the shortage of outcomes-linked research evidence concerning quality teaching and learning for Pasifika students at all levels of schooling. It also highlights the importance of a culture of care. How this translates into quality outcomes for Pasifika students requires the attention of New Zealand researchers.

Roberta Hunter
Senior Lecturer
School of Education Studies
Massey University, Albany Campus

Chief Education Adviser, BES

The Effective Pedagogy in Mathematics/Pāngarau BES sets out to uncover and explain the links between what we do in mathematics education and what the outcomes are for learners. The result is a valuable resource that can be used to enhance a wide range of outcomes for diverse learners. These include the ability to think creatively, critically, strategically and logically; mathematical knowledge; enjoyment of intellectual challenge; self-regulatory, collaborative and problem-solving skills; and the disposition to use, enjoy and build upon that knowledge throughout life.

The BES reflects the outstanding scholarly work and professional leadership of co-authors Drs Glenda Anthony and Margaret Walshaw of Massey University. They are the first to use the new *Guidelines for Generating a Best Evidence Synthesis* and follow the collaborative development process that is central to the Iterative BES Programme. They have consulted tirelessly and responsively with a wide range of early childhood teachers, primary and secondary teachers, principals, advisers, researchers, policy workers and teacher educator colleagues from across New Zealand, and with international colleagues. The Ministry of Education acknowledges and values all these contributions—and those of the formative quality assurers, whose affirmations and challenges have been so helpful in optimising the quality and potential usefulness of this BES.

The BES celebrates and returns to early childhood educators, teachers, teacher educators and researchers a record of their professional work, highlighting the complexity of that work, and suggesting how research evidence can be a valuable resource to inform their ongoing work and that of their colleagues. From the first vignette explaining how mathematical learning can be embedded in waiata (Māori song) and dance, the vignettes bring children's learning in mathematics to life. The underlying explanations and theoretical findings have the power to inform practice in ways that are relevant and responsive to the learners in any particular centre or classroom.

The challenge now is for us all is to use this resource in ways that will support further systemic development in mathematics education, with strengthened outcomes for diverse learners. In many cases, the BES will affirm what is already happening, but it will be the points of challenge that take us forward. Individual teachers have already engaged with the BES in its draft form, and some report remarkable insights and developments in their practice. But it is only through the wider and systemic development of the conditions that support effective practice for diverse learners that improvements will proliferate and become self-sustaining. The findings emerging from the outcomes-linked professional learning and development BESs^{1,2} should be an invaluable resource in determining how to generate changed practice on such a scale.

Many teachers and early childhood educators have indicated that they want to read this BES for themselves, and to do this they need time. They need time to read, discuss and consider how they can use relevant BES findings in response to diagnostic information about the mathematical understandings of the children and young people they teach. They also need time to participate in professional learning communities. The Teacher Professional Learning and Development BES³ finds that such participation doesn't guarantee better outcomes for students, but it is a consistent feature of teacher professional learning that does have a strong positive impact.

The same BES highlights the important role that external expertise with strong pedagogical content knowledge can play in facilitating and supporting changes in practice that impacts positively on student outcomes. Such expertise can be vital in engaging teachers' theories and challenging problematic discourses. The findings do, however, caution that 'experts' need more than good intentions—in the worst-case scenario, teacher professional development can actually impact negatively on student achievement. This finding calls for careful and iterative evaluation of the effectiveness of all professional development.

The teacher education community in New Zealand has already made a foundational contribution to this BES with its engagement in the research and development reported in this BES, and its advice to the BES writers. As the Teacher Professional Learning and Development BES⁴ will show, some of our most effective professional development has been taking place as part of the Numeracy Development Projects (NDP)—with effect sizes twice those attained in England^{5,6,7}. The primary and early childhood teachers’ union, NZEI, confirms what the evaluation reports have been saying: that teachers who have been involved in the NDP value the transformational experiences this professional learning has afforded them. Two teachers from a Hawkes Bay school explained to me recently that, as a result of professional learning undertaken through the NDP, they have changed the way they work across the curriculum—they now listen more, are more diagnostic, and they place much more emphasis on children articulating and sharing their learning strategies. The dynamic, reflective, nation-wide learning community of researchers, teacher educators, teachers, and other educators created by the NDP and its Māori-medium counterpart, Te Poutama Tau, has been inspirational for BES.

If the mathematics BES is to serve New Zealand education well, the teacher education and research communities must make it a ‘living’ BES by building on the powerful insights and exemplars it makes available, addressing the gaps, and ensuring a cumulative and increasingly dynamic shared knowledge base about what works for learners in New Zealand education. To assist in this collaborative work, the New Zealand Council for Educational Research is creating a database of relevant New Zealand education theses. It has already built a database to support this document, with live links to the electronic version so that readers can quickly access either the full text or bibliographic details for some of the most helpful articles that have informed the synthesis. These links are also listed in the print version.

It is our hope that this BES will stimulate readers to let the Iterative Best Evidence Synthesis Programme know of other/new research and development that should feature in future iterations of the synthesis. Such research needs to clearly document demonstrated or triangulated links to student outcomes (see the Guidelines for Generating a Best Evidence Synthesis Iteration, found on the BES website⁸), and preferably show larger positive impacts on desired outcomes for diverse learners. We are especially seeking studies of research and development in New Zealand contexts, but we are also interested in information on overseas studies that show particularly large impacts on diverse learners. Please send details to best.evidence@minedu.govt.nz.

In the New Zealand context, where schools and centres are self managing, principals and centre leaders have a critical role to play in supporting their staff to realise the potential of this BES. The Teacher Professional Learning and Development BES indicates that, in the case of the most effective school-based interventions, principals and others in leadership roles have actively supported the development of a learning culture amongst their teachers.

For centuries, societies have required their education systems to sort children into successes and failures. Knowledge societies, such as our own, require much more. Our challenge is to ensure that all our children flourish as learners, strong in their own identities, and confident global citizens.

To achieve such goals, we need to value, build upon, and go beyond the craft practice traditions that require each teacher to ‘rediscover the wheel’. The Effective Pedagogy in Mathematics/Pāngarau BES has been designed to serve as a resource and catalyst for strengthened practice, innovation, and systemic learning. By using it, and by making learner outcomes our touchstone, we can work together to give our children a mathematics education that prepares them well for the opportunities and challenges that will be their future.

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- ² Mitchell, L., & Cubey, P. (2004). *Characteristics of Professional Development Linked to Enhanced Pedagogy and Children’s Learning in Early Childhood Settings: Best Evidence Synthesis Iteration*. Wellington, NZ: Ministry of Education. educationcounts.edcentre.govt.nz/goto/BES
- ³ Ibid.
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- ⁵ Young-Loveridge, J. (2006). Patterns of Performance and progress on the Numeracy Development Project: Looking back from 2005. In *Findings from the Numeracy Development Projects 2005* (F. Ell, J. Higgins, K.C. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge). Wellington, NZ: Ministry of Education.
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- ⁷ Earl, L., Watson, N., Levin, B., Leithwood, K., Fullan, M., Torrance, N., Jantzi, D., Mascal, B., & Volante, L. (2003). *Watching and learning 3: Final report of the external evaluation of England’s National Numeracy and Literacy Strategies*. London, UK: DfES
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Authors' Preface

What is a Best Evidence Synthesis in Mathematics?

A best evidence synthesis draws together available evidence about what pedagogical approaches work to improve student outcomes in Mathematics/Pāngarau. This synthesis is part of the Iterative Best Evidence Synthesis (BES) Programme established late in 2003 by the Ministry of Education to deepen understanding of what works in education. The programme involves policy, research, and practice in collaborative knowledge building, aimed at maximising desirable outcomes for the diverse learners in the New Zealand education system.

This best evidence synthesis in Mathematics/Pāngarau plays a key role in knowledge building for New Zealand education. As a capability tool, it identifies, evaluates, analyses, and synthesises what the New Zealand evidence and international research tell us about quality mathematics teaching. It shows us how different contexts, systems, policies, resources, approaches, practices, and influences all impact on learners in different ways. Importantly, it illuminates what the evidence suggests can optimise outcomes for diverse mathematics learners.

The importance of dialogue

The development of this BES has been shaped by the *Guidelines for Generating a Best Evidence Synthesis Iteration* (Alton-Lee, 2004) and informed by dialogue amongst policy makers, educators, researchers and practitioners. Right from the very early stages of its development, the health-of-the-system perspective taken in this synthesis has ensured that we have listened to and responded to the viewpoints of a wide range of constituencies. Our interactions with these multiple communities have revealed to us the key roles that infrastructure, context, settings, and accountabilities play in a system that is functioning effectively for all its learners. Our various stakeholders have challenged us not only to produce better and more relevant educational research but to consider how this knowledge base might best be used. It is our hope that this discussion across sectors will be ongoing.

We have received a strong and positive response to the best evidence synthesis work from New Zealand's primary and post-primary teacher associations. Both have reported on how helpful the synthesis is to their core professional work. For example, the New Zealand Educational Institute (NZEI) writes: "In our view, the writers have drawn on national and international research which contributes to an understanding of what works in mathematics education; they have identified the significance of the context and ways in which to strengthen practice ... We liked the ... underpinning view that all children can learn mathematics" (p. 2). The representative for the Post Primary Teachers' Association at the Quality Assurance Day is reported as saying: "There are numerous wonderful ideas in the synthesis, and I found myself repeatedly jolted into possibilities for my own classroom resources." In addition, a group of initial and ongoing mathematics teacher educators have welcomed the "sophisticated treatment of diversity" and the way in which "the complexity of pedagogical approaches is preserved" (Victoria University of Wellington College of Education, 2006, p. 1).

Writing for multiple audiences

Our task was to make the findings of the synthesis accessible to and of benefit to a range of educational stakeholders. At one level of application, it is intended to provide a strengthened basis of knowledge about mathematics pedagogical practices in New Zealand today. The evidence it produces is expected to inform teacher educators within the discipline of mathematics education about effective pedagogical practice. At another level, the synthesis attempts to make transparent to policy makers and social planners an evidential basis for quality pedagogical approaches in mathematics. At a third level, the synthesis is expected to benefit practitioners and assist them in doing the best possible job for diverse learners in their classrooms.

Our approach to the “almost overwhelming task” (Cobb, 2006) of writing with several levels of application in mind has been to draw on both formal and informal approaches. We have offset the ‘academic’ language of the BES by including a series of vignettes that expand upon broad findings. We have received feedback from a range of sources that these vignettes bring the reality of classroom life to the fore and, in particular, do not minimise the complexities of actual practice. We hope that researchers, policy makers and practitioners alike will see in the vignettes theoretical tools that have been adapted and used by actual teachers.

The BES as a catalyst for change

This best evidence synthesis in mathematics does more than synthesise and explain evidence about what works for diverse learners. By bringing together rigorous and useful bodies of evidence about what works in mathematics, the project plays an important function as a catalyst for change. It is designed to help strengthen education policy and educational development in ways that effectively address both the needs of diverse learners and patterns of systemic underachievement in New Zealand education. It is written with the intent of stimulating activity across practitioners, policy makers, and researchers and so to strengthen system responsiveness to educational outcomes for all students.

The writers anticipate that reflection on the findings will lead to sustainable educational development that has a positive impact on learners. It will create new insights into what makes a difference for our children and young people. Reflection on the findings will also spark new questions and renewed, fruitful engagement with mathematics education. These new questions, in turn, will render the BES a snapshot in time—provisional and subject to future change.

Key features

Key features of the BES are:

- Its teacher orientation. Its view is towards a strengthened basis of knowledge about instructional practices that make a difference for diverse groups of learners.
- Its cross-sectoral approach. Its scope takes in the teaching of children in early childhood centres through to the teaching of learners in senior secondary school classrooms.
- Its inclusiveness. It documents research that reveals significant educational benefits for a wide range of diverse learners. It pays particular attention to the mathematical development of Māori and Pasifika students and documents research that captures the multiple identities held by New Zealand learners.
- Its breadth of search coverage. It reports on the characteristics of effective pedagogy, following searches through multiple national databases and inventories as well as masters’ projects and theses. It provides comprehensive information about effective teaching as evidenced from small cases, large-scale explorations, and short-term and longitudinal investigations.
- Its local character. It makes explicit links between claims and bodies of evidence that have successfully translated the intentions and spirit of the Treaty of Waitangi. It identifies research relevant to the particular conditions and contexts in New Zealand, both in mathematics education in particular and in education in general, in relation to the principles and goals of *Te Whāriki* for early childhood settings and of *The Curriculum Framework*, for teachers in English or Māori-medium settings.
- Its global linkages. It connects local sources with the international literature. It identifies important Australian and international work in the area and evaluates that wide-ranging resource in relation to similarities and differences in cultures,

populations and demographics between the country of origin and New Zealand.

- Its responsiveness to concerns about democratic participation. It heeds the concern about the development of competencies that equip students for lifelong learning. This orientation coincides with the national mathematics curriculum objective of developing those knowledges, skills, and identities that will enable students to meet and respond creatively to real-life challenges.
- Its quality assurance measures. It is guided by principles of transparency, accessibility, relevance, trustworthiness, rigour, and comprehensiveness. These principles form the backdrop to the selection and systematic integration of evidence.
- Its strategic focus on policy and social planning. It uses a health-of-the-system approach to address one of the most pressing problems in education, provide a direction for future growth, and push effective teaching beyond current understandings.
- Its provisional nature. The project is an important knowledge-building tool, creating new insights from what has gone before, and will be updated in the light of findings from new studies. The findings are, above all, 'of the moment' and open to future change.

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Executive Summary

The *Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration [BES]* was funded by a Ministry of Education contract awarded to Associate Professor Glenda Anthony and Dr Margaret Walshaw at Massey University. The synthesis is part of the Iterative Best Evidence Synthesis (BES) Programme, established by the Ministry of Education in New Zealand, to deepen understanding from the research literature of what is effective in education for diverse learners. The synthesis represents a systematic and credible evidence base about quality teaching in mathematics and explains the sort of pedagogical approaches that lead to improved engagement and desirable outcomes for learners from diverse social groups. It marks out the complexity of teaching and provides insight into the ways in which learners' mathematical identities and accomplishments are occasioned by effective pedagogical practices.

