Learning the work of ambitious mathematics teaching: Pedagogies and instructional activities to support prospective and experienced teachers

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Internationally, calls for improved outcomes for diverse learners are matched by calls for improvement in the quality of teaching and learning in schools. The intellectually and socially ambitious goal of mathematical proficiency for all learners demands that we rethink teachers’ work. We need to teach students not only to do mathematics competently, but also to make sense of it and be able to use it to solve authentic problems. Lampert and colleagues (2010) define this kind of work as “ambitious mathematics teaching”. Drawing on current research projects, this paper discusses two new approaches used to support teachers learning to enact core practices of ambitious mathematics teaching in principled ways, using knowledge of subject matter and knowledge of learners appropriately. The two approaches—(i) rehearsals and coaching, and (ii) teacher inquiry using a communication and participation framework—both target the learning of high-leverage practices associated with equitable and effective pedagogies in mathematics classrooms.

INTRODUCTION

Internationally, calls for improved outcomes for diverse learners are matched by calls for improvement in the quality of teaching and learning in schools. Nowhere are the calls more vocal than in mathematics education where curriculum reforms aim to transform mathematics pedagogies to ensure that all mathematics students are active problem solvers and sense makers. Mathematics educators in New Zealand (see Anthony & Walshaw, 2007; 2009) and elsewhere (e.g., Kazemi, Franke, & Lampert, 2009; Boaler, 2011) contend that countering the widespread systemic effects of underachievement and disengagement in mathematics education requires significant reforms in classroom teaching approaches. New pedagogies must respond to the needs of an increasingly diverse student cohort alongside changing educational targets in terms of knowledge and competencies.

In many countries the goal of mathematics education has been characterised as “mathematical proficiency”—a proficiency that includes both cognitive and dispositional/participatory components (Anthony & Walshaw, 2007; Kilpatrick, Swafford, & Findell, 2001; Sullivan, 2011). Kilpatrick et al.’s (2001) seminal use of this term defines mathematical proficiency as: a way of knowing in which conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition are intertwined in mathematical practice and learning at every level for every student. Importantly, this intellectually and socially ambitious goal demands a rethinking of teachers’ work. Mathematics teachers must teach learners not only to do mathematics competently, but also to make sense of it and be able to use it to solve authentic problems. Lampert, Beasley,

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1 An Arabic translation of this Educational Practice Series document is available at www.ibe.unesco.org/en/services/publications/educational-practices.html
Ghousseini, Franke, and Kazemi (2010) define this kind of work as “ambitious mathematics teaching”.

In New Zealand, the demands for educational reforms that assist all students to learn, succeed, and develop the capabilities needed to be mathematical proficient lifelong learners, coupled with concerns about significant levels of underachievement of Pasifika and Māori students (Crooks, Smith, & Flockton, 2010), have sharpened our attention towards those professional development models that can be implemented with scale and effect. In mathematics education, effect has been variously defined as the impact on teacher knowledge (Clarke, Clarke, & Roche, 2011) or beliefs (Grootenboer, 2008) on teacher practices (Kaur, 2011; Zevenbergen, Niesche, Grootenboer, & Boaler, 2008), on student participatory practices (Hunter & Anthony, 2011a), and on student outcomes (Young-Loveridge, 2010).

**INVESTING IN TEACHER LEARNING**

Internationally, there exists a resounding agreement that investment in teacher learning is a “major engine for academic success” (Wei, Andree, & Darling-Hammond, 2009, p. 28). Investment aimed at the development of quality teachers can occur in many ways and in a range of different contexts—including initial teacher education, beginning teacher mentor and guidance programmes, school based and external professional development experiences, and further study/research contexts. However, as noted by Alton-Lee (2011) any investment designed to act as a critical lever for systemic improvement requires “careful attention to evidence about the necessary and sufficient condition under which teacher professional learning and development translates into improved outcomes for students in a local context” (p. 316).

In New Zealand, current professional development models sponsored by the Ministry of Education and associated research partner New Zealand Council of Educational Research reflect a movement from professional development to professional learning as a form of systematic inquiry that places students at the centre of the process. As Timperley (2011) states, “improvements in student learning and well-being are not a by-product of professional learning but rather its central purpose” (p. 5). Another important shift in practice concerns the role of participants within the professional learning experiences. As is seen internationally (Rickinson, Sebba, & Edwards, 2011), collaboration between researchers and teachers is becoming increasing common, and frequently promoted by funding criteria.