The search of the literature focused attention on different contexts, different communities, and multiple ways of thinking and working. Priority was given to New Zealand research into mathematics in early childhood centres and schools, both English- and Māori-medium. Personal networks enhanced the library search and facilitated access to academic journals, theses and reports, as well as other local scholarly work. The New Zealand literature was complemented by reputable work undertaken in other countries with similar population and demographic characteristics. Indices, both print and electronic, were sourced, and the search covered relevant publications within the general education literature as well as specialist educational areas. In the end, 660 pieces of research, ranging from very small, single-site studies to large scale, longitudinal, experimental studies, found their way into the report.

Key findings highlight practices that relate specifically to effective mathematics teaching and to positive learning and social outcomes in centres/kōhanga and schools/kura. The findings stress the importance of interrelationships and the development of community in the classroom. They also reveal that both the cognitive and material decisions made by teachers concerning the mathematics tasks and activities they use, significantly influence learning. The findings demonstrate the importance of children's early mathematical experiences and stress that constituting and developing children's mathematical identities is a joint enterprise of teacher, centre/school, and family/whānau.

Key findings

In this section, key findings are organised and presented according to five themes: the key principles underpinning effective mathematics teaching, the early years, the classroom community, the pedagogical task and activity, and educational leadership and centre–home and school–home links.

Key principles underpinning effective mathematics teaching

Teachers who enhance positive social and academic outcomes for their diverse students are committed to teaching that takes students' mathematical thinking seriously. Their commitment to students' thinking is underpinned by the following principles:

- an acknowledgement that all students, irrespective of age, have the capacity to become powerful mathematical learners;
- a commitment to maximise access to mathematics;
- empowerment of all to develop mathematical identities and knowledge;
- holistic development for productive citizenship through mathematics;
- relationships and the connectedness of both people and ideas;
- interpersonal respect and sensitivity;
- fairness and consistency.

The early years

Young children are powerful mathematics learners. Quality teaching guarantees the development of appropriate relationships and support as well as an awareness of children's mathematical understanding. Research has consistently demonstrated how a wide range of children's everyday activities, play and interests can be used to engage, challenge and extend children's mathematical knowledge and skills. Researchers have found that effective teachers provide opportunities for children to explore mathematics through a range of imaginative and real-world learning contexts. Contexts that are rich in perceptual and social experiences support the development of problem-solving and creative-thinking skills.

There is now strong evidence that the most effective settings for young learners provide a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive, play activities. Opportunities for learning mathematics typically arise out of children's everyday activities: counting, playing with mathematical shapes, telling time, estimating distance, sharing, cooking, and playing games. Teachers in early childhood settings need a sound understanding of mathematics to effectively capture the learning opportunities within the child's environment and make available a range of appropriate resources and purposeful and challenging activities. Using this knowledge, effective teachers provide scaffolding that extends the child's mathematical thinking while simultaneously valuing the child's contribution.

The classroom community

Research has shown that opportunities to learn depend significantly on the community that is developed within centres and classrooms. Thus, people, relationships, and classroom environments are critically important. Whilst all teachers care about student engagement, research quite clearly demonstrates that pedagogy that is focused solely on the development of a trusting climate does not get to the heart of what mathematics teaching truly entails. Teachers who truly care about their students have high yet realistic expectations about enhancing students' capacity to think, reason, communicate and reflect upon their own and others' understandings. Research has provided conclusive evidence that effective teachers work at developing inclusive partnerships, ensure that the ideas put forward by learners are received with respect and, in time, become commensurate with mathematical convention and curricular goals.

Studies have provided conclusive evidence that teaching that is effective is able to bridge students' intuitive understandings and the mathematical understandings sanctioned by the world at large. Language plays a central role. Mathematical language involves more than vocabulary and technical usage; it encompasses the ways that expert and novice mathematicians use language to explain and to justify concepts. The teacher who has the interests of learners at heart ensures that the home language of students in multilingual classroom environments connects with the underlying meaning of mathematical concepts and technical terms. Teachers who make a difference are focused on shaping the development of novice mathematicians who speak the precise and generalisable language of mathematics.

Mathematics teaching for diverse learners creates a space for the individual and the collective. Whilst many researchers have shown that small-group work can provide the context for social and cognitive engagement, others have cautioned that students need opportunities and time to think and work quietly away from the demands of a group. There is evidence that some students, more than others, appear to thrive in class discussion groups. Many students, including limited-English-speaking students, are reluctant to share their thinking in class discussions. Research has also shown that an over-reliance on grouping according to attainment is not necessarily productive for all students. Teachers who teach lower streamed classes tend to follow a protracted curriculum and offer less varied teaching strategies. This pedagogical

practice may have a detrimental effect on the development of a mathematical disposition and on students' sense of their own mathematical identity.

Pedagogical tasks and activities

From the research, it is evident that the opportunity to learn is influenced by what is made available to learners. For all students, the 'what' that they do is integral to their learning. The 'what' is the result of sustained integration of planned and spontaneous learning opportunities made available by the teacher. The activities that teachers plan, and the sorts of mathematical inquiries that take place around those activities, are crucially important to learning. Effective teachers plan their activities with many factors in mind, including the individual student's knowledge and experiences, and the participation norms established within the classroom. Extensive research in this area has found that effective teachers develop their planning to allow students to develop habits of mind whereby they can engage with mathematics productively and make use of appropriate tools to support their understanding.

Choice of task, tools, and activity significantly influences the development of mathematical thinking. Quality teaching at all levels ensures that mathematical tasks are not simply 'time fillers' and is focused instead on the solution of genuine mathematical problems. The most productive tasks and activities are those that allow students to access important mathematical concepts and relationships, to investigate mathematical structure, and to use techniques and notations appropriately. Research provides sound evidence that when teachers employ tasks for these purposes over sustained periods of time, they provide students with opportunities for success, they present an appropriate level of challenge, they increase students' sense of control, and they enhance students' mathematical dispositions.

Effective teaching for diverse students demands teacher knowledge. Studies exploring the impact of content and pedagogical knowledge have shown that what teachers do in classrooms is very much dependent on what they know and believe about mathematics and on what they understand about the teaching and learning of mathematics. Successful teaching of mathematics requires a teacher to have both the *intention* and the *effect* to assist pupils to make sense of mathematical topics. A teacher with the intention of developing student understanding will not necessarily produce the desired effect. Unless teachers make good sense of the mathematical ideas, they will not have the confidence to press for student understanding nor will they have the flexibility they need for spotting the entry points that will move students towards more sophisticated and mathematically grounded understandings. There is now a wealth of evidence available that shows how teachers' knowledge can be developed with the support and encouragement of a professional community of learners.

Educational leadership and links between centre and home/school

Facilitating harmonious interactions between school, family, and community contributes to the enhancing of students' aspirations, attitudes, and achievement. Research that explores practices beyond the classroom provides insight into the part that school-wide, institutional and home processes play in developing mathematical identities and capabilities. There is conclusive evidence that quality teaching is a joint enterprise involving mutual relationships and system-level processes that are shared by school personnel. Research has provided clear evidence that effective pedagogy is founded on the material, systems, human and emotional support, and resourcing provided by school leaders as well as the collaborative efforts of teachers to make a difference for all learners.

Teachers who build whānau relationships and home–community and school–centre partnerships go out of their way to facilitate harmonious interactions between the sectors. There is convincing evidence to suggest that these relationships influence students' mathematical development. The home and community environments offer a rich source of mathematical experiences on which to build centre/school learning. Teachers who collaborate with parents, families/whānau and

community members come to understand their students better. Parents benefit too: through their purposeful involvement in school/centre activities, by assisting with homework, and in providing suitable games, music and books, they develop a greater understanding of the centre's or school's programme. Their involvement also provides an opportunity to scaffold the learning that takes place within the centre or school.

Overall key findings

This Best Evidence Synthesis examines the links between pedagogical practice and student outcomes. Consistent with recent theories of teaching and learning, it finds that quality teaching is not simply a matter of 'knowing your subject' or 'being born a teacher'.

Sound subject matter knowledge and pedagogical content knowledge are prerequisites for accessing students' conceptual understandings and for deciding where those understandings might be heading. They are also critical for accessing and adapting task, activities and resources to bring the mathematics to the fore.

The importance of building home–community and school–centre partnerships has been highlighted in a number of studies of effective teaching.

Early childhood centre researchers have provided evidence that the most effective settings offer a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive, play activities.

Within centres and classrooms, effective teachers care about their students and work at developing interrelationships that create spaces for learners to develop their mathematical and cultural identities.

Extensive research on task and activity has found that effective teachers make decisions on lesson content that provide learners with opportunities to develop their mathematical identities and their mathematical understandings.

Studies have provided conclusive evidence that teaching that is effective is able to bridge learners' intuitive understandings and the mathematical understandings sanctioned by the world at large.

Gaps in the literature and directions for future research

The synthesis provides research information about effective mathematics teaching. Although the scope of researchers' studies is broad and far-reaching, a number of gaps in the literature are apparent. Research has so far provided only limited information about effective teaching in New Zealand at the secondary school level. Additionally, there is little reported research that focuses on quality teaching for Pasifika students. Few researchers in New Zealand are exploring mathematics in early childhood centres. The New Zealand literature lacks longitudinal, large-scale studies of teaching and learning. Also missing are studies undertaken in collaboration with overseas researchers. Such research is crucially important for understanding teachers' work and the impact of curricular change. The scholarly exchange of ideas made possible through joint projects with the international research community contributes in numerous ways to the capability of our local researchers.

It is important to keep in mind that, as a knowledge building tool, the synthesis provides insights based on what has gone before. A snapshot in time, it is subject to change as new kinds of evidence about quality teaching become available. Important mathematics initiatives are underway in New Zealand schools and centres. The Numeracy Development Projects, new assessment methods, projects involving information technology, and a greater focus on statistics in the curriculum are just three examples of changes that are currently taking place. All new initiatives require research that monitors and evaluates their introduction and 'take up' by centres/schools and the changes in teaching and learning that take place as a result. Such research is necessary to guide future directions in schools, educational policy, and curriculum design.

6. Mathematics Practices outside the Classroom

Quality pedagogical practice is constituted in social practices (Wenger, 1998). The sorts of practices that teachers participate in to enhance students' cognitive capacities are, of course, not confined to the classroom. Students' mathematical identities and capacities are shaped in ongoing ways by interrelationships between the teacher and the family, whānau, wider school, and professional organisations in which the teacher is involved (Alton-Lee, 2003). These relationships that exist beyond the classroom, and the systems and practices associated with them, provide us with another context for exploring how mathematical knowledge and identities are developed.

Teachers' engagement with mathematics is significantly influenced by the opportunities made available to them within the wider school. In this chapter, we look first at how mathematics teachers practise their profession in ways that are responsive to the school leadership, structures, and organisational processes. We explore the role that school-wide relationships, systems, and professional development initiatives play in the core dimensions of effective classroom practice. Secondly, we investigate the ways in which quality teaching is enhanced by factors outside the school. As documented earlier, home and community represent the two major domains in which the development of young children takes place. We investigate collaborative partnerships between schools and various groups of people from the home and community, "work[ing] together to create better programmes and opportunities for students" (Epstein, 1995, p. 701). What will become apparent are the ways in which links forged between the home culture and the school environment can have beneficial effects on student attitude and academic achievement.

The school

Mathematics instructional practices are informed by teachers' active engagement with processes and people. Precisely because teaching takes place within nested systems of people and structures and develops "in situations where the available information is often partial or incomplete and where the consequences of actions are not always immediate" (Doerr & Lesh, 2002, p. 132), institutional settings are fundamental to the way teachers enact pedagogical practice in the classroom. As such, the school operates as a community of practice in shaping how mathematics is taught and learned.

Within this community, teachers are professionals who modify and transform their pedagogical practice in a generative fashion; they accommodate mandated curriculum in relation to the system-level processes at the school (Spillane, 1999). Teachers are *interpreters and adapters* of curriculum (Shulman & Shulman, 2004; Walshaw, 1995) within the constraints and affordances offered by the school. Goos and Jolley (2004) document how one school governed the ways in which mathematics was taught: its school-wide approach was founded on the common objective of enhancing student learning and placed a strong emphasis on mathematical thinking.

A School-wide Mathematics Philosophy

The school promoted pedagogical effectiveness through its consistent classroom philosophy to mathematics across levels, from kindergarten onwards. Specifically, the school emphasised the strategic use of mathematical knowledge. In junior classrooms teachers emphasised mathematical concepts such as place value and patterning through number and word games. They also encouraged students to make and verify conjectures. For example, pre-school teachers introduced the idea of estimation by asking the children to guess the number of small items in a jar and then think about how they could verify their guess. The students were then asked to compare estimates for two jars of different dimensions. Ways of verifying their results included using the objects to create bar graphs,

as well as simply counting. Throughout, the teachers talked about mathematical thinking, drawing on past experience with similar tasks, and choosing among a variety of acceptable ways to solve the problem.

By the time the students reached year 5 they appeared to be comfortable with the idea that mathematical activity entails attention to strategies and reflection on learning. This was evident in the way they were required to set out their workbooks to make their problem solving strategies explicit. Students at the school were expected to write notes in a double page titled 'Mathematical Thinking'. Sub-headings were included and these were expressed as 'Restatement of question in own words', 'Working', 'Reflection (what did I learn?)', and 'Extension (what if?)'. Across a range of problem solving activities and across the school students used this page as a heuristic to articulate and support their thinking. Evidence of this claim is supported by students' written comments such as: "I think my answer is accurate because I picked up a pattern" and "I learnt that when something is right in front of me I don't have to take the hard way."

From Goos and Jolley (2004)

School leadership

Principals who are respectful of the professional expertise and change intentions of the school's mathematics teaching community significantly influence how reform efforts are implemented (Millett & Johnson, 2004). Coburn (2005) has found that principals, through their greater access to policy messages, directly influence teachers' practice. Principals receive the directives and participate in networking events associated with reform efforts, learning about new materials, approaches, and ideas associated with changing policy" (pp. 499–500). In their discussions with teachers and the provisions they make for learning, principals emphasise certain aspects of curriculum while downplaying others, based on their own understandings.

In a study involving mathematics programmes in New Zealand primary schools, Wood (2003) found that in terms of making professional development opportunities available to staff and providing support to teachers engaged in professional development programmes, the support of principals was critical. Principals in the study consistently reported tensions when trying to balance individual teachers' needs for personal growth with whole-school improvement priorities. Time was a constant theme. For the principals and their teachers, lack of it was a barrier to both the planning and the implementation of new programmes. One principal spoke of it in this way: "That whole thing of having enough time for the amount of information that you are expected to digest and then to actually put that information into practice" (p. 107).

From research, we know that the school leadership team is a central engine of school curricular development (Spillane, 2005). The team has also been shown to be an important factor in teachers' implementation of standards-based assessment. Hipkins et al. (2004) provide evidence that when schools set aside school time for professional discussion and course development, the tasks that accompanied and followed the implementation of NCEA were deemed more manageable by the school. The Education Review Office (ERO, 2004) reported that most teachers valued the time allocated for professional development in connection with NCEA and valued the extra professional dialogue with colleagues.

Continuity within the leadership team has been shown to influence teachers' practice. In her work with disenfranchised students in the US, Gutierrez (2004) points out that teachers who teach mainly socially disadvantaged students, with whom they lack shared life experiences, often have to deal with frequent turnover of leadership and students as well as low staff morale and a disinclination to implement curricular changes. Part of a teacher's practice in these schools "will necessarily entail buffering herself and her students from conflicting practices and maintaining energy to sustain a practice that does not fit with the school context" (p. 3). Gutierrez maintains that a school leadership community that actively contributes to the preparation of students for leadership and active and responsible participation in a democratic society has the following characteristics: "a rigorous and common curriculum, an active

commitment to students, a commitment to a collective enterprise, and innovative instructional practices” (p. 5).

In his investigation into primary school leadership, Spillane (2005) found that school leaders talked with similar emphasis across curriculum areas of leadership functions such as teacher development, programme implementation, and monitoring of teaching practice. But, in his in-depth analysis of leadership practice, he found distinct variations in the way in which leadership is arranged and put into practice in different subject areas. His longitudinal investigation into eight high-poverty schools in the Chicago area revealed that fewer school leaders involved themselves in mathematics-related leadership routines, (e.g., meetings for mathematics teams or to discuss grade levels, curriculum and planning) compared with similar meetings and events for literacy. Typically, school leaders devolved their responsibility for mathematics routines to lead mathematics teachers. Whilst school leaders tended to view the school as the primary change agency for language arts, they looked to external programmes to bring about improvement in mathematics. Similarly, school leaders involved in subject-specific advice networks were less likely to involve themselves in mathematics than literacy.

McClain and Cobb (2004) have documented the critical role that the school leadership team plays in middle grade teacher development. For three and a half years, the researchers worked collaboratively in a school with teachers of 12- to 15-year-old students. They interviewed the teachers to determine their professional development participation, their understanding of the district’s mathematics policies, their accountability lines, their perception of high-stakes test scores, their understanding of assistance given to them and the people who influenced their mathematics teaching. The research found that the school leadership community perceived its role to be one of improving teacher capability in mathematics through the provision of teacher support while keeping an eye on students’ achievement results and teachers’ fidelity to the intent of the curriculum.

In particular, the school provided classroom support by making release time available, resourcing personnel, providing equipment and space, and assisting teachers with professional aspects of their role, such as planning, reflection, and assessment. The leadership team changed the lesson observation forms used in the school so that they became part of a teacher-initiated process instead of a tool for top-down assessment. They rearranged schedules so that teachers could collaborate with each other. They also organised regular meetings between school leaders and mathematics lead teachers to bridge gaps in knowledge and understanding.

In another project involving three schools, Cobb and McClain (in press) looked at how the leadership teams supported teachers in their efforts to improve the quality of teaching and learning. The leaders regularly visited classrooms, focusing specifically on how well the lesson objectives on the boards matched mandated curriculum objectives. They also observed students’ level of engagement and behaviour (Cobb, McClain, Lambert, & Dean, 2003). The leadership teams linked classroom curriculum and lesson structure to test scores, maintaining a watchful eye on student achievement in state-mandated tests. Any fall in test results was brought to the attention of the mathematics leaders. But responsibility for analysing reasons and putting in place remedial strategies was delegated to mathematics leaders. The leadership teams viewed fidelity to the curriculum as evidence of quality pedagogical practice. In one principal’s words, “If you teach the curriculum then the test scores will go up. My job is to make sure they teach the curriculum” (Cobb & McClain, in press, p. 18). In the following vignette, Cobb and McClain expand further on the role of the leadership team.

Active School Leadership

The school leaders have a relatively deep understanding of the general intent of current reform proposals in mathematics education. The team’s vision for mathematics teaching and learning was compatible with that of the leaders of mathematics in their schools. Both viewed teaching as complex and demanding, requiring a deep understanding of both mathematical ideas and students’

mathematical thinking processes. Their agendas involved quality pedagogical strategies as well as the forms that student engagement might take to facilitate outcomes. In turn, classroom teachers who were consistently supported by the leadership team were not reluctant to solicit advice and to develop and refine their practice. In the circumstance when a teacher's practice was perceived to be problematic, the teacher and a member of the school leadership team jointly constructed an improvement plan.

The school leadership team's vision of mathematics teaching and learning developed, in part, as a result of their engagement in numerous professional development seminars during the prior 4 years. In this process the school leaders had experienced teaching consistent with the vision articulated in reform documents. Furthermore they had come to see these competencies as crucial to their role as pedagogical leaders in their schools. For example, the school leaders devoted a proportion of their district-wide biweekly meetings to mathematics. In these settings they completed a pedagogical activity from the curriculum to both develop their own mathematical reasoning and to appreciate better the mathematical intent of the curriculum. These experiences supported their belief that fidelity to the curriculum was the primary means of improving student learning as indicated by test scores.

From Cobb and McClain (2006)

The role of the mathematics lead teacher or head of department in establishing productive teaching communities

The immediate professional community has a marked effect on teacher effectiveness and hence on learner outcomes. It is often the lead mathematics teacher who receives reform directives and who is encouraged to participate in networking events associated with reform efforts or learning about new materials, approaches, and ideas associated with changing policy. Lead teachers bring others 'on board' by working closely with professional development providers. They promote reform ideas by purchasing curriculum materials that focus on one teaching and learning approach to the exclusion of others and by integrating those materials into their mathematics planning.

In a landmark study in the UK, researchers (Millett, Brown, & Askew, 2004) demonstrated that lead mathematics teachers were key players in sustaining a major initiative. In particular, the personal resources of the lead mathematics teacher determined how—and indeed if—reform ideas were taken up by mathematics teachers in a school. Crucially, what was done in classrooms could be attributed in no small way to the lead mathematics teacher's degree of enthusiasm and inclination to make the project work. Effective classroom practice was also attributed to the sharing of ideas and resources amongst teachers. Practical support was important; so too was the emotional support teachers received from each other.

Garden, Wagemaker, and Mooney (1987) report that the support of colleagues and the kind of discussion that takes place during mathematics meetings both have an impact on student outcomes. In pointing out the influences on the performance of the 199 year 9 students who formed the New Zealand sample in the Second International Mathematics Study of the International Association for the Evaluation of Educational Achievement (IEA), Garden et al. highlighted the fact that when teachers centred their mathematics meeting agendas around curriculum content or teaching strategies, their students were more likely to record high gains from pre- and post-testing. Schools that focused on organisational and administrative matters in their meetings were less likely to report high student gains.

Cobb and McClain (in press) report that successful mathematics leaders take the time to analyse the performance of their students. Typically, this analysis is carried out in collaboration with the school leadership team (principal and assistant principal) with the primary purpose of monitoring achievement levels at each grade as well as identifying potential weaknesses in

the curriculum and the adequacy of content coverage. The lead mathematics teachers and leadership teams in the study saw student outcomes as an indicator of both student progress and teacher effectiveness in implementing the curriculum. But it was the mathematics lead teacher who played the key role in conveying curriculum intent to the classroom teachers. “In th[eir] pivotal role as brokers between their own and the other communities, the [lead teachers] had at least partial access to the practices of both the professional teaching and the school leadership community” (McClain & Cobb, p. 285).

In the following vignette, Cobb and McClain provide us with further insights into the critical leadership role played by the lead mathematics teacher in creating effective teacher communities. Teachers in their research drew on their colleagues for support as classroom resources. In placing students and their needs at the centre of their work, they developed and shared instructional materials, they regularly communicated and reflected on students and teaching, they sometimes planned their lessons jointly, they took turns at writing course assignments, and they panel-marked assignments. Pedagogical practice at these schools was to some extent influenced by the relationships and the systems within the institution.

An Effective Teachers’ Learning Group

As part of their process of organising for mathematics teaching, three mathematics teacher leaders conducted biweekly meetings with their teachers working at a particular grade level in three different middle-grade schools. Although the mathematics teacher leaders gave priority to the implementation of the curriculum and adherence to the State Standards in these grade-level meetings, their larger goal was to support the teachers’ development of pedagogical practices that would support students’ development of mathematical understanding. To achieve this goal, the mathematics teacher leaders focused on the teachers’ understanding of the mathematical intent of pedagogical activities as they addressed implementation issues. To this end they and the teachers worked together to complete pedagogical activities and examined student work on these and similar activities.

One classroom teacher described this emphasis in the grade-level meetings as follows:

“I would call it a grade-level learning group. It’s a grade-level maths meeting where you go in and you usually pick the topic at the prior meeting [based] on where you’ll be. That’s where you go in and really look at what you are studying, how close your students are getting. We take a bit of time doing that, then we may tear apart the [mathematics] book. We may sit and look at a fraction book, we have two fraction books. We may say, this is really redundant, these are the same lessons—let’s do one and take the other out for expediency’s sake. Or we may say, you know, this is really crucial so we need more lessons. I know people are reading this and that, but look at this lesson over here and what relates to it. And this one and this one. And this one. We may bring in articles that we found were valuable. Or we may say, you know what, I have no idea what the point is, I have no idea what the form or the function is. We can sit and discuss how important that is, and how it works.”

From Cobb and McClain (2006)

Making time available within the school day for teachers to engage in such fruitful discussions is fundamental to teacher community development and hence to student outcomes. Time plays another key role in student achievement: time apportioned by the institution to the teaching and learning of mathematics significantly influences student performance (Garden et al., 1987). The more time there is available for the teaching and learning of mathematics, the higher the achievement of students. The way in which a teacher allocates time to mathematics is also a good predictor of student gains. Reporting on secondary school student achievement in the IEA Study, Garden et al. (1987) found that teachers of high-gain classes spent more time teaching mathematics. While both high- and low-gain classes devoted some of the allocated class time to routine administration, teachers of high-gain classes tended to spend less time than others maintaining order and discipline within the class.

Support from the professional community

A study undertaken by Steinberg, Empson, and Carpenter (2004) found what proved to be a powerful means of accessing student thinking: the presence of a ‘knowledgeable’ mathematics resource person in the classroom who could observe, describe, and unpack critical moments that the classroom teacher had overlooked. The presence of such a resource person also gave the teacher the confidence to try out new ideas and new pedagogical approaches to exploring student thinking. Unfortunately, resource people are not always on hand in the classroom. Another approach that Kazemi and Franke, 2004 and others (e.g., Ball & Cohen, 1999; Lin, 2002) have found conducive to teachers’ reflecting-in-action is their participation in a professional community. There is a wealth of evidence that shows how reflecting-in-action is developed through the support and encouragement received within a professional community of learners (Dufour, 2004; Higgins, Irwin, Thomas, Trinick, & Young-Loveridge, 2005; Sherin & Han, 2004). Within such a community, teachers engage in and reflect on mathematical teaching experiences (Shulman & Shulman, 2004) and “participate in a professional discourse that includes and does not avoid critique” (Wilson & Berne, 1999, p. 195). The professional learning community might involve researchers and teachers (Little, 1999) engaged in a collaborative learning endeavour. Equally, it might be defined through the interactions between the teacher and a mathematics resource teacher, numeracy facilitator, or mathematics syndicate or department (Jaworski, 1994)—all of whom can provide the teacher with observational or written material for reflecting on students’ mathematical understandings.

Expanding on the advantages of collaborative pedagogical work, Little (1999) proposes that reflection on teaching practice is most profoundly effective when the professional learning community engages in the “systematic, sustained study of student work, coupled with individual and collective efforts to figure out how that work results from the practices and choices of teaching” (p. 235). Hammerness, Darling-Hammond, and Bransford (2005) highlight the point in the following vignette.

Attending to Students’ Thinking

A second grade teacher asked students to solve $3 + 3$. One boy, whom we’ll call Jimmy, excitedly answered that the answer was 8. After asking him to rethink and still hearing the same answer, the teacher held up three fingers on each hand and asked Jimmy to count them. This time he got the answer “6.” Great,” said the teacher, “so what is $3 + 3$?” Jimmy again said “8,” leaving the teacher perplexed.

Eventually it was discovered that Jimmy was highly visual and considered “8” to be the answer because a 3 and a reversed 3 made 8 visually. Initially it took considerable time for the teacher to understand the reasons for Jimmy’s answer (which was far preferable than simply saying “you are wrong” and not helping him understand why).