Mathematics teacher professional learning models in New Zealand are informed by two strands of research from the Ministry of Education Iterative Best Evidence Synthesis (BES) programme—(i) a synthesis on teacher professional learning and development (Timperley, Wilson, Barrar, & Fung, 2007, 2008) and (ii) a synthesis on effective mathematics pedagogy (Anthony & Walshaw, 2007, 2009). Combining the findings from these synthesis has led to new programmes of teacher learning (e.g., professional development for Mathematics Specialist Teachers) that include partnerships between universities (providing postgraduate specialist courses), school advisors, and teachers. Professional learning experiences within the partnership programmes are linked to valued students outcomes (in this case providing equitable and cultural responsive learning experiences for struggling students in mathematics) through the utilisation of teaching inquiry and knowledge building cycles (see Figure 1 adapted from Timperley (2008)). Another source of professional learning in New Zealand occurs within the research and development context. The Teaching and Learning Research Initiative (TLRI) administered by the New Zealand Council of Educational Research is a funded research programme designed to enhance the links between educational research and

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2 See <http://www.educationcounts.govt.nz/topics/BES>
teaching practices to improve outcomes for learners. Established by the Ministry of Education in 2003 the TLRI community now involves more than 400 researchers and practitioners. The TLRI programme aims to:

(i) develop new knowledge about teaching and learning that is useful to practice,
(ii) grow research capability and capacity in the areas of teaching and learning, and
(iii) enhance the links between research and teaching practices—and researchers and teachers.

The focus on projects that involve partnerships between researcher and practitioners enable teachers to gain expertise in systematic enquiry.

![Figure 1. Teacher inquiry and knowledge-building cycles to promote valued student outcomes](image)

**LEARNING THE WORK OF AMBITIOUS MATHEMATICS TEACHING**

Current conception of what it takes to do the work of ambitious mathematics teaching is informed by a growing body of research aimed at both understanding what teachers need *to do* to successfully accomplish ambitious mathematical goals (see Anthony & Walshaw, 2007, 2009; National Mathematics Advisory Panel, 2008) and what they need *to know* to do it (Ball, Thames, & Phelps, 2008).

In classrooms where one would observe ambitious mathematics teaching we would find that teachers have specialised knowledge for teaching mathematics, alongside skills in orchestrating instructional activities and the relational work involved in creating classroom inquiry communities (Hunter & Anthony, 2011a). The ability of teachers to notice, elicit, interpret students’ mathematical reasoning, language, and arguments and to adjust their instruction accordingly to promote learning (Ciengiz, Kline, & Grant, 2011, Franke, Turro, & Webb, 2010; Lampert, 2001; Parks, 2010) is key to such an approach. As Cobb argues (foreword of Anthony & Walshaw, 2007, p. ix) it is only when teaching “places students’ reasoning at the centre of instructional decision making” will we see a shift from a traditional...
transmission mode of instruction to a practice that is more inclusive and culturally responsive.

In addition to this agenda of proficiency that acknowledges and builds on students’ thinking, ambitious mathematics teachers promote an ethic of care, building relationships that support and expect all students’ to engage in serious intellectual activity (Anthony & Walshaw, 2009; Cohen, 2011). Supporting students’ capability as legitimate participants in the production of mathematical knowledge requires teachers to develop pedagogical practices that are both adaptive to students’ mathematical needs and understandings and maintains an expectation of high standards of achievement. To do this teacher must take seriously the participatory practices within the classroom, ensuring equitable and culturally responsive opportunities are afforded all students (Averill, 2012, Hunter & Anthony, 2011a) such that “classroom relationships become a resource for developing [students’] mathematical competencies and identities” (Anthony & Walshaw, 2009, p. 7).

Given that this vision of ambitious teaching is both complex and challenging and outside of the many teachers’ own experience as a mathematics learner (Lawrence, Anthony, & Ding, 2009) supporting teachers—both new and experienced—to teach in a ways that is “more socially and intellectually ambitious than the current norm (Lampert et al., in press) must be an urgent priority for policy makers and the education community generally.

In the remaining sections of the paper, I will draw on two Teaching and Learning Research Initiative projects (TLRI) that I am involved in to illustrate ways in which we can support teachers learning to enact core practices of ambitious mathematics teaching in principled ways, using knowledge of subject matter and knowledge of learners appropriately. The two approaches, broadly categorised as: (i) rehearsals and coaching, and (ii) teacher inquiry using a communication and participation framework, both target the learning of high-leverage practices (e.g., noticing, eliciting, and responding to students’ thinking, orchestrating class discussion, (Ball, Sleep, Boerst, & Bass, 2009; Kazemi et al., 2009)) associated with equitable and effective pedagogies in mathematics classrooms.