Once the teacher understands Jimmy’s reasoning, it should become much easier (more efficient) for her to diagnose similar answers from others who might also have a proclivity to think visually about these kinds of problems. Adding this information to the teachers’ repertoire of familiar (routine) problems helps her become more likely to handle new sets of novel (non-routine) teaching problems that may occur subsequently.

From Hammerness et al. (2005)

Kazemi and Franke (2004) document key shifts that occurred in the course of a year as the result of monthly workgroups. Their research is informed by the precepts of Cognitively Guided Instruction (CGI), a research and professional development programme (Carpenter & Fennema, 1992; Carpenter, Fennema, Franke, Levi, & Empson, 1999). In their work with teachers, the researchers found that the most powerful change occurred when their research teachers were engaged in investigations into students’ thinking. Franke and colleagues have distinguished

five levels of sophistication in teachers' engagement with students' mathematical thinking. Their hierarchical order identifies degrees of pedagogical aptitude when it comes to negotiating between instructional approaches, teaching principles, and students' contributions:

- Level 1. The teacher does not believe that the students in his or her classroom can solve the problems unless they have been taught how. Does not provide opportunities for solving problems. Does not ask the students how they solved problems. Does not use students' mathematical thinking when making instructional decisions.
- Level 2. The teacher begins to view students as bringing mathematical knowledge to learning situations. Believes that students can solve problems without being explicitly taught a strategy. Talks about the value of a variety of solutions and expands the types of problems they use. Is inconsistent in beliefs and practices related to showing children how to solve problems. Issues other than student's thinking drive the selection of problems and activities.
- Level 3. The teacher believes it is beneficial for children to solve problems in their own ways because their own ways make more sense to them and the teacher wants the students to understand what they are doing. Provides a variety of different problems for students to solve. Provides an opportunity for the students to discuss their solutions. Listens to students talking about their thinking.
- Level 4A. The teacher believes that students' mathematical thinking should determine the evolution of the curriculum and the ways in which the teacher individually interacts with students. Provides opportunities for students to solve problems and elicits their thinking. Describes in detail individual students' mathematical thinking. Uses knowledge of thinking of students as a group to make instructional decisions.
- Level 4B. The teacher creates opportunities to build in students' mathematical thinking. Describes in detail individual students' thinking. Uses what he or she learns about individual students' mathematical thinking to drive instruction.

(Franke et al., 2001, p. 662)

In their research, Fennema and colleagues (1996) found that at the end of a four-year intervention, 19 out of 21 teachers were teaching at level 3 on this scale or higher. At the beginning of the intervention, the researchers had identified nine teachers at level 1 and seven at level 2. Using the same scale, Franke et al. (2001) showed that movement through the levels requires a supportive community who will pose questions about students' thinking for teachers to reflect upon. Later, teachers will initiate this questioning themselves and it becomes part of their own pedagogical practice. Teachers in their research who engaged in reflection-in-action were able to change their teaching in ways that were both sustainable and self-generative, and that enhanced the quality of mathematical interactions.

Whole-school partnerships

Trinick (2005) explored whole-school effects on Māori-medium mathematics teaching. Like other researchers (e.g., Bishop, Berryman, & Richardson, 2001; Hohepa, 1993) investigating Māori-medium education, he noted the key role of culture and the importance of whole-school partnerships. In his investigation into system-wide processes, Trinick studied two schools that had participated in Te Poutama Tau in 2003. Both had achieved positive mean stage gains on the Number Framework and both were highly committed to the project. Findings revealed that lead mathematics teachers played a significant role in the way in which the Numeracy Project was implemented. In 2003, Te Poutama Tau facilitators and teachers reported results from initial and final diagnostic interviews involving 1339 students. The results reveal encouraging gains in student achievement when compared with achievements recorded during the previous year's pilot programme. Teachers' enthusiasm for, and success with, the project

was influenced by their “willingness to change and a desire to improve teaching practice; a workload that allowed adequate time for teachers to focus on the programme; and good classroom management skills” (Christensen, 2004, p. 23). Teachers were less likely to engage with the intent of the reform if school-wide support was not available.

The senior staff in Trinick’s (2005) study agreed that their success with the Te Poutama Tau initiative in their Māori-medium classrooms was also due in part to their schools’ focus on cooperative learning, which aligned well with the teaching and learning philosophy of Te Poutama Tau. In collaboration with staff, leadership teams monitored student performance and developed clear goals and expectations. The two principals concerned provided release time and financial support for classroom and lead teachers as well as additional support for a number of classroom teachers. But a significant factor in the success of the programme was the principals’ hands-on involvement, which initiated a shared sense of purpose amongst staff. This was important, given the complex changes to teaching practice required by the project. Both principals attended professional development and progress meetings with the numeracy facilitators and worked directly alongside teachers. They modelled dispositions, language and actions symptomatic of the programme and both were able to generate enthusiasm and enhance teachers’ belief in their own capabilities. Pedagogical change became a collaborative problem-solving activity for the principals, classroom teachers, and lead teachers at the two schools.

From research, we know that such collaborative processes are not always at work in schools. Kensington-Miller (2004) documents difficulties in implementing professional development strategies in low-decile secondary school classrooms. Ten teachers in four schools worked with researchers involved in The Mathematics Enhancement Project. The researchers report that meetings with teachers were difficult to organise because communication by email regularly went unanswered and phone messages were often not passed on. The four schools operated according to their own individual timetables for sporting events, student examinations, ERO visits, and report writing. These different schedules made it difficult for the researchers to co-ordinate teacher meetings. Organising peer observations also proved difficult, particularly because teachers were reluctant to take up the offer of classroom release time for this purpose. Mentoring teachers was less than straightforward in the case of teachers who were sensitive to the need to justify their pedagogical approach. Although teachers reported that they valued the opportunity to access literature, some found reading an onerous task and none implemented any of the ideas in their own practice.

Balfanz, MacIver, and Byrnes, (2006) also report on the implementation of mathematics reforms in a high-poverty district. In a four-year effort to develop comprehensive and sustainable reforms in three US middle schools, the researchers report that only a moderate level of implementation was attained. Principally, the impact of the project was constrained by high levels of teacher mobility, both within schools (between grades and subjects) and between schools. As the researchers say, successful implementation is “very difficult to achieve and sustain on a broad scale when every year a substantial number of students are being taught by teachers who are new to the curriculum or the grade” (p. 58). They add that when many students in a school are behind grade level, substantial assistance needs to be found to provide effective extra help.

Clarke (2001) finds that effective teacher development hinges on school and district leadership teams providing substantive rather than merely tacit support for mathematics teaching and learning. Specifically, “the support of the school and district administration, students, parents and the broader school community” (p. 22) was a feature that enhanced learning for students in the Australian state of Victoria’s Early Numeracy Research Project (ENRP). The professional teaching community also directly influences teachers’ development. In an evaluation of the impact on teachers of the Count Me In Too early numeracy programme in New South Wales, Bobis (2004) reports that one of the real impediments to successful implementation was the expressed disapproval of the programme by other teachers. Opposition had a more marked

effect on teachers who were in the early stages of their teaching career. Where support was forthcoming, the professional community provided a climate for sharing knowledge and for sharing thinking about what counts as effective instruction.

Walshaw and Anthony (2006) provide evidence of an empowering team of teachers who developed their own subject knowledge as they worked at facilitating their students' social and cognitive development. The teachers were engaged in collaborative action in their implementation of the Numeracy Development Project (NDP). The NDP is part of the Ministry of Education's Numeracy Strategy, which has as its primary objective the raising of student achievement in numeracy through lifting teachers' professional capability. The NDP has been shown to be a particularly effective development model (Alton-Lee, 2006, p. 5). Teachers' enthusiasm for and success with the project is influenced by the support and encouragement they receive from each other and from the leadership teams in their schools. Within the supportive community of colleagues, teachers have engaged in professional discourse that has confronted the hard issues that surround new approaches to teaching.

Walshaw and Anthony interviewed principals, lead mathematics teachers, numeracy teachers and new teachers in 12 schools across the decile rankings. Based on teacher reports, Walshaw and Anthony show that system-level support, as well as collegial debate, discussion, and reflection on pedagogical practice, allows classroom teachers to improve practice in specific ways. For the teachers in their study, this support provided a lens through which they were able to interpret, adapt, and transform the intent of the reforms. Their sense making of the reforms emerged through patterns of social interaction with colleagues and the norms that shaped and structured priorities. The way in which teaching was organised, and the human, material, and financial support offered by the school, significantly influenced pedagogical effectiveness. Conversations with peers, grounded in everyday efforts to enact curriculum ideas and exemplifying a 'norm of collaboration and deliberation' (Spillane, 1999) enabled classroom teachers to grasp what the numeracy reforms meant for classroom practice. The following vignette illustrates one school's effective school-wide approach to teacher practice and student performance.

A School-wide Approach to Numeracy Teaching

The school had made significant commitment to the New Zealand Numeracy Development Project in terms of finance, time and resourcing. School-wide expectations and accountability measures at the decile 3 school, to some extent, pressed classroom teachers to attend carefully to the reform proposals. The principal noted that "twice a year we collect information from school-wide assessment and from that we set our targets. At Year 8 I want them to be at a certain stage. I looked at our data and thought, 'that's not good enough for them to be going off to high school'."

The principal explained how the school had organised extensive support for teachers in his school. Professional support was centred initially on an expert working in isolated classrooms and modelling lessons. Support didn't stop there: the expert "came back to check on teachers. We had a teacher who wasn't fulfilling the obligation and she came and worked alongside that teacher. It wasn't just 'Go off, have a day, and then go back and do it'," he explained. "There was that on-going thing."

Colleagues as well as expert facilitators were central to the school's reform efforts. Still centred on the classroom, a school-wide system was developed whereby individual teachers chose one senior teacher in the school to "come in and observe a specific aspect of the mathematics programme that the teachers decided on." Feedback was provided immediately afterwards. As the principal said, "that way we're actually continuing the professional development." Peer observation not only allowed teachers to determine if practice was consistent with the numeracy programme intent and to sort out pedagogical or content problems, it also provided teachers with the motivation to improve their practice.

The school's lead mathematics teacher explained that they took a whole-school approach to mathematics teaching: "We share across the school the different things that we are doing. And so we

do things like that to help our planning and to help our organisation.” Collegial support and feedback on practice, as well as the sharing of individual classroom efforts, created incentives for teachers to think seriously about and formalise their ideas about effective practice. From their team meeting deliberations the teachers had produced a document that synthesised their collective ideas about effective numeracy classrooms. The schedule established for them the characteristics of effective teaching and the features of an empowering classroom environment for their particular students and for their particular school.

Teachers at the school were able to exert a degree of control over their professional development. Their involvement in the core professional business of the school gave them a sense of affiliation and mission. And the support and encouragement they gave to and received from each other was recognition that they were all valued as professionals.

From Walshaw and Anthony (2006)

School–home partnerships

Effective and sustainable relationships between the home, community, and school are based on the premise of shared responsibilities and mutual investment in students’ well-being. The Program for International Student Assessment (PISA) demonstrated that parental involvement affects students’ academic attainment in no small measure. “When parents interact and communicate well with their children they can offer encouragement, demonstrate their interest in their children’s progress, and generally convey their concern for how their children are faring, both in and out of school” (OECD, 2004, p. 166). Research has highlighted the importance of providing families, irrespective of socio-economic background, with information about and support for their participation as ‘active partners’ in the production of their children’s education (Alton-Lee, 2003; McNamara et al., 2000; Sheldon & Epstein, 2005).

Biddulph et al. (2003) make particular mention of the Johns Hopkins University Centre on School, Family and Community Partnerships and its groundbreaking research on the development of effective collaboration between these groups (e.g., Epstein, Simon, & Salinas, 1997; Henderson & Berla, 1994). Epstein (1992) has argued that “students at all levels do better academic work and have more positive school attitudes, higher aspirations and other positive behaviours if they have parents who are aware, knowledgeable, encouraging and involved” (p. 1141). Research conducted by Epstein (1987) demonstrates that teachers who collaborate with parents come to understand their students better and negotiate meanings with parents and students. Parents benefit too: they develop a greater understanding of the school’s programme and an appreciation of their children’s understandings (McBride, 1991).

As the principal agents of mathematics education, teachers have an obligation to work with parents and community to develop an understanding of the relevance for future, informed citizenship of mathematics at school and in the home. Conclusive research has shown that parents tend to value teaching in the school and in the home if it makes a significant contribution to the social and emotional as well as cognitive aspects of their children’s development (de Abreu & Cline, 2005). Building school–home/community partnerships is at the heart of the New Zealand Literacy and Numeracy Strategy (Ministry of Education, 2001) and parent and community involvement in the teaching of mathematics has been an important factor in the successful implementation of the Numeracy Projects in many schools.

Parents’ beliefs and expectations for their children concerning mathematics, and parent–teen mathematical discussions have both been associated with student achievement in elementary, middle and high school mathematics (Entwisle & Alexander, 1996; Gill & Reynolds, 1999; Ho & Willms, 1996). Wang and Lin (2005) explored this association via a systematic analysis of comparative research focusing on Chinese and other specific groups of students within the US. They wanted to tease out the factors and social conditions that contribute to the well-documented mathematical success of students from China (see OECD, 2004; Hiebert, 2003).

The study highlighted the importance of family values and processes and of parents' high expectations of and support for their children's mathematical education.

Like that of Ran (2001, cited in Biddulph et al., 2003), the study undertaken by Wang and Lin found that Chinese families actively engage in informal and formal instructional approaches with their children in the home. Huntsinger et al. (2000) provide first-hand evidence of this practice. They tracked matched groups of 40 Chinese American and 40 Caucasian American students and their parents from preschool to fourth grade. On the basis of yearly test scores, parent interviews and observations of parent-child interactions, Huntsinger and colleagues report that the Caucasian students were outperformed by the Chinese American students, whose parents had allocated more time to formal and systematic instruction in the home. This is not to suggest that home instruction *caused* the higher gains but to note that home involvement, along with many other factors and conditions, does play a part in mathematical success.

There are, of course, many good reasons why parents might not be involved in, or might even show resistance to, their children's mathematics education. Parental hesitancy to participate is sometimes influenced by their own unhappy mathematical experiences and lack of confidence in their ability to help their child (Bryan, Burstein, & Bryan 2001). Gal and Stoudt (1995) have found that parents frequently do not have the requisite content knowledge, that they may not be familiar with the culture of the mathematics programme or the school's teaching approaches, and that they may not receive guidance from teachers about the ways in which they can help. Parents know what it means to *read* with children, yet they are often unclear about what it means to *do mathematics*. OECD (2004) has also signalled that single parents coping with the dual responsibilities of work and children's education often lack the time and resources to help with homework, volunteer in the classroom, or attend meetings with teachers or principals.

Sukon and Jawahir (2005) carried out a survey involving 1800 fourth grade students in 60 primary schools and found that parental education, availability of reading material in the home, family possessions, and parental support for education in the home all influence students' numeracy achievement. Results from their survey, conducted in Mauritius, showed that home-related factors explained 24.6% of variation in numeracy achievement. Parental income has also been associated with student success. OECD (2004) noted that primary-aged children whose parents have a low level of education and low incomes or are working in low-prestige occupations or are unemployed are unlikely to achieve as well as children living in socio-economically advantaged situations.

This finding needs to be set alongside evidence from a study undertaken by Lubienski (2002) in the US. This research recorded no differences in terms of support from parents of diverse SES. Parents of low SES are just as keen as other parents to encourage and support their children in their mathematics education. Similarly, a New Zealand study (Walshaw, 2006) found that, irrespective of school decile ranking, parents were seriously concerned that their children should develop the mathematical skills and know-how that would equip them to take advantage of post-school opportunities. In their BES of community and family influences on student achievement, Biddulph et al. (2003) report on McKinley's (2000) study that showed that Māori families "wanted their children to do well at school ... [and that] their children required an education to get a job and hoped that their efforts would be enough to achieve that" (p. 118). Biddulph et al. note, too, that "many Tongan parents were prepared to make considerable sacrifices to enhance their children's educational achievement because they believed in socio-economic mobility through education" (p. 118). The same was true for the parents of the secondary school Pasifika girls in Jones' (1986) study. Parents in that study viewed education as a means to access a wider range of post-school options than had been available to the parents themselves.

Pritchard (2004) surveyed the parents of a small, inner-city, New Zealand primary school and found they were positive about helping their children with mathematics and had many ideas on

how they could assist their children at home. These ideas included shopping, cooking, measuring, counting, and making comparisons. The parents also suggested discussing strategies, revisiting mathematics problems, playing board games, and making links with music. In addition, they offered practical examples for developing conceptual understandings for fractions, patterns, and relationships and assistance with modelling and rewriting mathematical questions. Tizard and Hughes (1984), as well as Savell and Anthony (2000), claim that home cultures that have a close affinity to the school culture tend to contribute to higher student achievement. Savell (2000) noted that, for families where children were experiencing mathematical difficulties, school newsletters offering information and mathematical activities for families were seen as a ‘test’ or an exercise, while the families of high achievers saw them as an opportunity for incorporating mathematics into their shared daily lives.

This is an important finding, given that parental attitudes and perceptions of mathematics have been found to influence not only student learning outcomes (Hall & Davis, 1999; Horne, 1998), but also the development of student self-efficacy (Lehrer & Shumow, 1997; Tiedemann, 2000). Savell and Anthony (2000) note: “It is possible that rather than levelling the playing field (so that high- and low-contact parents can all be informed about classroom programmes) the mathematics newsletters may have a differential effect: they may be advantaging some children and not others, thereby widening the gap between high- and low-achieving children” (p. 53).

The issue of differential effect is, however, complex and it concerns, among other things, the issue of motivation. D’Amato (1992) has provided conclusive evidence of differential motivation amongst students. D’Amato defines motivation as a relation between students and the practice established in the classroom rather than an inherent, stable trait of students. Students’ development of mathematical interests reflects one of two ways identified by D’Amato for how learning mathematics in school can have value to students. D’Amato refers to this source of value as *situational* significance, where students come to view engagement in mathematical activities as a means of gaining experiences of mastery and accomplishment, and a means of maintaining valued relationships with peers and teachers. D’Amato contrasts situational significance with what he terms *structural* significance. In this case, students come to view achievement in mathematics as a means of attaining other ends, such as entry to college and high-status careers or acceptance and approval in household and other social networks.

It has been well documented that not all students have access to a structural rationale for learning mathematics in school. Gutiérrez (2004) observes, for example, that many urban students do not see themselves going to college. They may hold activist stances, have more pressing daily concerns (e.g., housing, safety, healthcare), or may not believe that hard work and effort will be rewarded by future educational and economic opportunities. D’Amato (1992), Erickson (1992), and Mehan, Hubbard, and Villanueva (1994) all document that students’ access to a structural rationale varies as a consequence of family history, race, or ethnic history, and class and caste structures within society. Failure of schools and classroom teachers to recognise diversity and to provide all students with access to a situational rationale for learning mathematics creates a barrier to the formation of fruitful mathematical identities.

Facilitating school, family, and community partnerships

There is evidence that parental conversations with mathematics educators can encourage parents to be more positive towards mathematics and towards investigating their children’s mathematical thinking (Civil, 2002). Goldring (1991) goes as far as to say that some teachers spend more time and energy on children of high-contact parents. Sanchez and Baquedano (1993) showed that students whose parents met with teachers and were informed about ways they might help at home registered higher mathematical achievement than those whose parents did not receive information. Parents who attended training and information workshops about how to help their preschool (Starkey & Klein, 2000) or primary school children at home (Shaver & Walls, 1998) appeared to contribute more significantly to their children’s mathematics performance than did the parents who did not attend such sessions.

Sheldon and Epstein (2005) described the kinds of home practices that promote students' mathematical understanding. The following vignette illustrates how effectively implemented activities mobilise family involvement and contribute to students' success at school.

Home Activities that Contribute to Mathematical Understanding

Sheldon and Epstein explored the efforts of schools to develop relationships between family, community and school, and the effect of those involvement activities on student performance in mathematics. Drawing on longitudinal data from 18 primary and secondary schools with largely economically disadvantaged student populations, these researchers found that, after controlling for previous levels of mathematics achievement, successful implementation of targeted supporting practices in the home correlated positively to greater student mathematics gains on standardised tests. Mathematics-focused learning-at-home activities were consistently associated with improved student performance.

School, family and community partnerships were forged for 18 schools in an initiative to improve students' mathematical understanding. All of the schools reported that they (a) provided parents with information on how to contact mathematics teachers, (b) scheduled meetings with parents of students who were struggling with mathematics and (c) reported to parents on student progress and problems in mathematics. Those common practices were rated among the most effective for helping students improve their mathematics achievement.

Schools used a range of activities for varied purposes, and the activities for each type of involvement varied in effectiveness. For example, evening workshops for parents were rated more effective than daytime workshops. Teacher-designed interactive homework and mathematics materials for families and students to use at home were rated more positively for boosting students' skills than were videotapes. Certificates were issued to students to recognise mastery of specific mathematics skills by fewer schools than those who employed the more traditional lines of communication. However, the strategy was rated highly by the schools that used it.

School leaders expressed high levels of confidence that family and community involvement activities can help improve student learning and achievement in mathematics. Activities that supported mathematics learning included homework assignments that required students and parents to interact and talk about mathematics. Mathematics materials and resources made available to families to use at home also supported mathematics learning. The relationships between implementation of these activities and mathematics achievement were strong and positive, even after accounting for the influential variables of schools' prior achievement or level of schooling. The reported quality of implementation, rather than mere use of material or activity, was strongly and consistently associated with changes in levels of student mathematics achievement.

From Sheldon and Epstein (2005)

Sheldon and Epstein urge schools to plan strategically for family-involvement activities that will encourage and enable curriculum-relevant interactions between students and family members. In what ways might schools make it possible for parents to participate actively in their children's mathematics education? Epstein (1995) proposes a typology of six categories of involvement and support:

- Parenting: Helping all families establish supportive home environments for children.
- Communicating: Establishing two-way exchanges about school programmes and children's progress.
- Volunteering: Recruiting and organising parent help at school, home, or other locations.
- Learning at home: Providing information and ideas to families about how to help students with homework and other curriculum-related materials.
- Decision making: Having parents from all backgrounds serve as representatives and

leaders on school committees.

- Collaborating with the community: Identifying and integrating resources and services from the community to strengthen school programmes.

Communication

Walshaw (2006) found that, where specific school and community initiatives are lacking, New Zealand parents are less able to fulfil an effective supporting role. Peters (1998) reported that parents of new entrants felt uninformed and would welcome information about the school and about how their children were dealing with a new environment, the skills they were developing, and the content they were learning. They preferred teachers to schedule time with them for formal meetings rather than meet with them on a casual basis. Similarly, the six parents interviewed by Eyers and Young-Loveridge (2005) expressed their frustration at a lack of information about what their children were doing at school and how they, as parents, could help. “Initially, all the parents ... believed that their children were doing well at mathematics, but on reflection ... three of them felt that they had no evidence on which to base this assumption” (p. 46). “Two parents had ‘absolutely no idea’ about what their children were achieving” (p. 45), and “five out of the six parents said that they knew ‘nothing’ or ‘not a lot’” (p. 45) about the mathematics their children were doing in class.

Two-way sustainable relationships and open communication pathways are necessary if the content and teaching and learning approaches of the classroom are to be familiar to families. Mutual relationships also ensure that the needs and experiences of the home community are made relevant in the classroom. In a study undertaken by Carey (1998), communication was found to be a key lever for encouraging parents to engage with their children’s mathematical development. Carey found that the parents in one multi-ethnic classroom were keen to assist in any way possible, but many considered themselves lacking the skills to engage in parent–student dialogue about mathematics. The teacher in this urban school recognised that these parents needed greater fluency with English, so began by assisting them to improve their general English language skills and specialist mathematics vocabulary. Only then did she ask them to help their children with mathematical problems. She kept up the communication with parents, informing them of the particular skills their children were working on.

Parents want to be informed of their children’s classroom experiences. Parent–teacher communication through newsletters was listed as the best first step by parents in a study undertaken by Cattermole and Robinson (1985). Parents in that research more readily assisted in mathematical activities when they received information about the kinds of mathematical activities the class were involved in. Pamphlets that describe the Numeracy Projects and give some simple yet effective ways in which parents can encourage and support their children’s mathematics learning (Ministry of Education, 2004, 2005) have been particularly useful for many parents. Written materials such as these help parents become familiar with the pedagogical practices in their child’s classroom, as do hui or meetings with the classroom teacher and hands-on experience with the mathematics equipment being used (Eyers & Young-Loveridge, 2005).

Mothers in the study by Savell (1998), however, reported that teachers avoided them rather than communicated with them: “It is made quite clear that you are not allowed to waltz in and out of the classroom ... [The teacher] doesn’t even want you sitting on the seat outside waiting. She says that if the child sees you, that distracts them and she doesn’t want that happening” (p. 530). Peressini (1998) has underlined the issue of parent alienation from school mathematics activities and shown how school practices that deter parents from active participation contrast markedly with the rhetoric of partnership found in educational initiatives. Teachers and governance structures that conceptualise the home as subservient to the school when it comes to mathematics development make it difficult for parents to be involved, except as spectators, in their child’s education. Hornby (2000) goes as far as saying that some teachers have negative attitudes towards parents, viewing them more as problems or adversaries than collaborative partners.

Salway and Winter (2003), in their Home School Knowledge Exchange Project, undertaken in the UK, attempted to draw on the rich experiences, skills, and knowledge of the communities of four schools. A home-school book was established, in which students filed weekly information sheets describing the mathematics currently being taught and outlining related activities that parents and children could collaborate on in the home. In some schools, parents participated in class with their children, and some parents attended mathematics workshops. Board games played at home were shared with others in the classroom. Some schools utilised student profiles as a means of finding out about students' out-of-school interests and activities.

In the following vignette, Goos and Jolley (2004) document a school-generated partnership between teachers, parents, and students across year levels. In this school, it is the principal who, with a long-time interest in community partnerships, is instrumental in forming productive working partnerships across the community. She is supported in her goals by like-minded and enthusiastic people in the community.

Creating Productive Partnerships between Teachers, Parents and Students

An urban middle-class school worked at creating productive partnerships between teachers, parents and students. The school's perspective on parental involvement emphasised communicating and learning at home through strategies such as the inclusion of a "Maths Corner" in the school newsletter and the provision of individualised "take-home packs" of mathematics activities to parents who requested additional materials to use with their children. However, the most enriching initiatives for some parents were the fortnightly Maths for Parents sessions, where the Principal and teachers discussed topics identified by parents (usually involving current curriculum issues and pedagogical approaches) and participation in a mock literacy and numeracy test after which the Principal worked through the paper and discussed the kind of mathematical knowledge and skills being assessed.

At these sessions parents were concerned they were not familiar with current mathematics teaching approaches that differed from their own experiences at school. For example, some parents expressed anxiety that their children were not learning 'tables' by rote. There was a clear feeling from the parents that children should be drilled (and by implication tested) on 'tables' and given more homework. This was despite the fact that they remembered hating learning tables and said that they themselves never mastered mathematics. Teachers spent some time demonstrating to parents that there are many efficient strategies for mental and written computations. They emphasised that developing this kind of flexibility and fluency was encouraged in today's schools. They also offered suggestions for ways in which parents might meaningfully incorporate mathematical thinking into everyday activities such as sharing out food or comparing the shapes of traffic signs.