Case 1: Supporting novice teachers learn the work of ambitious mathematics teaching

Despite a strong evidence base characterising effective mathematics pedagogy in schools (Anthony & Walshaw, 2007, 2009) and professional learning within schools (Timperley, 2011) initial teacher education lacks a common practice that enables prospective teachers to learn to enact core practices of ambitious mathematics teaching (Anthony, Beswick, & Ell, 2012; Ball & Forzani, 2011; Tatro et al., 2012). The three year longitudinal TLRI study Learning the Work of Ambitious Mathematics Teaching3 (see http://www.tlri.org.nz/tlri-research/research-progress/post-school-sector/learning-work-ambitious-mathematics-teaching), currently in the second year, looks at new ways we can support novice teachers learn not just to be aware of ambitious teaching practices for the mathematics classroom but also to develop sufficient capability and disposition towards ambitious forms of pedagogy.

Building on the seminal work of a team of U.S. researchers in the Learning in, from and for Teaching Practice (LTP) project we are trialling “appropriate ways to develop, fine-tune and coach novice teachers’ performance over a variety of settings” (Kazemi et al., 2009, p. 12) using a pedagogical approach that is based on rehearsal of purposefully designed Instructional Activities (IAs). Our mathematics methods classes provide opportunities for

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3 The project team is comprised of mathematics researchers and educators from both Massey University (Glenda Anthony, Tim Burgess, Jodie Hunter Roberta Hunter, Peter Rawlins) and Victoria University of Wellington (Dayle Anderson, Robin Averill, Michael Drake, & Roger Harvey).
novice teachers to observe (teacher educator or video model), collectively analyse, prepare, and rehearse specified IAs.

IAs are chosen to have legitimacy in school classrooms and known to positively and equitably support student achievement outcomes. These activities (e.g., choral counting, quick images, solving word problems) serve as containers that carry principles, core practices, and knowledge into practice. Specifically, they are designed to support novice teachers to “get deep enough into authentic interactions with specific learners to practice inventing education responses while not being overwhelmed with the unpredictability and complexity of creating improvised interaction” (Lampert et al., 2010, p. 135). Some IAs (e.g., choral counting) could be used as warm up activities adaptable across a wide range of class levels, and other IAs (e.g., orchestrating discussions of rich task) could form the core of a mathematics lesson.

In-class rehearsals involve the novice teacher deliberately practising in public how to teach an IA. The learners comprise a group of the novice teacher’s peers and the teacher educator acting in the role of simulated classroom students. During and after the rehearsal process the novice teacher can pause to question, reflect or seek guidance from the teacher educator. In all phases of the process (planning for rehearsal, rehearsing with a peer group, and reflective analysis of the rehearsal) the interactions between peers and teacher educator educators support the learning community investigation of ambitious teaching by naming and analysing the interrelated teaching practices, the normative principles which shape teacher judgement in the use of the practices, and the mathematical knowledge. Also under analysis is how these are used in relationships among the teacher, students (in this situation their peers) and the mathematical content to be learned. This experience provides the novice teacher with opportunities to do what Grossman and McDonald (2008) term approximations of ambitious teaching, within a supported setting.

The introduction of the IAs required us, as teacher educators/researchers, to model the activity, and then orchestrate the public rehearsals. During the rehearsal the PT is responsible for planning and teaching the IA. The teacher educator in the role of a classroom student ‘acts back’ in ways that intentionally represent the intellectual and social range of actions that might be anticipated in an actual lesson (Lampert & Graziani, 2009). In this way the teacher educator can scaffold enactment by deliberately increasing or reducing complexity of the ongoing engagement. Additionally the teacher educator acts as a coach—stopping the rehearsal to coach the novice as he or she deliberately practices moves that are responsive to student responses. For example, the guidance may take to form of directive feedback, where the teacher educator suggests a next move or seeks suggestions as to what could be done next (e.g., “Ask students to pair share their thinking first”). Other forms of feedback include an evaluative comment that highlights to the PT and peers what was productive about the instance or what could be improved on (e.g., “Nice questioning to unpack Joe’s thinking”). The teacher educator can also lead a discussion with the wider group of novice teachers concerning the value of possible approaches. An important part of the rehearsals has been the opportunity to try out approximations of ambitious teaching within a supportive community of learners, to develop shared language about teaching practice, to develop common points of reference re activities and routines, and to develop capacity for professional reflection on teacher approaches related to learner outcomes.