From Goos and Jolley (2004)

Decision making and collaboration: curriculum development and activities

A small number of international research projects highlight the ways in which parents and the wider community have participated in the development of mathematics curriculum (e.g., Lipka, 1994). Meaney (2001) provides evidence of a New Zealand community's involvement in curriculum development. She documents how she facilitated a Māori school as they negotiated a curriculum for their kura kaupapa Māori. Parents at the school were expected to take an active role in decision-making processes. The research was premised on the understanding that collaboration between parents and teachers over mathematics content and teaching approaches in the kura would build productive links between the home and school cultures. Sharing curriculum decision making with the school community went a significant way towards creating sustainable, meaningful collaboration.

The framework that Meaney designed served as a catalyst for deliberations on the mathematics curriculum that were "based on, but not exclusively, Māori preferred teaching and learning

methods” (Smith, 1990, p. 148). Sufficiently open to allow for modification, the framework consisted of two parts: process and issues. The process aspect included discussion, reflection, research, and decision making by consensus. The issues aspect involved mathematics in the school and the community, teaching and learning mathematics, the sequence of student learning, the language of instruction, assessment, teacher professional development, and resources.

The Māori community discussed and debated how mathematics should best be taught to their children. Using the framework as a starting point, most of the dilemmas raised were relevant to the community’s own particular circumstances and conditions. Since these dilemmas were context-driven, they did not necessarily have application for other communities. In the main, the discussions were focused not so much on the production of a new curriculum as on purposeful engagement with others and sharing understandings and beliefs with a view to ensuring that the curriculum would be relevant and meaningful for the community. As a consequence of the discussions, two teachers assumed responsibility for organising ongoing meetings with the aim of producing a mathematics curriculum policy document.

Civil (in press) reports on the work she has carried out on curriculum development for working-class, mostly Latino communities. Central to her ongoing work is a belief that community knowledge and experience are strategic resources for schooling. Civil has gathered evidence that students contribute actively to household functioning by interpreting for parents and other family members, helping care for younger siblings, assisting in the economic development of the household (through, for example, assisting with the repair of cars and appliances), and by participating in cultural customs and traditional ceremonies. Because students’ participation and learning is often at odds with their experiences at school, the teacher of a year 4 and 5 class developed a module designed to allow students to experience classroom learning that was more closely aligned to their out-of-school learning. In the following vignette, Civil records a successful implementation that preserves the purity of the students’ funds of knowledge while maintaining fidelity to the mathematics.

The Garden Module

A garden module was developed by the researchers and teachers after interviews with families revealed that parents held considerable knowledge in relation to gardening. The module became an opportunity for students to build on their gardening experiences while they were engaged in mathematically rich tasks (e.g., exploring how area varies given a fixed perimeter and discussing different ways to graph and scale the growth of an *Amaryllis*).

The teacher was well-liked and respected by the parents. She viewed the parents as intellectual resources and drew them in, in ways that were very different from typical parental involvement, allowing them to become co-constructors of the curriculum. The families contributed actual resources (such as seeds and soil) as well as offering personal expertise with gardening. The teacher learned from the parents and families and had some of them come in as experts. She engaged with them in conversations and these conversations helped her shape the curriculum.

In one activity, each group of 4 or 5 students made a garden enclosure using a 3 foot long string that each student glued to paper in their chosen shape. The challenge for them was to find the area of that shape, using tools (if they so chose) such as cubes, tiles and rulers to help them. The different shapes and their areas were displayed and the students discussed what shape would yield the largest area. Despite the artificiality of the activity the students were intrigued and curious about the problems of how to find the area of an irregular shape and how to maximise the area while keeping the perimeter constant.

Their experience with the in-class activity made its way into their gardens as many of the groups started working towards making their garden circular—although some acknowledged that in bigger gardens a circular design could be problematic in terms of access to plants.

From Civil (in press)

A large number of studies undertaken in a range of countries have demonstrated that parent involvement in classroom instructional tasks [see also Early Years chapter] leads to positive learning outcomes (e.g., Campbell & Mandel, 1990; Coleman, 1998; Rosenholtz, 1989; Sanders & Epstein, 1998). Fullan (1991) provides evidence that parent involvement (in the capacity of either a volunteer or an assistant) has an impact on instruction over and above all other forms of parental involvement. What these studies have found is that student learning benefits from parent involvement in the classroom when parents are given the opportunity to act as ‘subteachers’, providing instruction that models, reinforces, and supports the teacher’s instructional purposes. Such involvement also helps develop student attitudes and behaviours that are typically associated with productive school performance.

Kyriakides (2005) documents the successful implementation of a partnership policy in Cyprus that encouraged parents to work with their children in the classroom while teaching was taking place. The project involved parents and teachers as collaborators, with parents being assigned the role of advisor, learner, and teacher aide rather than the role of ‘house-keeper.’ The teachers drew on the life experiences of the parents and used them as instructional resources. The parents’ classroom activities were incorporated into the planning and every effort was made to clarify for the parents the intent of the lesson. Time was scheduled for parent feedback. When compared with the control group six months later, the students in the experimental school showed greater improvement in attainment in mathematics and in other subjects. The partnership fostered positive attitudes among parents and students. Parents reported that home–school links had been consolidated and that student behaviour in the home had improved.

Parent–teacher collaboration was a key factor in the success of a mathematics programme in a Māori-medium mathematics classroom. Trinick (2005) presents a compelling example of two-way communication between family/whānau and the school.

Kura A Implements a Māori-medium Mathematics Programme

Kura A is a rural full primary decile 2 school of 245 students, 94% are Māori. The school has a long history of productive home–school relationships, of strong bonds with the local community and of an identification with a vibrant iwi. The school has an even distribution of Māori-medium and English-medium classes. Many of the parents attended the school as students and expressed a desire for their children to be immersed in the reo and traditions of their own iwi. In some cases, children live with grandparents who still reside on the local area so that they can go to this particular school.

The school actively promoted community involvement in Te Poutama Tau, in order to develop a shared sense of purpose and direction. The school reported regularly to families on students’ progress and recorded significant changes in teachers’ and students’ attitudes to pangarau. A large number of whānau attended the hui held to introduce Te Poutama Tau and were fully supportive of the initiative in the school. In this process the Te Poutama Tau facilitators played a significant role. Parents/caregivers were invited to participate in the development of aspects of the teaching and learning programme, particularly on te reo and tikanga. The principal and other senior staff visited the local contributing kōhanga to provide information to staff and parents about classroom routines and offer an introduction to the Number Framework. The principal felt that this strategy assisted in preparing kōhanga graduates for entry into primary school.

From Trinick (2005)

Homework

One way in which “a culture of parents as partners within the school community” (Merttens, 1999, p. 80) can be created is through parent involvement in home activities. Indeed, often the only link between home and school is through formal homework (Anthony & Knight, 1999). Despite the fact that many parents recognise regular homework more for its character-

building potential than for its educational value (Merttens, 1999), generations of parents have been involved in sustained, collaborative partnerships with schools through their support for homework (Peressini, 1998). For example, parents have monitored and assisted their children as they have worked on mathematics homework tasks assigned by teachers for out-of-classroom completion. New Zealand year 9 students work on mathematics homework for up to one hour per day (Garden, 1996), and students aged 15 years spend slightly more time on English, mathematics, and science homework combined than do many other students (OECD, 2003).

Over a half of all students in the IEA Study (Garden, Wagemaker, & Mooney, 1987) received help with mathematics from other family members. Students in the year 9 study who demonstrated high gains between pre- and post-testing were less likely to receive regular assistance than students who achieved low gains. It has been found that as students move into secondary school and deal with more advanced mathematical content, parents are less likely to assist with homework. Meredith (2005) surveyed New Zealand secondary school students for their views on homework and found that class friends were the primary source of assistance for year 11 and 13 students. Only 13% of year 13 parents were in a position to help with homework activities. In contrast, year 9 students reported that they received help with homework—mostly from their mothers.

De Abreu and Cline (2005) report that new immigrant families in their UK study tend to rely on siblings to help with homework: “we don’t have much time for the children with regards maths ... because it’s an English medium here ... They won’t understand, so therefore the brothers and sisters help more at home. If there’s a deficiency in them, that they don’t understand then they obtain help from their brothers and sisters rather than us” (p. 714). Homework centres have proven to be another form of successful community support. Tongan families in New Zealand benefited from such an initiative. Biddulph et al. (2003) report on Fusitu’a and Coxon’s (1998) study of a successful venture set up to assist the Tongan secondary school students of one high school with their homework.

Research has found that homework that precipitates family interactions opens up communication lines between home and school. It ensures parents’ participation in their children’s mathematical schooling and helps raise student performance. However, time spent on homework is not a reliable indicator of student achievement. “How [homework] is done is more important than that it is done, because the how will make the difference between supporting children’s learning and facilitating the collaboration of their parents, or it becoming yet another element in an education system in which the benefits are differentially available, according to socio-economic class, gender or ethnicity” (Merttens, 1999, p. 79).

Homework that is purposeful and engaging, rather than homework that demands sustained periods of time, lends itself to family interactions and discussion (Crystal & Stevenson, 1991). Families that encourage and support mathematics at home are likely to advance students’ performance on mathematics at school. Sheldon and Epstein (2005) conducted research in primary, middle, and secondary schools. Students from the 18 research schools recorded higher proficiency levels when the take-home mathematics activities encouraged parent-child interactions. After accounting for prior levels of mathematics proficiency in the school, the researchers found that mathematics-focused, learning-at-home activities that required students to talk about and interact with their families were consistently and positively related to improvements in mathematical proficiency as measured by achievement tests.

Merttens (1999) and Peressini (1998) provide clear evidence that mathematics homework that relates directly to current school topics and that invites productive dialogue between parent and child plays a key role in students’ cognitive and affective development. Parents’ own experiences and knowledge act as catalysts in developing students’ understanding, particularly in problem-solving activities in realistic contexts (Ford, Follmer, & Litz, 1998). Involvement like this, made possible by the school, enables the parent to become an insider, “a participant in a discipline, rather than someone viewing the discipline entirely from the outside” (Bereiter, 1994, p. 22).

Home activities

A number of researchers have found that mathematical activities made available in the home are conducive to students' cognitive and affective development (e.g., Bragg, 2003; Cutler et al., 2003; Ernest, 1997). As we noted in chapter 4, pre-schoolers engage from an early age and in varying degrees (Young-Loveridge, 1989) with their parents in number-based activities and conversations when interacting with puzzles, toys, television, computer programs, and other games (Tudge & Doucet, 2004). Particular activities have been shown to enhance cognitive development. For example, Biddulph et al. (2003) document evidence that reveals a positive association between card and board game playing and early mathematical competence.

Street, Baker and Tomlin (2005) report a mother's comment on numeracy development as a consequence of her year 2 daughter's involvement in a game of Monopoly.

She suddenly knows \$8, five, a one and a two, or two threes and a two to make eight, because she had to pay you \$8. She soon worked it out, three and three is six and two is eight. It's very small numbers but it's amazing how quickly a lot of children her age can't cope with going round the board and the money. (p. 69)

The daughter played a variety of games, including Monopoly, Snap, and a shopping game, at home with her mother, younger sister, and older cousins. The mother had previously told researchers of her own concern about helping her daughter with mathematics: "My help with maths is going to be very limited ... I've already told my husband that's his department, maths" (p. 64). However the mother (Trisha) was able to use Monopoly to help her daughter (Anne) develop confidence with counting.

Trisha: Can you make 12 with 3s, Anne?

Anne: Oh yeh, I can. Well these 3s go like you've got 3, and 3 makes 6, and another 3 will make 9, and another 3 makes 12. With that it makes 15.

Trisha: [to researcher] I didn't know she could do 3s.

Trisha: [to Anne] Can you count in 5s?

Anne: [quickly] 5, 10, 15, 20, 25.

Trisha: Did you do that in school?

Anne: No, you taught me!

(Street, Baker, and Tomlin 2005, p. 69)

Money

Home activities and practices come into play in students' mathematical development. One particular home activity may assist in enhancing classroom learning. Abranovitch, Freedman and Pliner (1991) found that six- to ten-year-old Canadian children who were given pocket money or allowances seemed more sophisticated about money than those who were not. Students who received pocket money in the form of a weekly or monthly allowance—given either unconditionally or for some household work—revealed a more developed understanding about saving, planning, and keeping to money plans. Berti and Monaci (1998) implemented a 20-hour instructional session on banking with 25 third grade middle-class students over a two-month period. While students in a control group showed very limited understanding, most of those in the experimental group revealed a grasp of banking fundamentals. The researchers developed the programme around the teaching of new arithmetic skills and suggest that banking offers a relevant and meaningful context for number work.

In her study involving Hawaiian students from early years to year 2, Brenner (1998) explored the money activities engaged in at home, out shopping, and in classroom lessons. The investigation showed that students constructed different knowledge at different levels. More importantly, differences between school and everyday money experiences led the researcher to question

the purpose of everyday mathematics in the curriculum. Brenner says: “the goal of bringing mathematics into the classroom is not to recreate the everyday experience” (p. 153). Rather, she argues, everyday topics should be used in the classroom to harness and reconcile the powerful mathematical knowledge students learn at school.

Different home activities are practised within different cultures. Guberman (2006) investigated the association between students’ out-of-school activities and arithmetical achievements and the role that ethnicity plays in these. The study revealed that year 1–3 Latin American students engage in instrumental activities with money, while Korean students tend to engage in money activities that are expected to support their learning. The students’ achievements on arithmetic tasks matched their out-of-school activities: Latin American students were more successful with tasks that involved money while Korean American students were more successful with tasks that involved the use of denominational chips. Guberman suggests that “a culturally relevant pedagogy ... is likely a more appropriate approach, especially for teachers working in multicultural settings” (p. 145).

Games and books

Games and books provided at centre or school have been shown to enhance numeracy levels (Young-Loveridge, 2004). Young-Loveridge’s initiative examined the effectiveness of commercially published books and games on new entrants’ numeracy development.

New Entrants Using Mathematical Books and Games

In a New Zealand intervention based on tutoring pairs of students, 151 new entrants from low decile schools used number stories, rhymes and games to enhance numeracy development. Students were initially directly involved either as intervention or control students. 44% were Māori, 8% were from Pasifika or other cultures, and 48% were Pākehā. The immediate and long-term benefits of using commercial publications of books and games were closely examined for 23 students, and compared to 83 non-users. The intervention session took place each weekday for 30 minutes over a seven-week period. The intent of the programme was to develop children’s numeracy skills across ethnic groupings. In particular the programme focused on developing children’s knowledge of number word sequences, on improving their accuracy, reliability and automaticity in using the enumeration process, on enhancing their experience with forming collections of particular sizes, and on developing their knowledge of stylised (spatial) number patterns and numerals.

The games library was extensive and included commercial games such as Snakes and Ladders, Bingo, Make a Set, Tiddly Winks, and dice with dot patterns or numerals. Levels of difficulty were adjusted to ensure appropriate challenge for each child. The extensive books library included *The Very Hungry Caterpillar*, *A Weta has Six Legs*, and *Ten Silly Sheep*. The session was introduced with a number rhyme and was followed by a number story. The children were encouraged to check the quantities described in the story and predict the next quantity. A familiar board game, chosen by the children, was played and followed by participation in an unfamiliar game. The games ordinarily involved reading the numeral or recognising the number pattern displayed in the dice, and counting out the movement of counters in the game. In some games children counted small wooden sticks or the numerals on playing cards. A contrast group of students used the resources of their classroom BSM programme rather than the intervention programme games and books.

Children in the intervention group, using real authentic story books and games, made substantially greater gains than the contrast group over the intervention period. Effect sizes were as high as 1.99 immediately after the programme ceased, and most students sustained the knowledge 15 months after the intervention. Significant gains were recorded for students’ knowledge of number sequence, stylised number patterns, numeral identification, and forming and adding small collections. The specialist teacher was fully occupied in managing the session and her guidance and support was critical to the success of the intervention. Indeed a key feature of the programme was the role that this teacher played in supporting and structuring levels of challenge to advance students’ learning.

From Young-Loveridge (2004)

The support and guidance provided by the specialist teacher in Young-Loveridge's study might also be provided by parents. Of course, the instructional value of authentic games and books found in the home or sent home from school with students is dependent on parents' availability and willingness to provide guidance and support. It is also crucially dependent on parents' understanding of the mathematical purpose of the activity, the role they play, and the ways in which the activity builds on classroom work (Peters, 1994). Parents need to know the rationales for activities and approaches undertaken at school. De Abreu and Cline (2005) report on parents' reluctance to encourage specific classroom activities in the home. As one parent in the multi-ethnic study said, "When we were young, in school, we didn't use these calculator things ... if you learn by heart, you always remember" (p. 709).

Jennings et al. (1992) found that children's mathematics story books provide an effective means to teach mathematics. Working with five-year-olds from predominantly single-parent families, the researchers reported a statistically significant improvement in students' numerical understanding. Peters (1994) examined the effectiveness of a mathematics initiative with year 3 children and their parents. Once a week for 15 weeks, parents participated in classroom mathematical games with small groups of children. Peters reports on the difficulty experienced in getting parents to engage; those who did participate actively often lacked the skills to enhance student learning. These findings suggest that teachers should be made aware of the critical importance of providing skills development and support for parents. An initiative that has focused on developing parents' skills so that they can work with their children is the Family Math program. The EQUALS programmes at Berkeley University, California, established Family Math to help parents support their children's learning by engaging together in mathematical activities.

Hughes and Greenhough (1998) observed five- to seven-year-olds playing one version of a mathematical game with their teacher and another version with a parent at home. The game was based on one used in the IMPACT project (Merttens & Vass, 1993). The two versions of the game, the Snail Game and the Train Game, required simple addition skills as well as simple ideas about chance. All children played the two versions: half commenced with the Snail Game and the other half commenced with the Train Game. Most children made connections between the two activities: some were concerned with game-playing procedures and others with appropriate strategies. What the study revealed was that similar contextual and content factors inherent in an activity will provide students with sufficient information to make connections between home and school environments. As Hughes and Greenhough note, the study lends support to the classroom practice of sending home mathematics activities, with the proviso that the activities have sufficient similarity in both locations.

Ell (1998) reports on one school's approach, which allowed parents to become directly involved with their children's mathematics education. Parents of five-year-olds and their teachers at this primary school established a games library, which acted as a bridge between learning at home and learning at school. In this investigation, one group of parents joined in the games, taking turns along with the children and talking about the progress of the game in the way that a young player would. A second group of parents took on more of an instructional role, scaffolding learning and introducing more advanced concepts. Ell reports that the nature of the interactions and the extent of engagement during game playing were both critical to the level of student learning that took place.

Conclusion

In exploring pedagogical practices and their impact on student performance, the literature has highlighted key aspects that lie outside the classroom walls. Taking Wenger's (1998) characterisation of community as our basis for effective practice, we have investigated factors and conditions that complement and enhance classroom practices. We have paid close attention to the wider school, home, and community and the role that these play in students' academic and social outcomes. These relationships, and the systems and practices associated with

them, have provided us with another context for exploring how mathematical knowledge and identities are shaped.

What this exploration has revealed is that quality teaching is a joint enterprise involving mutual relationships and system-level processes that are shared by school personnel. Quality teaching in mathematics is a resource rather than blueprint, adapted by teachers within the dynamics of the spaces they share with other professionals in their schools. Research has provided clear evidence that effective pedagogy is founded on the material, systems, human and emotional support, and resourcing provided by school leaders as well as the collaborative efforts of teachers to make a difference for all learners. Teacher relationships and capability, in turn, contribute to student performance (McClain & Cobb, 2004).

Empirical evidence also reveals the beneficial effects of collaborative and sustainable relationships with teachers, family, and whānau. The home, the school, and the community comprise the major domains in which students live, learn, and grow. It is in these domains that shared interests in, and responsibilities for, children are recognised. The relationships and processes that cross over within these spheres contribute in no small way to enhanced student achievement. They are fundamental to the creation of classroom environments that affirm the identities, experiences, aspirations, and knowledge of students. Cross-overs that are fruitful involve partnerships; these involve and require two-way communication, sustained mutual collaboration, encouragement, and support in effort and activities. Initiatives by schools that result in cultures of productive partnership have the effect of creating greater opportunities for students; they become important sources for students' cognitive and affective growth.

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Appendix 1: Locating and Assembling BES Data

Using the ‘health-of-the-system’ approach, we sought to examine the various factors implicated in the creation of an effective learning community. We investigated a number of measures that fell naturally from the ‘what’, ‘why’, ‘how’, and ‘under what conditions’ questions concerning pedagogical approaches that facilitate learning for all students. The task was a considerable one, involving information management, the engagement of advisory and audit groups, and the seeking of contributions from the education community in general and the mathematics education community in particular. This level of engagement ensured that the Best Evidence Synthesis would be inclusive of views from across the community.

Our initial search strategy required us to pay attention to different contexts, different communities, and multiple ways of thinking and working. With this in mind, we undertook a literature search that crossed national and international boundaries. We used a range of search engines as well as personal networks to help us find academic journals, theses, projects, and other scholarly work with a focus on mathematics in New Zealand schools and centres, and by selected authors worldwide. We searched both print indices and electronic indices, endeavouring to make our search as broad as possible within the limits of manageability. This search took into account relevant publications from the general education literature and from the literature that relates to specialist areas of education. The search covered:

- key mathematics education literature including all major mathematics education journals (e.g., *Journal for Research in Mathematics Education*, *Educational Studies in Mathematics*, *Journal of Mathematics Teacher Education*, *For the Learning of Mathematics*, *The Journal of Mathematical Behaviour*), international conference proceedings (e.g., PME, ICME), Mathematics Research Group of Australasia publications, and international handbooks of mathematics education (e.g., Bishop et al., 2003);
- relevant New Zealand-based studies, reports, and thesis databases, supported by input from the professional community and the Ministry of Education;
- education journals (e.g., *American Educational Research Journal*, *British Educational Research Journal*, *Cognition and Instruction*, *The Elementary School Journal*, *Learning and Instruction*, etc.) and New Zealand work (e.g., SAMEpapers, SET, NZJES);
- specialist journals and projects, especially those located within the wider education field (e.g., *New Zealand Research in Early Childhood Education*; *Journal of Learning Disabilities*);
- landmark international studies including TIMSS, PISA, the UK Leverhulme projects.

This search strategy led us to a large body of literature that had something to say about facilitating mathematics learning: the total number of sourced references was just over 1100. Table 1 categorises these references by source:

Table 1: Initial Database Composition by Source

Source of data	Relative frequency (n ~1100)
Mathematics education journals	24%
Mathematics education reports, books, handbooks	16%
Mathematics education conference proceedings	15%
Theses and projects	6%
General education reports, books, handbooks	10%
General education journals, reports, and proceedings	19%
Specialist journals	10%

All entries were stored and categorised using EndNote. To assist in the initial synthesis, we distinguished between ‘research’ and ‘discussion document’, and categorised entries according to (a) our ‘diversity’ descriptors (e.g., ethnicity, gender, socioeconomic), (b) centre/school level, and (c) country-of-origin of the data.

These categories and sub-divisions served as a useful starting point for overviewing the literature and allowed us to foreground our fundamental intent to be responsive to diversity. In addition, by classifying entries according to sector and country of origin, we gave ourselves a constant reminder of the need to be inclusive of all perspectives and interests. This inclusiveness gave us a body of literature comprising diverse frameworks and eclectic methodological and analytic approaches.

Selecting the evidence

Given the complexity of the teaching and learning process, it is not an easy matter to link specific outcomes with specific pedagogical approaches. In our first pass through the literature, we noted that studies could claim that student achievement was *influenced* by pedagogical practice much more readily than they could explain *how* that practice affected student achievement. Many studies offered detailed explanations of student outcomes yet failed to draw conclusive evidence about how those outcomes related to specific teaching practices. Others provided detailed explanations of pedagogical practice yet made unsubstantiated claims about, or provided only inferential evidence for, how those practices connected with student outcomes.

Granted, we were not looking for linear explanations. As Sfard (2005) points out, the complexity of the teaching–learning relationship “precludes the possibility of identifying clear-cut cause–effect relationships” (p. 407). What we were searching for were studies that were able “to offer a developing picture of what it looks like for a teacher’s practice to cultivate student [proficiency]” (Blanton & Kaput, 2005, p. 440). We were searching for studies that offered a “detailed look at how [teachers’] actions played out in the classroom and how students were involved in this” (Blanton & Kaput, 2005, p. 435) and the sorts of mathematical proficiency that resulted. Specifically, we were seeking studies that offered not just detailed descriptions of pedagogy and outcomes but rigorous explanation for close associations between pedagogical practice and particular outcomes.

Paying attention to diverse forms of research evidence required our serious consideration of the literature relating to disparate factors from different sectors and representative of different time periods. Luke and Hogan (in press) note that what is distinctive about the approach undertaken in the New Zealand Best Evidence programme “is its willingness to consider all forms of research evidence regardless of methodological paradigms and ideological rectitude, and its concern in finding contextually effective appropriate and locally powerful examples of ‘what works’... with particular populations, in particular settings, to particular educational ends” (p. 5). We have included many different kinds of evidence that take into account human volition, programme variability, cultural diversity, and multiple perspectives. Each form of evidence, characterised by its own way of looking at the world, has led to different kinds of truth claims and different ways of investigating the truth. Our pluralist stance left us free to consider the relative strengths and weaknesses of different methodological approaches.

A fundamental challenge for this BES has been to demonstrate a basis for knowledge claims. We are absolutely aware that, like data selection, assessment of evidential claims from secondary sources is a highly perspectival activity. “Even those gazing down a microscope are as capable of finding what they expect to find, or want to find, as anyone else” (Davies, 2003). In response to this challenge, studies have been reported in a way that will make the original evidence as transparent as possible. Informed by the *Guidelines for Generating a Best Evidence Synthesis Iteration 2004*, we included studies that:

- provided a description of the context, the sample, and the data;

- offered details about the particular pedagogy and the specific outcomes;
- connected research to relevant literature and theory;
- used methods that allow investigation of the pedagogy–outcome link;
- yielded findings that illuminated what did or did not work.

The *Guidelines for Generating a Best Evidence Synthesis Iteration* allowed us to deal not only with a diversity of research topics, approaches, and methods, but also to capture differences in the context, practices, and ways of thinking of researchers. The method employed in this BES for evaluating validity required us to look at the ways different pieces of data meshed together and to determine the plausibility, coherence, and trustworthiness of the interpretation offered.

Assessments about the quality of research depend to a large extent on the nature of the knowledge claims made and the degree of explanatory coherence between those claims and the evidence provided. What we were looking for was the explanatory power of the stated pedagogy–outcome link. When assessing the nature and strength of the causal relations between pedagogical approaches and learning outcomes, we were guided by Maxwell’s (2004) categorisations of two types of explanations of causality. The first type, the *regularity view of causation*, is based on observed regularities across a number of cases. The second type, *process-oriented explanations*, sees “causality as fundamentally referring to the actual causal mechanisms and processes that are involved in particular events and situations” (p. 4). Cobb argues (2006, personal communication) that regularity explanations are particularly useful for policy makers, while process-oriented explanations are relevant to teachers, who are concerned with “the mechanism through which and the conditions under which that causal relationship holds” (Shadish, Cook, & Campbell, 2002, p. 9, cited in Maxwell, 2004, p. 4). Attending to both types of explanation of causality meant including both large-scale and single-case studies. In many instances, we have found it useful to present a single case—a learner or teacher, a classroom, or a school—in the form of a vignette to exemplify the relations between learning processes and the means by which they are supported.

Research sources in this BES report

This BES report contains approximately 660 references. Included amongst these are research reports of empirical studies, ranging from very small, single-site settings (e.g., Hunter, 2002) to large-scale longitudinal studies (e.g., Balfanz, MacIver, and Byrnes, 2006). Some of the larger studies have multiple references because they include different papers/conference proceedings/book chapters or because they embrace work authored by different researchers (e.g., the New Zealand Numeracy Development Project). In addition, the references include reports containing educational statistics and policy, theoretical writings, and commentaries and reviews on multiple research findings (e.g., van Tassel-Baska, 1997) .