Following rehearsals in the university setting, the next phase of the project is for novice teachers to practice IAs in school settings with small groups of learners. Again this approximation of teaching would be followed by collective analysis from their peers and teacher educators. The cyclic model is based on the premise that novice teachers “learn through building an iterative and interactive relationship between knowledge and principles, on one hand, and practical tools, on the other” (Lampert et al., in press). The model integrates
what Grossman, Hammerness, and McDonald (2009) identify as ‘Pedagogies of Investigation and Pedagogies of Enactment’

Case 2: Supporting experienced teachers learn the work of ambitious mathematics teaching

The second case reports on a recently completed two year TLRI project—Learning to ‘friendly argue’ in a community of mathematical inquiry (see summary Hunter & Anthony, 2011b). This project comprised a ‘formative intervention’ (Engeström & Sannino, 2010) focused on supporting equitable communication and participation patterns in mathematics classrooms. Designed as a collaboration between the two researchers and four teachers the study utilised a Communication and Participation Framework (CPF) tool (Hunter, 2008; Alton-Lee, Hunter, Pulegatoa-Diggins, & Sinnema, 2011) to support teachers’ development of inquiry-based practices within the mathematics lesson. Structured around two components—communication and participation—The CPF (see Appendix 1) details possible actions teachers could scaffold students to use in order to provide them with opportunities to learn and use mathematical practices within the collective inquiry discourse. The vertical progression of the CPF outlines “a set of collective reasoning practices matched to the communicative and performative actions teachers might require of their students in learning to and using mathematical practices” (Hunter, 2008, p. 32). Whereas the horizontal flow over three phases sketches “a possible sequence of communicative and participatory actions teachers could scaffold their students to use as they went about establishing communicates of mathematical inquiry” (p. 32). As such, the CPF tool can be used by the teachers as a flexible and adaptive tool to map out and reflect upon their development of an inquiry learning environment.

In using the CPF to support their professional learning and classroom activity the elementary level teachers in our study placed their initial focus on a set of communicative and participatory actions that required students to:

- provide a mathematical explanation using the context of the problem not just the numbers
- provide mathematical reasons/justification for their thinking rather than just description of their thinking
- as a group, explore two or more ways to explain a strategy solution and agree on the construction of one or more solution strategies that all group members understand
- analyse explanations and construct ways to revise, extend, and elaborate on sections others might not understand
- ask questions which clarify an explanation. (What do you mean by? What did you do in that bit? Can you show us what you mean by? Could you draw a picture of what you are thinking?)
- work together to check, explain and re-explain in different ways the group explanation.

The CPF also served as a reflective tool for the teachers to analyse and plan culturally responsive ways to support their students’ development of additional communicative and participatory actions. Teacher modelling and explicit expectations included requiring the students to:

- indicate agreement or disagreement with mathematical reasoning within an explanation
- justify an explanation using language like “I know $3 + 4 = 7$ because $3 + 3 = 6$ and one more is 7”
use exploratory language like, ‘so’, if’, ‘then’, ‘because’ to justify and validate an explanation
ask questions which lead to justification (e.g., ‘How do you know it works?’), ‘Can you convince us’, ‘Why would that tell you to’, ‘Why does that work like that?’, ‘Are you sure it’s...?’, ‘So what happens if...?’
use questions which lead to generalisations (e.g., ‘Does it always work?’, Can you make connections between...?’, ‘Can you see any patterns?’, ‘How is this the same or different to what we did before?’).

The gradual establishment of expectations for students to engage in mathematical argumentation discourse created opportunities for students to critically evaluate and build on the thinking of their peers alongside the development of their own mathematical understandings. Developing confidence in these ways of interacting with mathematics as a community had significant impact on students’ mathematical identity and disposition (Hunter & Anthony, 2011a).

DISCUSSION and CONCLUSION

Both of the TLRI projects highlighted in this paper are underpinned by principles and findings of the two Best Evidence Iterative Syntheses documents (Anthony & Walshaw, 2007, 2009; Timperley, 2008). In both Cases a collaborative partnership with the researchers/teacher educators and the teachers was a key design feature. In Case 1 the public rehearsal process helped the novice teacher and their peers develop a shared conceptual framework and language about mathematics teaching and learning. In Case 2 the CPF provided a way for teachers to share progress and challenges related to students’ developing communication and participation practices. Timperley (2008) argues that a professional learning community that is focused on becoming responsive to students supports teacher learners through “opportunities to process new information while helping them keep their eyes on the goal” (p. 9). This affirms other studies that found that mathematics teacher communities that value students’ thinking, treating students like sense-makers, and adapting teaching to learners, are more likely to accomplish ambitious teaching practices (Horn & Little, 2010; Strutchens, Quander, & Gutierrez, 2011).

Our experiences, related to Case 1 and Case 2, confirm that professional learning and development must do more than build teacher knowledge of mathematics and mathematics teaching. Rather than using a prescriptive approach, professional learning opportunities must promote inquiry. Within the shared professional learning communities novice and experienced teachers were supported to develop knowledge about ‘why’ and ‘how’ to promote students’ learning of mathematics. This knowledge is crucial to the development of adaptive performance required for ambitious mathematics teaching. According to Timperley (2011) adaptive experts are constantly attentive about the impact of teaching and learning routines on students’ engagement, learning, and wellbeing. In both of these Cases inquiry was supported by use of a purpose designed ‘tool’. In Case 1 the teacher educator’s ability to scaffold adaptive performance was enhanced by the use of the IA, a common tool that afforded opportunities for the novice teacher to approximate the complexity of teaching. Moreover, the repeated use of the rehearsal process provided multiple opportunities for the novice teacher to reconsider, retry, and to receive feedback about their practice. Likewise, in Case 2 the CPF tool provided a series of markers for the teachers to self or jointly evaluate their own and peers’ inquiry classroom practices.
Most importantly, in each of these cases participants—be they researchers, novice or experienced teachers—were all involved in a cycle of learning in, from, and for practice that was explicitly connected to student learning outcomes.

Acknowledgements

The author sincerely thanks co-researcher and colleagues and teachers that have contributed to the two projects (funded by the New Zealand Ministry of Education’s Teaching and Learning Research Initiative (TLRI) discussed in this paper). Special thanks is given to Dr Roberta Hunter who developed the Communications and Participation Framework in Case 2 and who is the co-principal investigator for Case 1 project.

REFERENCES


### The Communication and Participation Framework (From Hunter, 2008)

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<tr>
<th></th>
<th>Phase One</th>
<th>Phase Two</th>
<th>Phase Three</th>
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<tbody>
<tr>
<td>Making conceptual</td>
<td>Use problem context to make explanation experientially real.</td>
<td>Provide alternative ways to explain solution strategies.</td>
<td>Revise, extend, or elaborate on sections of explanations.</td>
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<td>explanations</td>
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<tr>
<td>Making explanatory</td>
<td>Indicate agreement or disagreement with an explanation.</td>
<td>Provide mathematical reasons for agreeing or disagreeing with solution strategy. Justify using other explanations.</td>
<td>Validate reasoning using own means. Resolve disagreement by discussing viability of various solution strategies.</td>
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<td>justification</td>
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<td>Making generalisations</td>
<td>Look for patterns and connections. Compare and contrast own reasoning with that used by others.</td>
<td>Make comparisons and explain the differences and similarities between solution strategies. Explain number properties, relationships.</td>
<td>Analyse and make comparisons between explanations that are different, efficient, sophisticated. Provide further examples for number patterns, number relations and number properties.</td>
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<td>Using representations</td>
<td>Discuss and use a range of representations to support explanations.</td>
<td>Describe inscriptions used, to explain and justify conceptually as actions on quantities, not manipulation of symbols.</td>
<td>Interpret inscriptions used by others and contrast with own. Translate across representations to clarify and justify reasoning.</td>
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<td>Using mathematical</td>
<td>Use mathematical words to describe actions.</td>
<td>Use correct mathematical terms. Ask questions to clarify terms and actions.</td>
<td>Use mathematical words to describe actions. Reword or re-explain mathematical terms and solution strategies. Use other examples to illustrate.</td>
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<td>language and definitions</td>
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<td>Participatory actions</td>
<td>Active listening and questioning for more information.</td>
<td>Prepare a group explanation and justification collaboratively.</td>
<td>Indicate need to question during and after explanations.</td>
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<td></td>
<td>Collaborative support and responsibility for reasoning of all group members.</td>
<td>Prepare ways to re-explain or justify the group explanation.</td>
<td>Ask a range of questions including those which draw justification and generalised models of problem situations, number patterns and properties.</td>
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<td>Discuss, interpret and reinterpret problems.</td>
<td>Provide support for group members when explaining and justifying to the large group or when responding to questions and challenges.</td>
<td>Work together collaboratively in small groups examining and exploring all group members reasoning.</td>
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<td>Agree on the construction of one solution strategy that all members can explain.</td>
<td>Use wait-time as a think-time before answering or asking questions.</td>
<td>Compare and contrast and select most proficient (that all members can understand, explain and justify).</td>
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<td>Indicate need to question during large group sharing.</td>
<td>Indicate need to question and challenge.</td>
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<td>Use questions which clarify specific sections of explanations or gain more information about an explanation.</td>
<td>Use questions which challenge an explanation mathematically and which draw justification. Ask clarifying questions if representation and inscriptions or mathematical terms are not clear.</td>
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