The *Guidelines for Generating a Best Evidence Synthesis Iteration* point to the importance of drawing on New Zealand research in order to illuminate what works in the New Zealand context. However, despite an exhaustive search for New Zealand work, it is apparent (see chapter 8 for further discussion) that the strengths and foci of New Zealand research are not evenly distributed. In some areas—for example, early years education—there are relatively few New Zealand (or Australian) researchers working with a specific focus on mathematics education (Walshaw & Anthony, 2004). Table 2 shows the country of origin of the literature included in this BES. The numbers reflect New Zealand’s relatively new positioning within the international mathematics education research community.

Table 2: Database composition according to country

Country	Relative Frequency
New Zealand	27%
Australia	17%
United Kingdom	11%
United States	49%
Other (e.g., Africa, Netherlands, Spain)	6%

Table 3 shows the proportion of the items included in the BES (both empirical studies and commentaries) that relates to each of the different sectors. Publications relating specifically to intermediate schools have been classified with the literature on primary schools.

Table 3: Database composition according to sector

Sector	Relative Frequency (n~520)
Preschool	18%
Primary school	48%
Secondary school	21%
Teacher education	13%

Synthesising the data

Our conceptual framework, outlined in chapter 2, offered a way of structuring the data. Within the community of practice frame in and beyond the classroom, we identified the following components: (a) the organisation of activities and the associated norms of participation, (b) discourse, particularly norms of mathematical argumentation, (c) the instructional tasks, and (d) the tools and resources that learners use. We began the iterative chapter-structuring process by outlining a number of key areas. These included mathematical thinking and identities, scaffolding and co-construction, tasks, activities, assessment, educational leadership, home-school/centre links, and wider school communities. Each of these served as a starting point for our exploration and was found, in the course of the investigation, to be a useful initial category for addressing questions of equity and proficiency in relation to effective mathematics teaching.

In time, we organised these categories more cohesively into groups. What we endeavoured to do was organise multiple elements, types, and levels and varying temporal conditions in line with the critical dimensions of a community of practice and the guiding principles established in chapter 2. The content of the subsequent chapters is shaped according to these dimensions and principles. Chapter 3 focuses on all three dimensions in a search for understanding of how pedagogy influences early years outcomes. Chapters 4 and 6 explore interrelationships that are centred on the joint enterprise of developing mathematical proficiency for all learners. Chapter 5 explores the role of mathematical tasks and the part that they play in enhancing students' learning.

Reminding ourselves and readers that this BES synthesis is a product of currently accessible research, we concur with Atkinson's (2000) view that "the purpose of educational research is surely not merely to provide 'answers' to the problems of the next decade or so, but to continue to inform discussion, among practitioners, researchers and policy-makers, about the nature, purpose and content of the educational enterprise" (p. 328). Rather than offering broad answers that promise much and achieve little, it is our hope that the structure we have used will foster understanding, reflection, and action concerning the characteristics of effective pedagogical approaches in mathematics.

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Appendix 2: URLs of citations

The following 22 papers/articles/chapters/books are suggested as potentially useful sources for teachers to engage more deeply with the range of issues raised in this best evidence synthesis iteration. Readers are encouraged to source and read them. Several are available online; the others can be sourced through libraries.

The full citations are hyperlinked in the online PDF. For the convenience of those using a hard copy of the text, the URLs are listed below.

Carpenter, Thomas P ; Franke, Megan L ; Jacobs, Victoria R

A longitudinal study of invention and understanding in children's multidigit addition and subtraction

<http://nzcer.org.nz/BES.php?id=BES001>

Clarke, Barbara ; Clarke, Doug

Mathematics teaching in Grades K-2: painting a picture of challenging supportive, and effective classrooms

<http://nzcer.org.nz/BES.php?id=BES002>

Cobb, Paul ; Boufi, Ada ; McClain, Kay ; Whitenack, Jor

Reflective discourse and collective reflection

<http://nzcer.org.nz/BES.php?id=BES020>

Empson, Susan B

Low performing students and teaching fractions for understanding: An interactions analysis

<http://nzcer.org.nz/BES.php?id=BES021>

Fraivillig, Judith L ; Murphy, Laren A ; Fuson, Karen C

Advancing children's mathematical thinking in everyday mathematics classrooms

<http://nzcer.org.nz/BES.php?id=BES003>

Gifford, Sue

A new mathematics pedagogy for the early years: in search of principles for practice

<http://nzcer.org.nz/BES.php?id=BES004>

Goos, Marilyn

Learning mathematics a classroom community of inquiry

<http://nzcer.org.nz/BES.php?id=BES005>

Houssart, Jenny

Simplification and repetition of mathematical tasks: a recipe for success or failure?

<http://nzcer.org.nz/BES.php?id=BES006>

Irwin, Kathie ; Woodward, J (paper available online)

A snapshot of the discourse used in mathematics where students are mostly Pasifika (a case study in two classrooms)

<http://nzcer.org.nz/BES.php?id=BES007>

Kazemi, Elham ; Stipek, Deborah

Promoting conceptual thinking in four upper-elementary mathematics classrooms

<http://nzcer.org.nz/BES.php?id=BES008>

Latu, Viliami (paper available online)

Language factors that affect mathematics teaching and learning of Pasifika students

<http://nzcer.org.nz/BES.php?id=BES009>

O'Connor, Mary Catherine

"Can any fraction be turned into a decimal?" A case study of the mathematical group discussion

<http://nzcer.org.nz/BES.php?id=BES010>

Rietveld, Christine M.

Classroom learning experiences of mathematics by new entrant children with Down syndrome

<http://nzcer.org.nz/BES.php?id=BES011>

Savell, Jan ; Anthony, Glenda Joy

Crossing the home-school boundary in mathematics

<http://nzcer.org.nz/BES.php?id=BES049>

Sheldon, Steven B ; Epstein, Joyce L

Involvement counts: family and community partnerships and mathematics achievement

<http://nzcer.org.nz/BES.php?id=BESo12>

Smith, Margaret Schwan Smith ; Henningsen, Marjorie A

Implementing standards-based mathematics instruction: a casebook for professional development

<http://nzcer.org.nz/BES.php?id=BESo13>

Steinberg, Ruth M ; Empson, Susan B ; Carpenter, Thomas P

Inquiry into children's mathematical thinking as a means to teacher change

<http://nzcer.org.nz/BES.php?id=BESo14>

Watson, Anne ; De Geest, Els

Principled teaching for deep progress: Improving mathematical learning beyond methods and material

<http://nzcer.org.nz/BES.php?id=BESo15>

Wood, Terry (paper available online)

What does it mean to teach mathematics differently?

<http://nzcer.org.nz/BES.php?id=BESo16>

Yackel, Erna ; Cobb, Paul

Sociomathematical norms, argumentation, and autonomy in mathematics

<http://nzcer.org.nz/BES.php?id=BESo17>

Young-Loveridge, Jenny (paper available online)

Students views about mathematics learning: a case study of one school involved in Great Expectations Project

<http://nzcer.org.nz/BES.php?id=BESo18>

Zevenbergen, R

The construction of a mathematical habitus: implications of ability grouping in the middle years

<http://nzcer.org.nz/BES.php?id=BESo19>

Appendix 3: Glossary

The page reference for the first and/or most important occurrence of the term is given in brackets.

- Cognitive engagement** (p. 2). The state of being engaged in thinking
- Community of Practice** (p. 6). The complex network of relationships within which teachers teach and students learn
- Connectionist teachers** (p. 97). Teachers who consistently make connections between different aspects of mathematics
- Decile** (p. 9). In New Zealand, a 1–10 system used by the Ministry of Education to indicate the socio-economic status of the communities from which schools draw their students; low-decile schools receive a higher level of government funding
- Developmental progressions** (p. 47). Sequential learning pathways categorised as a series of steps
- Empirical evidence** (p. 24). Data that has been collected systematically for research purposes
- Equity** (p. 9). The principle based on the belief that social injustices should be redressed by allocating resources according to need, not power; in education, this may mean, amongst other things, the provision of different pedagogical approaches depending upon the needs of the learners
- Family Math** (p. 171). A US initiative designed to develop parents' skills so they can work with their children on their mathematics
- Feed the Mind** (p. 45). A media campaign funded by the New Zealand Ministry of Education and designed to support family involvement in children's learning
- High or low press for understanding** (p. 121). Differing levels of cognitive engagement demanded of students by teachers for clarification of thinking
- Kahoa** (p. 36). A festive necklace (Tongan)
- Kōhanga reo** (p. 9). Māori-medium early childhood centres
- Kura kaupapa Māori** (p. 10). Māori-medium schools (kura = school), based on a Māori philosophy of learning (see pp. 54–5)
- Manipulatives** (p. 133). Any concrete materials used by students to model mathematical relationships
- Mathematical argumentation** (p. 123). Presenting a case to support or refute a premise developed by mathematical thinking
- Mathematical identity** (p. 19). How a student sees him/herself as a learner and doer of mathematics
- Metacognition** (p. 38). The knowledge and processes involved in thinking about and regulating one's own thinking, which is essential for reflecting, self-monitoring, and planning
- Norms of participation** (p. 54). The rules, spoken or unspoken, that govern the way students behave and contribute in the classroom
- Number Framework** (p. 109). A model, structured in 8 stages, showing how students typically develop understanding of number and number operations (New Zealand, NDP)
- Number sense** (p. 98). An understanding of the relationships, patterns, and fundamental reasonableness that lie behind all mathematical operations
- Numeracy** (p. 28). The ability to use mathematics effectively, fluently, and with understanding in a wide variety of contexts
- Numeracy Development Project (NDP)** (pp. 9, 17). The central part of the New Zealand Ministry of Education's Numeracy Strategy, which has as its primary objective the raising of student achievement in numeracy through lifting teachers' professional capability
- NumPA** (p. 9). A structured, diagnostic interview used by teachers to place students on the early stages of the Number Framework (New Zealand, NDP)
- Open-ended tasks** (p. 106). Tasks that require students to engage in problem definition and formulation before beginning to think about a solution
- Pasifika students** (p. 9). Students whose families have come from Sāmoa, Tonga, the Cook Islands, Niue, Tokelau, Tuvalu, and some other, smaller Pacific nations
- Pedagogical Content Knowledge** (p. 199). In this context, knowledge about mathematics and how to teach it as well as knowledge about how to understand students' thinking about mathematics
- Pedagogy** (p. 5). The processes and actions by which teachers engage students in learning
- Poi** (p. 26). A small ball, often made of woven flax, on the end of a length of string; swung rhythmically by women when performing action songs (Māori)
- QUASAR** (p. 95). A programme developed to help urban students develop understanding of mathematical ideas through engagement with challenging mathematical tasks
- Revoicing** (p. 78). The repeating, rephrasing, or expansion of student talk in order to clarify or highlight content, extend reasoning, introduce new ideas, or move discussion in another direction
- Scaffolding** (p. 27). Temporary, structured support designed to move learners forward in their thinking

- School–home or home–school partnership** (p. 160). The deliberate nurturing of relationships between the school and the home, in the interests of better supporting student learning
- Sociocultural practices** (p. 19). Practices relating to the social and cultural aspects of participation in the classroom
- Sociocultural theory** (p. 24). The theory that learning arises out of social interaction
- Socio-economic status (SES)** (p. 30). Categorisation of individuals or communities, based on income, family background, and qualifications
- Sociomathematical norms** (pp. 61–62). Shared understandings of the processes by which students and teacher contribute to a mathematical discussion
- Tasks** (p. 94). Defined by Doyle (1983) as “products that students are expected to produce, the operations that students are expected to use to generate those products, and the resources available to students while they are generating the products”
- Te ao Māori** (p. 54). The Māori world
- Te Poutama Tau** (p. 59). The Numeracy Project (New Zealand) as developed for implementation in Māori-medium schools
- Te Whāriki** (p. 24). The New Zealand early childhood curriculum (for children aged 5 or under)
- Tukutuku panels** (p. 115). A Māori craft form consisting of ornamental lattice-work panels woven together with strips of flax into intricate designs
- Waiata** (p. 26). A song (Māori)
- Whānau** (p. 41). Extended family (Māori)
- Wharekura** (p. 9). Māori-medium secondary schools, which are based on a Māori philosophy of learning
- Zone of Proximal Development (ZPD)** (p. 36). Vygotsky (1986) describes the ZPD as the “distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers”

Abbreviations

CGI	Cognitively Guided Instruction Project	pp. 17, 105
EAL	English as an Additional Language	p. 116
EFTPOS	Electronic Funds Transfer at Point of Sale	p. 115
EMI-4s	Enhancing the Mathematics of Four-Year-Olds.	p. 28
ENRP	Early Numeracy Research Project	p. 158
EPPE	Effective Provision of Pre-school Education Project.	p. 25
ERO	Education Review Office.	p. 158
IAMP	Improving Attainment in Mathematics Project	pp. 18, 99
ICME	International Congress on Mathematics Education	p. 20
ICT	Information and Communication Technologies	p. 27
IEA	International Association for the Evaluation of Educational Achievement	p. 154
IMPACT	Increasing the Mathematical Power of All Children and Teachers	p. 73
MEP	Mathematics Enhancement Project.	p. 60
NCEA	National Certificate of Educational Achievement	pp. 10, 66
NEMP	National Education Monitoring Project	p. 9
NNS	National Numeracy Strategy	p. 17
PISA	Program for International Student Assessment.	p. 8
REPEY	Researching Effective Pedagogy in the Early Years	p. 25
RME	Realistic Mathematics Education	p. 113
TIMSS	Third International Mathematics and Science Study.	p. 14
VAMP	Values and Mathematics Project	p. 58

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